Nonlinear dynamic analysis of guyed mast under random aerodynamic loading

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Abstract. In this research, a guyed mast with non-linear behavior under random aerodynamic loading is modeled. It is a slender lattice metal structure, articulated at the base, stabilized by cables. An equivalent mass of the system concentrated at its upper end is admitted, whose displacements are the generalized coordinates of the system. The model admits large displacements, leading to the need to consider geometric nonlinearity, that is, dynamic equilibrium is always imposed on the displaced position. The non-linear equations of motion are derived via direct dynamic equilibrium. The numerical algorithm for simulating the response is based on step-by-step time integration using Runge-Kutta's Method. The random aerodynamic loading is simulated by a Monte Carlo-type procedure with superposition of harmonic series whose amplitudes are obtained from Power Spectrum Density Functions with randomly chosen phases. For application to the model under study, an equivalent point load is applied to the mass at the top of the mast. This procedure is inspired by the well-known Synthetic Wind Method.

Keywords: guyed mast, nonlinear dynamics, random vibrations, synthetic wind.

1 Introduction

In this paper we present results of an ongoing research on the nonlinear dynamics of guyed masts subjected to wind forces of random nature. Here, a simple two degrees of freedom plane model is considered, as shown in Fig. 1, comprising a straight, prismatic, massless mast stabilized by two tensioned straight inclined cables. In section 2 we derive the two nonlinear equations of motion via direct dynamic equilibrium. The two adopted generalized coordinates are the horizontal and vertical (large) displacements of the point mass considered at the top of the mast.

The random aerodynamic point force applied at this mass is computed via the so called "Synthetic Wind Method", as presented by Franco [5-8]. In this method, detailed in Section 3, generates load time histories by superposition of several harmonic components whose amplitudes and frequencies are given by a chosen PSD with randomly set phase angles.

The time response of this model is obtained by step-by-step time integration via Runge-Kutta's method, as implemented in Matlab software. Graphical results are displayed in Section 4.

2 The mathematical model

Figure 1 displays our plane Mathematical Model. A h tall straight, massless mast with EA_1 axial stiffness and a M point mass at its top, whose horizontal and vertical large u and v displacements are our two generalized coordinates. It is stabilized by two straight cables under T_i initial traction forces, with EA_2 and EA_3 axial stiffness, whose horizontal projections are d_1 and d_2. The point aerodynamic point force at the top mass is $W(t)$

Figure 2 displays the computation of the L'_i deformed length of the originally L_i long three members of our model.

$$L'_i = \sqrt{(d_i + u)^2 + (h + v)^2} \tag{1}$$

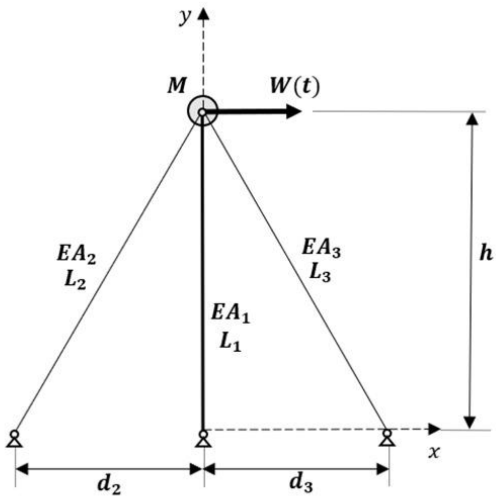

Figure 1. The guyed mast simplified model

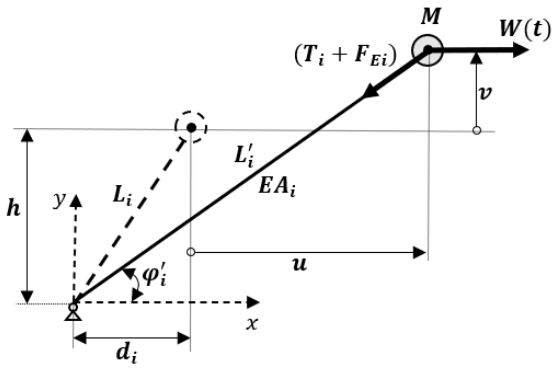

Figure 2. Computation of deformed length of the members

The member's change in length is

$$\Delta L_i = L'_i - L_i \tag{2}$$

and the corresponding axial force is

$$F_{Ei} = K_i \, \Delta L_i \qquad \text{where} \quad K_i = EA_i/L_i \tag{3}$$

By direct dynamic equilibrium, according to Newton's second law, we derive the two nonlinear Equations of Motion, adopting linear viscous damping:

$$M\ddot{u} = W(t) - C_1\dot{u} - \sum_{i=1}^{3}(T_i + F_{Ei})\cos\varphi'_i \quad \text{where} \quad \cos\varphi'_i = (d_i + u)/L'_i \tag{4}$$

$$M\ddot{v} = -C_2\dot{v} - \sum_{i=1}^{3}(T_i + F_{Ei})\,\text{sen}\,\varphi'_i \qquad \text{where} \quad \text{sen}\,\varphi'_i = (h + v)/L'_i \tag{5}$$

