A discrete and explicit representation of tendons based on coupling finite elements for numerical analysis of prestressed concrete structures

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Abstract. This work presents a new strategy for representing tendons with a discrete and explicit approach to numerical analysis of prestressed concrete structures via finite element method. The prestressing tendons are represented by one-dimensional truss finite elements with the behavior described through a bilinear elastoplastic model with a prescription for initial deformations. The concrete and prestressing tendons are inititally discretized in finite elements in a totally independent way. Then, coupling finite elements are inserted to describe the interaction between tendons and concrete. A damage constitutive model is used for modeling unbonded postensioned cases. The proposed approach is validated through the numerical simulation of benchmarks available in the literature and a good agreement between them can be observed.

Keywords: Prestressed concrete, Coupling finite element, Tendon-concrete interaction, Unbonded post-tensioned, Bonded post-tensioned.

1 Introduction

In general, prestressed concrete structures are divided into three types of structural systems: pre-tensioned, unbonded post-tensioned, and post-tensioned with posterior bond. The numerical modeling of this type of structural members still represents a field of research and development due to the factors involved: the representation of the tendon; the application of prestressing; the immediate losses of prestressing; the long-term prestressing losses; and the faithful representation of the behavior of the prestressed structure subjected to external loads and carried to its rupture.

In the scenario of Finite Element Analysis, the success of modeling different prestressed concrete structure systems lies in the interface between the prestressing cable and the concrete surrounding it or its greased plastic sheath [1].

In the literature, different modeling approaches have been proposed to represent the bonding conditions at the interface. Some models consider empirical expressions and special procedures to determine deformations in unbonded tendons, as can be seen in Kang and Scordelis [2] and Van Greunen and Scordelis [3]. Link or spring elements are also usually used to connect unbound tendons and concrete [4–6]. These models allow a unified treatment of the problem in the context of finite element analysis; however, this method is suitable for small rotations and slips, since large rotations of the connecting elements lead to an imprecise balancing force due to prestressing [7]. Also, there is an approach using link elements and empirical relationships to determine the loss of prestressing along the tendon with partial bonding condition due to time-dependent effects due to load history, relaxation of prestress, and creep and shrinkage of concrete [8].

More recently, numerical strategies have considered formulating the bonding interface as a contact problem [7, 9]. Also, Huang and Kang [1] proposed a modified node-to-segment contact formulation that allows model sliding have also been proposed as an alternative to represent the interface modeling in prestressed concrete structures with different bond conditions. In this model, material and boundary nonlinearity are considered in the 2D formulation and five types of elements are employed to assemble a complete post-tensioned member: a two-node truss element for the tendon, a two-node Euler-Bernoulli beam element for the concrete host member of the beam or slab, a contact element to represent the interaction between prestressing tendon sheathings and its prestressing tendon element, embedding elements to ensure that prestressing loads and moments are properly transferred to the concrete beam element, and an anchorage element for the post-tensioning procedures and to model the anchorage zone of prestressing tendons.

The present work focuses on developing the formulation for a discrete and explicit representation of the tendons using coupling finite element (CFE) [10] for the analysis of prestressed concrete members. The proposed technique has the potential to be applied in 2D and 3D analysis, allowing to representation of any given bonding conditions between the prestressed tendons and the concrete, as perfectly bonded, partially bonded and unbonded.

2 Modeling prestressing tendons based on CFE

The proposed model is an embedded approach for modeling prestressing tendons in prestressed concrete members. The same strategy has been successfully applied recently for the numerical modeling of reinforcing bars (rebars) [11] and steel fibers [12–14] using a discrete and explicit representation scheme.

The strategy can be summarized by the following steps:

i) Definition of the geometry of the problem, including the tendon routing (Fig. 1 (a));

ii) Discretization of the tendon in finite elements (Fig. 1 (b));

iii) Discretization of the concrete bulk in finite elements (Fig. 1 (c));

iv) Mapping and insertion of Coupling Finite Elements (CFE) to describe the interaction between concrete and tendon (Fig. 1 (d)).

