Satellite ACS Design during Orbit Injection using the SDRE Method

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Abstract. The performance of the Satellite Attitude Control System (ACS) during the orbit injection phase is of fundamental importance for the success of the mission. In this phase the satellite leaves the launcher with high angular velocity and then the ACS needs to manoeuvre the satellite to its normal mode of operation, which is characterized by an attitude of small angles. One way to achieve this transition between these two phases is using gas jets followed by reaction wheels. In this paper one investigates and develops by simulation the ACS algorithm to minimize space mission costs by reducing the number of errors transmitted to laboratory prototypes project. The high angular velocities of the satellite in the injection phase makes its dynamics highly nonlinear introducing some level of perturbation into the system. As a result, application of linear control technique cannot be able to design the ACS with adequate performance to reach the required level of appointment. To mitigate this problem, one will use the State-Dependent Riccati Equation (SDRE) method which can deal with nonlinear system. The SDRE controller design algorithm is based on gas jets and reaction wheel torques to perform large angle manoeuvre to reduce the high angular velocities to attitude with small angles. The criterion for the transition between the two operating modes is based on the decrease of the system energy. This investigation serves to validate the numerical simulator model and to verify the functionality of the control algorithm designed by the SDRE method. It is intended to use in the next phase of this research the Federal University of ABC (UFABC) 3D simulator which supplies the conditions for implementing and testing the SDRE algorithm in terms of hardware and software.

Keywords: satellite control, nonlinear sdre method

1 Introduction

The design of a satellite Attitude Control System (ACS), that involves plant uncertainties [1] and large angle maneuvers followed by stringent pointing control, may require new nonlinear attitude control techniques to have adequate stability, good performance and robustness. Experimental ACS design using nonlinear control techniques through prototypes is one way to increase confidence in the control algorithm. Experimental design has the important advantage of representing the satellite dynamics in a laboratory setting, from which it is possible to accomplish different simulations to evaluate the satellite ACS [2]. However, the drawback of experimental testing is the difficulty of reproducing zero gravity and torque free space conditions. A Multi-objective approach [3] has been used to design a satellite controller with real codification. An investigated through experimental procedure has been used by Conti and Souza in [4] for simulator inertia parameters identification. An algorithm based on the least squares method to identify mass parameters of a satellite during attitude maneuvers has been developed by Lee and Wertz in [5]; a method with the same objectives but based on Kaman filter theory has been investigated by Souza in [6]. The H-infinity control technique was used in [7] to design robust control laws for a rigid-flexible satellite. The SDRE method can handle with nonlinear dynamics which is transformed to time-invariant, linear-like structure making use of the state-dependent coefficients (SDC) parametrization [8]. Infinite-horizon Linear Quadratic Regulator (LQR) technique is then applied to the linear-like structure with the coefficient matrices being evaluated at the current operational point in the state space. The process is repeated in the next sampling periods with a control law which is function of the state. The SDRE method was applied by Souza in [8] for controlling a nonlinear satellite model with six-degrees of freedom without incorporating the SDRE filter as a state observer. In this paper the SDRE technique [9] along with the associated Kalman filter [10] is applied to design a nonlinear controller for a nonlinear simulator plant where the unstructured uncertainties of the system are represented by process and measurement noise. As a result, the satellite attitude control algorithm design using the SDRE technique and SDRE filter can deal with large angle maneuvers and plant uncertainties. The control strategy is based on reaction wheel and gas jets as actuators which allow the design of two control algorithms related to the transition from high angular velocity mode to the normal mode of operation with stringent pointing using an optimal switching control algorithm based on minimum system energy. Several simulations have proven the computationally feasibility for real time execution of the SDRE control algorithm to be used in the satellite onboard computer [11].