As we will adopt Runge-Kutta's step-by-step time integration scheme, as implemented in Matlab, it is necessary to transform these two second order equations into four first order ones, via a change of variables:

$$x(1) = u, \quad x(2) = \dot{u}, \quad x(3) = v \quad \text{e} \quad x(4) = \dot{v} \tag{6}$$

Finally, the ODE system is:

$$\dot{x}(1) = x(2) \tag{7}$$

$$\dot{x}(2) = [\, W(t) - C_1 x(2) - \sum_{i=1}^{3}(T_i + F_{Ei})\cos\varphi'_i \,]/M, \qquad \text{where} \quad \cos\varphi'_i = \big(d_i + x(1)\big)/L'_i \tag{8}$$

and

$$\dot{x}(3) = x(4) \tag{9}$$

$$\dot{x}(4) = [-C_2 x(4) - \textstyle\sum_{i=1}^{3}(T_i + F_{Ei})\, sen\, \varphi'_i\,]/M, \qquad \text{where} \quad sen\, \varphi'_i = \big(h + x(3)\big)/L'_i \tag{10}$$

with

$$L_i = \sqrt{\big(d_i + x(1)\big)^2 + \big(h + x(3)\big)^2} \tag{11}$$

3 The Synthetic Wind Method

In this paper, we model the random aerodynamic forces acting on our structure via the so-called Synthetic Wind Method, a Monte Carlo type procedure, due to Franco [5-8]. The adopted wind parameters are those given by the Brazilian Standard for a hypothetic location, leading to “static” and “fluctuating” pressure parcels. For the later, a wind velocity PSD, $S_r(n)$, based on the Canadian Standard, is used to generate dynamic pressure amplitudes related to the frequencies of 11 harmonic time histories to be summed up for a total loading history, with randomly chosen phase angles.

$$S_r(n) \approx 4\,\frac{{X_1}^2}{\left(1+{X_1}^2\right)^{4/3}} \qquad \text{where} \quad X_1 = \frac{1220\, n}{U_0} \tag{12}$$

If a statistical evaluation is desired, this process will be repeated a large number of times, resetting phase angles for each summed up loading history generated.

Table 1 presents these frequencies and relative pressure amplitudes, c_k, for each harmonic time history for numerical data: basic wind velocity 45 m/s, “static” pressure 0.49 KN/m², total “fluctuating” pressure 0.73 KN/m². For a particular case, randomly chosen phase angles are also displayed for each harmonic signal.

$$c_k = \sqrt{\frac{S_r(n)}{6.125\ \sum_{k=1}^{m} S_r(n)}} \tag{13}$$

$$p'(t) \cong \sum_{k=1}^{m} \underbrace{c_k\, p_f}_{p_k} \cos[\omega_k\, t - \theta_k] \tag{14}$$

Table 1. Frequencies, PSD values, pressure amplitude and phase angle for each harmonic time history

Harmonic k	Frequency n (Hz)	Frequency ω_k (rad/s)	$S_r(n)$	c_k x 10^{-2}	p_k (KN/m²)	Phase angle θ_k (rad)
1	35.52	223.179	0.031	2.6	0.019	3.9309
(r_2) 2	16.98	106.703	0.050	3.4	0.025	4.9023
3	8.12	51.0148	0.082	5.7*	0.042	0.5097
(r_1) 4	3.88	24.3903	0.135	2.8*	0.020	5.8395
5	1.86	11.6610	0.220	8.4*	0.062	4.8739
6	0.89	5.57517	0.360	9.0	0.066	3.0586
7	0.42	2.66550	0.587	11.5	0.084	2.7386
8	0.20	1.27438	0.946	14.6	0.107	2.8072
9	0.10	0.60929	1.456	18.1	0.133	1.9249
10	0.05	0.29130	1.879	20.6	0.151	3.1951
(m) 11	0.02	0.13927	1.500	18.4	0.135	3.2093

$c^*_{(r_1)} = 0.5c_{(r_1)}$; $c^*_{(r_1)\pm 1} = c_{(r_1)\pm 1} \pm 0.25c_{(r_1)}$

Figure 3 displays the first 30 seconds of obtained wind force time history due to pressures in Table 1 applied to a 10 m² area in the top of the structure, and time step 0.006 s.