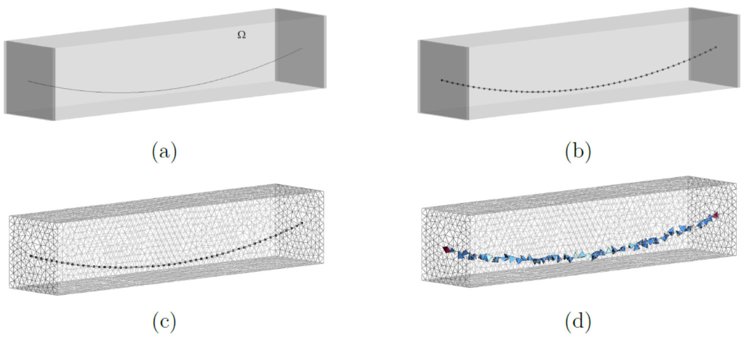

Figure 1. Creation of the numerical model for a prestressed concrete beam based on CFEs: (a) geometry of the problem; (b) discretion of the tendon in finite element; (c) discretization of the concrete domain; and (d) insertion of the coupling finite elements.

One of the main advantages of the proposed strategy is that the finite element meshes from concrete and tendons are initially independent, and the interaction between them is described after the insertion of the CFEs. Each CFE has the same nodes of the matching concrete element plus an additional node – herein designated coupling node – represented by the loose node of the rebar that belongs to the domain of the referred concrete element [10]. In addition, unlike the typical embedded approaches, where the effect of the cable is locally introduced in the stiffness matrix and internal force vector of the corresponding concrete elements, after the coupling procedure, the contribution of the cables is added directly to the global internal force vector (F^{int}) and stiffness matrix (K), i.e.:

$$\mathbf{F}^{\text{int}} = \underbrace{\mathsf{A}_{e=1}^{\text{nel}\,\Omega^C} \mathbf{F}^{\text{int}}_{e,\Omega^C}}_{\text{concrete elements}} + \underbrace{\mathsf{A}_{e=1}^{\text{nel}\,\Omega^T} \mathbf{F}^{\text{int}}_{e,\Omega^T}}_{\text{tendon elements}} + \underbrace{\mathsf{A}_{e=1}^{\text{nel}\,\Omega^{CFE}} \mathbf{F}^{\text{int}}_{e,\Omega^{CFE}}}_{\text{coupling elements}} \tag{1}$$

and

$$\mathbf{K} = \underbrace{\mathsf{A}_{e=1}^{\text{nel}\,\Omega^C} \mathbf{K}_{e,\Omega^C}}_{\text{concrete elements}} + \underbrace{\mathsf{A}_{e=1}^{\text{nel}\,\Omega^T} \mathbf{K}_{e,\Omega^T}}_{\text{tendon elements}} + \underbrace{\mathsf{A}_{e=1}^{\text{nel}\,\Omega^{CFE}} \mathbf{K}_{e,\Omega^{CFE}}}_{\text{coupling elements}} \tag{2}$$

where A stands for the finite element assembly operator. The first and second terms of Equations 1 and 2 are related to the subdomains of concrete, Ω^C, and reinforcement (tendon), Ω^T, respectively, and the third term is associated with the CFEs introduced in the model.

In this work, the concrete is discretized using constant strain triangles (CST) and four-noded tetrahedral finite elements for 2D and 3D analyses, respectively, with a linear elastic behavior. The tendons are discretized using two-noded truss finite elements with an elastoplastic behavior.

A simple and efficient way to represent prestressing is to consider it as an initial deformation in the elements of the tendon, which occurs before the application of external actions. Thus, the internal force vector of a tendon element (local coordinate system) considering the initial deformation can be written as:

$$\mathbf{f}_e^{int} = A_e L_e \mathbf{B}_e^T E_{tg}^e (\epsilon_x^e - \epsilon_0^e). \tag{3}$$

where A^e is the cross-section area, L^e is the length of the segment, $\mathbf{B}_e$ is the strain-displacement matrix, and ϵ_0^e is a constant that represents the initial deformation, which is applied with the same value for all tendon elements. Note that the relation between the global and local internal force vector is given by $\mathbf{F}_e^{int} = \mathbf{T}_e^T \mathbf{f}_e^{int}$, in which $\mathbf{T}^e$ is the transformation matrix.