2 SDRE Control Methodology

The LQR approach is well known, and its theory has been extended for the synthesis of nonlinear control laws for nonlinear systems [8, 12]. Several methodologies exist for the control design and synthesis of these highly nonlinear systems; these techniques include many linear design methodologies [13] such as Jacobian linearization and feedback linearization used in conjunction with gain scheduling [14]. Nonlinear design techniques have also been proposed including dynamic inversion and sliding mode control [15], recursive back stepping and adaptive control [16]. Compared to multi-objective optimization nonlinear control methods [3] the SDRE method has the advantage of avoiding intensive interaction calculations, resulting in simpler control algorithms that are more appropriate for implementation on a satellite´s onboard computer. The Nonlinear Regulator problem [18] for a system represented in the SDRE form with infinite horizon, can be formulated by minimizing the cost functional given by

$$J(x_0,u) = \frac{1}{2}\int_{t_0}^{\infty} (x^T Q(x) x + u^T R(x) u\,)dt \tag{1}$$

with the state x and control u subject to the nonlinear system constraints given by

$$\begin{aligned} \dot{x} &= f(x) + B(x)u \\ y &= C(x)x \\ x(0) &= x_0 \end{aligned} \tag{2}$$

where B and C are the system input and output matrices, and y is the output vector of the system with initial conditions x (0). Q(x) and R(x) are the weight matrix semi defined positive and defined positive.

Applying a direct parameterization to transform the nonlinear system into State Dependent Coefficients (SDC) representation, the dynamic equations of the system with control can be write in the form.

$$\dot{x} = A(x)x + B(x)u \tag{3}$$

where A(x) is the state matrix and B(x) is the input matrix both can be function of the state x. By and large $A(x)$ is not unique, in fact there are an infinite number of parameterizations for SDC representation. An important factor for this choice is not violating the controllability of the system. The feedback control law is given by

$$u = -R^{-1}(x)B^T(x)P(x)x \tag{4}$$

where P(x) solution of the state dependent algebraic Riccati equation (SDARE) given by

$$P(x)A(x) + A^T(x)P(x) - P(x)B(x)R^{-1}(x)B^T(x)P(x) + Q(x) = 0 \tag{5}$$

Then the SDRE nonlinear regulator produces a closed loop solution that is locally asymptotically stable. An important factor of the SDRE method is that it does not cancel the benefits of designing the nonlinear control law considering the nonlinearities of the dynamic system.

3 Simulator Model

The computational model developed in this work is based on the real model with reaction wheels and gas jet that is being developed at INPE, which is shown in the figure 1. The INPE 3-D simulator which has a disk-shaped platform, supported on a plane with a spherical air bearing. Considering that the INPE 3-D simulator is not complete build, one assumes that there are three reaction wheel and the gas jets configuration set capable to perform maneuver around the three axes and that there are three angular velocities sensor, like gyros. Apart from the difficulty of reproducing zero gravity and torque free condition, modeling a 3-D simulator, basically, follows the same step of modeling a rigid satellite with rotation in three axes free in space.

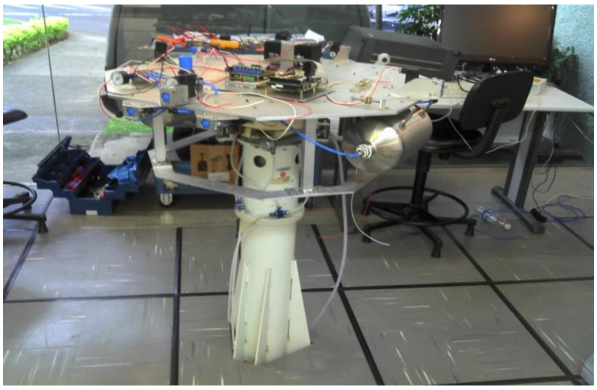

Fig. 1- INPE 3-D simulator with gas jets and three reaction wheels.