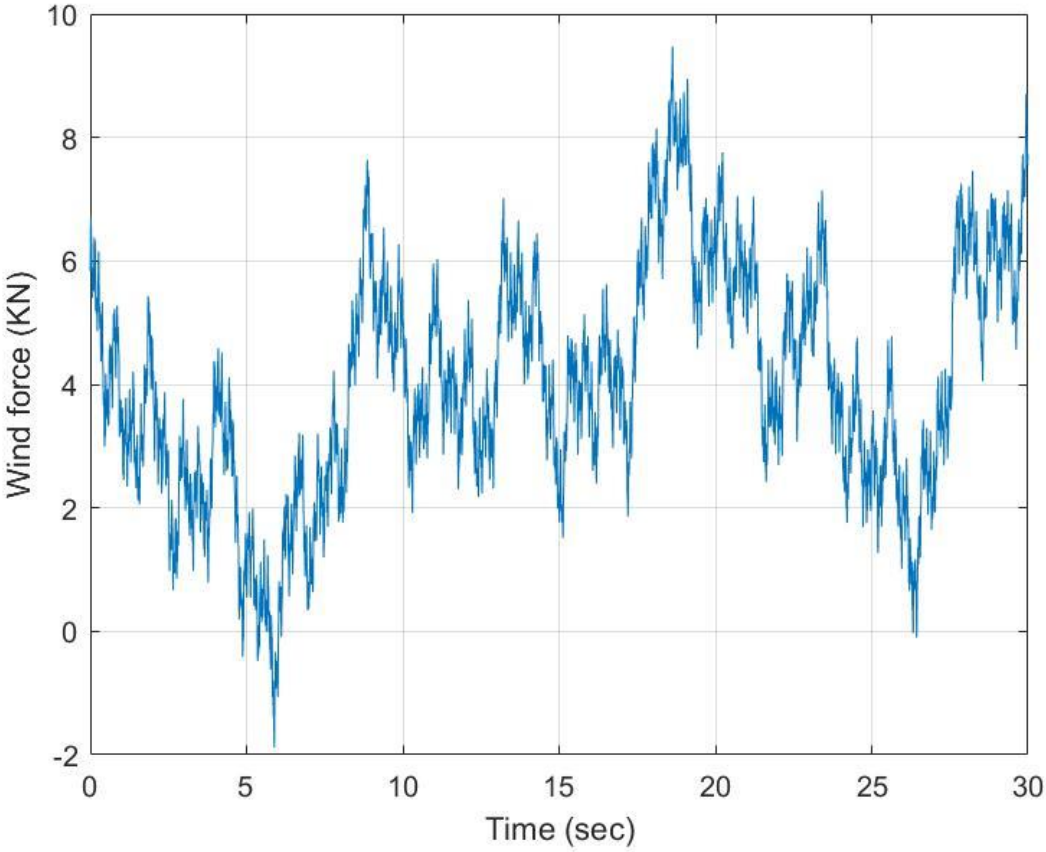

Figure 3. Wind force time history

4 Numerical Results

Adopted numerical values for our structural parameters are: h = 40m, d_1= d_2 = 30m, A_1 = 40 cm², A_2 = A_3 = 4 cm², E = 210 GPa, C_1 = 5,000 Ns/m, C_2 = 20,000 Ns/m, T_2 = T_3 = 20 KN. For these values, this structure has linearized frequencies 3.88 Hz and 16.98 Hz.

Figures 4 and 5 display the evolution for the first 30 seconds of horizontal displacements u and vertical displacements v under the random wind force computed in the previous section. Integration time step is 0.006 s.