According to Bitencourt Jr. et al. [10], the idea behind the CFE consists of the introduction of the concept of a reaction force due to a relative displacement, defined as the difference between the displacement of the coupling node and displacement of the correspondent material point, evaluated with the shape function of the concrete element used for its construction. Therefore, considering a linear elastic model to describe the relation between the reaction force and relative displacement, the internal force vector and stiffness matrix of a CFE can be written as:

$$\mathbf{F}_e^{int} = \mathbf{B}_e^T \mathbf{C_e} \mathbf{B_e} \mathbf{D_e} \tag{4}$$

and

$$\mathbf{K}_e = \mathbf{B}_e^T \mathbf{C_e} \mathbf{B_e}. \tag{5}$$

where $\mathbf{D_e}$ stores the displacement components of the CFE, and $\mathbf{C_e}$ is a diagonal matrix of elastic constants that assume high values ($\approx 10^9$ MPa/mm) for simulating bonded-postensioned cases. For unbonded post-tensioned simulation, the matrix of elastic constants can be written in the local coordinate system and only the penalty parameters related to directions transverse to the tendon assume high values, allowing the tendon to slide in the axial direction. More details about the non-matching mesh coupling scheme can be found in Bitencourt Jr. et al. [10].

3 Numerical Examples

In this section, the proposed formulation is assessed through the numerical simulations of two examples reported in the literature.

3.1 2D Simple beam with different bond conditions

To demonstrate the ability of the technique to represent the prestressed beam with different bonding conditions, the simple beam proposed by Lin and Burns [15] and analyzed by El-Mezaini and Çitipitioğlu [4] is considered. The beam geometry and the parabolic tendon layout are presented in Fig. 2. The structure was modeled in 2D for a plane stress analysis. The stresses along the tendon were obtained due to an imposed load of 10 kN/m under different bond conditions: no bond; perfect bond; partially bonded with equivalent spring constant k_s=10^5 kN/m/m; and perfectly bonded except in the middle third, which is left unbonded.

For the numerical analysis are adopted the following parameters for the concrete: modulus of elasticity $E_c = 28000$ MPa and Poisson ratio of ν=0.25, and for the prestressing tendon is assumed a modulus of elasticity $E_p = 224000$ MPa.

In the bond condition in which only the middle third has no bond and the rest of the tendon has a perfect bond, the numerical model was made using coupling elements with rigid (bonded) and non-rigid (unbonded) scheme for each tendon region, as illustrated in Fig. 3.

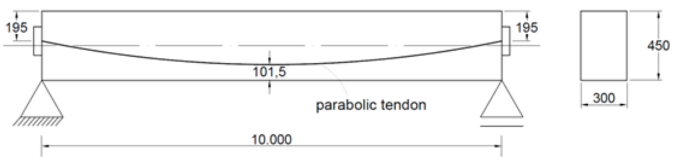

Figure 2. Prestressed beam with a parabolic tendon and different bonding conditions (dimensions in mm).

Figure 3. Prestressed beam with distinct coupling scheme along the length of the tendon.

The numerical results of the stresses in the tendon are shown in Fig. 4, and the results are also compared with the numerical results obtained by El-Mezaini and Çitipitioğlu [4]. There is a good agreement of the results, with the differences that exist between the two results caused by the eccentricity of the cable in the support section (above the center of gravity of the section) different from the reference numerical model analyzed.

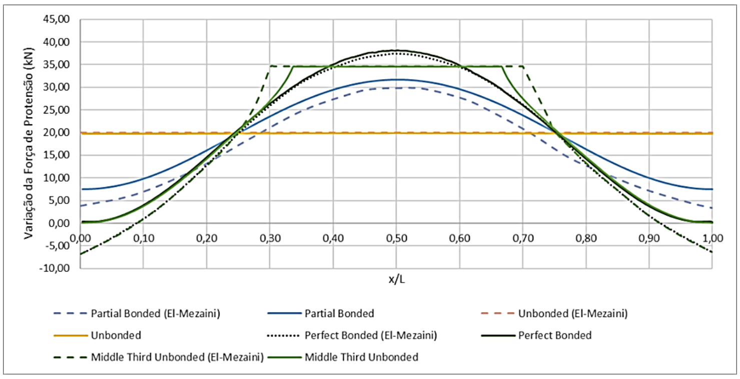

Figure 4. Comparison between numerical results along the length of the tendon.

3.2 Two-way prestressed slab

A two-way single-panel prestressed slab 6,10 m × 7,32 m analyzed by Nawy [16] is considered. It is simply supported on all sides, with negligible rotational restraint at these boundaries. The slab is 152 mm thick and has to carry a superimposed service dead load of 0,72 kN/m^2, its self-weight and a service live load of 3,59 kN/m^2.