The orientation of the platform is given by the body reference system F_b with respect to inertial reference system F_I considering the principal axes of inertia and using the Euler angles (θ_1, θ_2, θ_3) in the sequence 3-2-1, to guarantee that there is no singularity in the simulator attitude rotation. The equations of motions are obtained using Euler´s angular moment theorem given by

$$\dot{\vec{h}} = \vec{g} \tag{6}$$

where $\vec{g}$ and $\vec{h}$ are the torque and the angular moment of the system, which is given by

$$\vec{h} = I\vec{\omega} + I_w\left(\vec{\Omega} + \vec{\omega}\right) \tag{7}$$

where I = diag (I_{11}, I_{22}, I_{33}) is the system matrix inertia moment, $\vec{\omega}$ is the angular velocity of the platform, $\vec{I}_w$ = diag (I_{w1}, I_{w2}, I_{w3}) is the reaction wheel matrix inertia moment and Ω = (Ω_1, Ω_2, Ω_3) are the reaction wheel angular velocity. Differentiating Eq. (7) and considering that the angular velocity of F_b is $\vec{\omega}$ and that the external torque is equal to zero, given by

$$\dot{\vec{h}} + \vec{\omega}^x\vec{h} = 0 \tag{8}$$

Substituting Eq.(7) into Eq.(8), the angular velocity of the system is

$$\dot{\vec{\omega}} = \left(I + I_w\right)^{-1}\left[-\vec{\omega}^x\left(I + I_w\right)\vec{\omega} - \vec{\omega}^x I_w\vec{\Omega} - I_w\dot{\vec{\Omega}}\right] \tag{9}$$

The simulator attitude as function of the angular velocity is given by

$$\begin{pmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\end{pmatrix} = \begin{pmatrix}0 & \sin\theta_3/\cos\theta_2 & \cos\theta_3/\cos\theta_2\\ 0 & \cos\theta_3 & -\sin\theta_3\\ 1 & \sin\theta_3\sin\theta_2/\cos\theta_2 & \cos\theta_3\sin\theta_2/\cos\theta_2\end{pmatrix}\begin{pmatrix}\omega_1\\ \omega_2\\ \omega_3\end{pmatrix} \tag{10}$$

To design the attitude control system based on reaction wheel and gas jets actuators to perform a large angle manoeuvre, it is important to have in mind that each control algorithm is designed based on two different a set of equations of motions. In other words, the gas jets are applied to reduce the high angular velocity and the reaction wheel is used to control in the fine pointing accuracy mode. As a result, for each operation mode one has different matrices A(x) and the respective matrix B associated with it. The C matrix, although depend on the sensor type is assumed unity for simplicity.

In the fine pointing mode where the reaction wheel is the actuator, the state's x are $(\theta_1\ \ \theta_2\ \ \theta_3\ \ \omega_1\ \ \omega_2\ \omega_3)^T$ and the control u are $(\dot{\Omega}_1\ \ \dot{\Omega}_2\ \ \dot{\Omega}_3)^{\mathrm{T}}$, the matrices A(x) and B are given by

$$A(x) = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\sin\theta_3}{\cos\theta_2} & \frac{\cos\theta_3}{\cos\theta_2} \\ 0 & 0 & 0 & 0 & \cos\theta_3 & -\sin\theta_3 \\ 0 & 0 & 0 & 1 & \frac{\sin\theta_3\sin\theta_2}{\cos\theta_2} & \frac{\cos\theta_3\sin\theta_2}{\cos\theta_2} \\ 0 & 0 & 0 & 0 & \frac{I_{22}\omega_3 - I_w\Omega_3}{(I_{11}+I_w)} & \frac{-I_{33}\omega_2 + I_w\Omega_2}{(I_{11}+I_w)} \\ 0 & 0 & 0 & \frac{-I_{11}\omega_3 + I_w\Omega_3}{(I_{22}+I_w)} & 0 & \frac{I_{33}\omega_1 - I_w\Omega_1}{(I_{22}+I_w)} \\ 0 & 0 & 0 & \frac{I_{11}\omega_2 - I_w\Omega_2}{(I_{33}+I_w)} & \frac{-I_{22}\omega_1 + I_w\Omega_1}{(I_{33}+I_w)} & 0 \end{pmatrix} \tag{11}$$

$$B = \begin{pmatrix} \frac{-I_w}{(I_{11}+I_w)} & 0 & 0 \\ 0 & \frac{-I_w}{(I_{22}+I_w)} & 0 \\ 0 & 0 & \frac{-I_w}{(I_{33}+I_w)} \end{pmatrix} \tag{12}$$