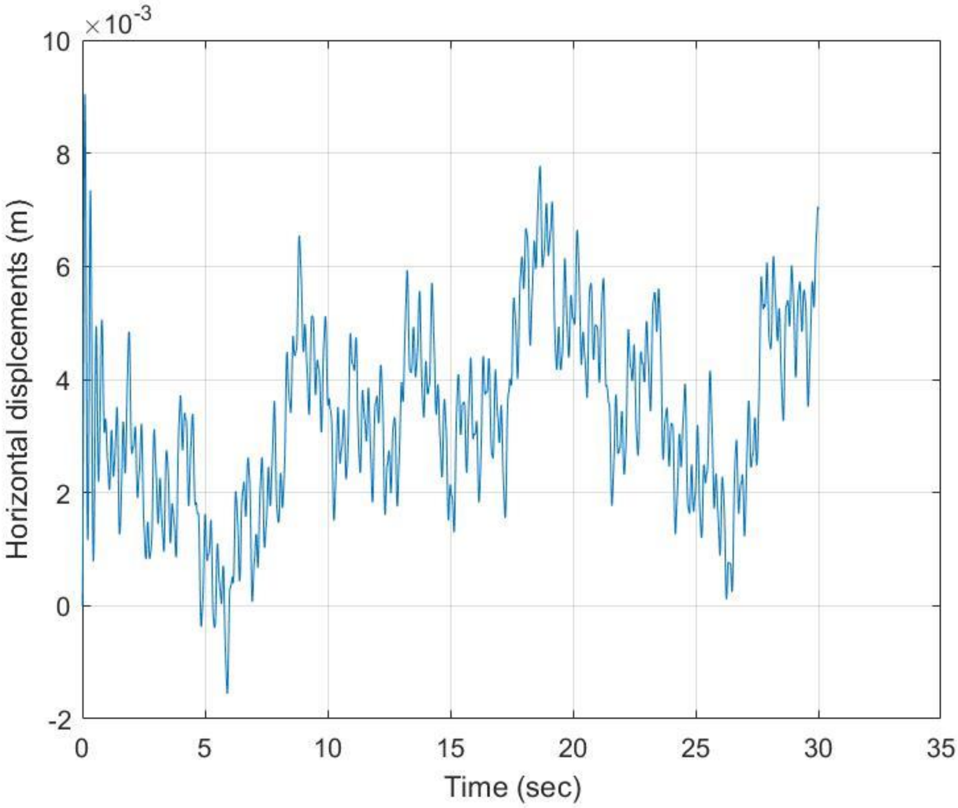

Figure 4. Horizontal displacements

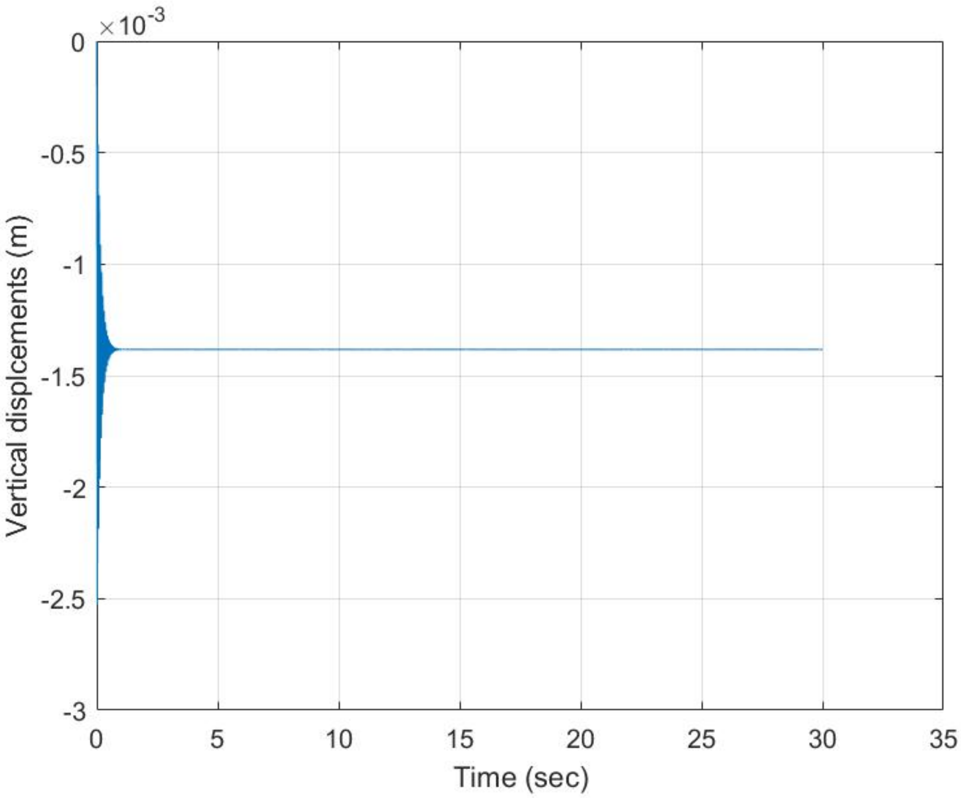

Figure 5. Vertical displacements

5 Conclusions

We presented a simple plane nonlinear dynamic mathematical model of a guyed mast under random aerodynamic wind forces modeled via the Synthetic Wind Method.

It is clear that, for normal engineering structural parameters, the vertical displacements of Fig. 5 are of a very small order compared to the horizontal displacements of Fig. 4. Thus, the used nonlinear formulation is not necessary and may be replaced by usual linear Matrix Structural Analysis.

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