The slab is designed as a post-tensioned unbonded prestressed two-way floor using 12,7 mm diameter strands (A_p = 99 mm^2), so that no deflection is allowed under full dead load for an effective prestressing force and stress in each tendon equals to P_e = 108,2 kN and σ_{pe}=1096 MPa, respectively. The spacing between tendons in the short direction is s$_S$=432 mm and in the long direction is s$_L$=515 mm. The modulus of elasticity assumed for concrete and tendons were: E$_c$ = 27800 MPa and E$_p$ = 200000 MPa, respectively.

The 3D numerical model created can be seen in Fig. 5. The numerical strand stresses are in very good agreement compared to the analytical expressions, which show the stress for each tendon with values close to 1096 MPa for full dead and service loads acting on the slab (Fig. 5).

The results for the concrete stresses at midspan section obtained for the numerical model shows good correlation with analytical expressions (Table 1).

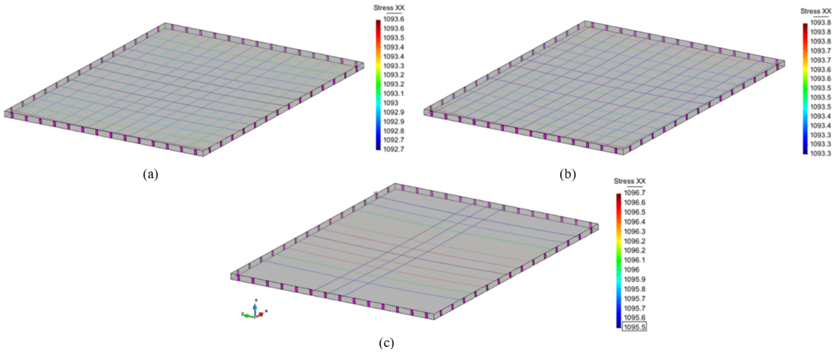

Figure 5. Stress variation in each strand for load steps: (a) prestressing and self-weight loads acting on the slab, (b) additional superimposed dead loads, and (c) additional service live load.

Table 1. Concrete stresses at midspan section.

Direction	Fiber location	Stress at Midspan section [MPa]	
		Analytical Expression	Numerical Model
Short	Top	-3.68	-3.21
σ_{yy}	Bottom	+0.39	+0.37
Long	Top	-3.05	-3.04
σ_{xx}	Bottom	+0.29	-0.25

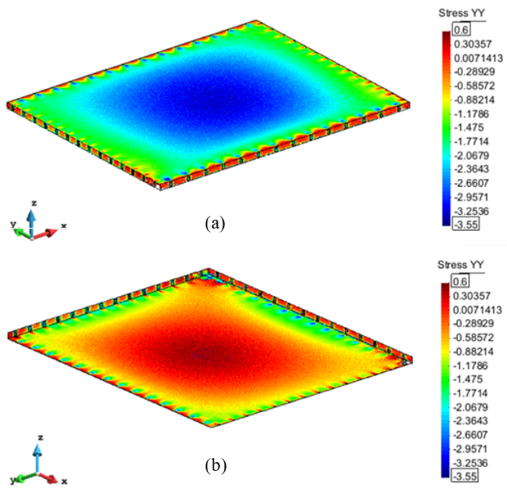

Figure 6. Normal stress on concrete (σ_{yy}) for full dead and service loads at short direction: (a) top surface, and (b) bottom surface

4 Conclusions

This paper presents an alternative numerical formulation for modeling prestressing tendons in prestressed concrete members. The main novelty lies in the discrete representation of the tendon with different bond conditions based on coupling finite elements. The formulation is generalized for the cases of perfect bond–rigid coupling scheme – along the bonded tendon for pretensioned structures and for the anchoring zone of the tendons of the post-tensioned structures; and loss of bond – non-rigid coupling scheme – along the unbonded tendon for post-tensioned structures. The prestressing is simulated in a simple and efficient way, adopting an initial deformation in the elements of the tendon.

The proposed numerical approach was presented and validated using two case studies from the literature. Different bond conditions to simulate pre-tensioned and post-tensioned structures were adopted. A simple beam with different bond conditions and a two-way prestressed slab with multiple tendons were considered. The results obtained are in good agreement with those numerical and analytical models available in the literature.

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