One knows that the reaction wheel generates internal torques and the attitude control is performed by exchange of angular moment between the reaction wheel and the satellite. On the other hand, gas jets generate external torque M given by

$$M_{Pi} = -T_i . d_i \tag{13}$$

where M_{Pi} is the torque around the "i" axis due to the force T_i applied at distance d_i from the rotation axis.

In the angular reduction mode using gas jets the reaction wheel is locked, therefore, its acceleration and angular velocity are zero and the satellite angular velocity is given by

$$\dot{\omega} = (I + I_w)^{-1}[-\omega^x(I + I_w)\omega - T_i . d_i] \tag{14}$$

The states x are $(\theta_1\ \theta_2\ \theta_3\ \omega_1\ \omega_2\ \omega_3)^T$ and the control u are $(T_1\ T_2\ T_3)^T$, the matrices A(x) and B are given by

$$A(x) = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\sin\theta_3}{\cos\theta_2} & \frac{\cos\theta_3}{\cos\theta_2} \\ 0 & 0 & 0 & 0 & \cos\theta_3 & -\sin\theta_3 \\ 0 & 0 & 0 & 1 & \frac{\sin\theta_3\sin\theta_2}{\cos\theta_2} & \frac{\cos\theta_3\sin\theta_2}{\cos\theta_2} \\ 0 & 0 & 0 & 0 & \frac{I_{22}\omega_3}{(I_{11}+I_w)} & \frac{-I_{33}\omega_2}{(I_{11}+I_w)} \\ 0 & 0 & 0 & \frac{-I_{11}\omega_3}{(I_{22}+I_w)} & 0 & \frac{I_{33}\omega_1}{(I_{22}+I_w)} \\ 0 & 0 & 0 & \frac{I_{11}\omega_2}{(I_{33}+I_w)} & \frac{-I_{22}\omega_1}{(I_{33}+I_w)} & 0 \end{pmatrix} \tag{15}$$

$$B = \begin{pmatrix} \frac{-d_1}{(I_{11}+I_w)} & 0 & 0 \\ 0 & \frac{-d_2}{(I_{22}+I_w)} & 0 \\ 0 & 0 & \frac{-d_3}{(I_{33}+I_w)} \end{pmatrix} \tag{16}$$

4 Criterion for Exchanging the Actuator

The implementation of the SDRE algorithm in real time has becomes more realistic because of the commercial microprocessor is getting faster [11]. Here, the control system has to deal with two operation modes where the first one is the reduction of high angular velocity using gas jets and the second one is the control in three axes with fine pointing accuracy using reaction wheel. As a result, it is necessary to establish a criterion to change from one actuator to another. This criterion of course is function of the satellite space mission and the control system algorithm and equipment's. For example, from the angular velocity reduction mode to the normal mode of operation the criterion could be associated with the amount of energy that the reaction wheel can support before being saturated or with the minimum and maximum values of the gas jets capacity. The criterion used here is based on the total potential and kinetic energy of the system, which means that when the system reaches a certain level of energy the control algorithm changes the type of actuator. Therefore, the potential energy associated with the angular displacement is given by

$$U = K_u \Delta\theta^2 \tag{17}$$

where K_u is a constant and $\Delta\theta$ represent the angular displacement of the simulator.
The simulator kinetic energy is given by

$$K = K_c w^2 \tag{18}$$

where K_c is a constant and w is the angular velocity of the simulator. It is important to say that the constants K_u and K_c must be such to maintain the total system energy compatibles. Besides, the level of energy can be changed according with the kind of control system to be evaluated. Here one assumes certain level of energy just for simulation purpose.

5 Simulation Results

The superiority of the SDRE method to perform a regulation and tracking large angle manoeuvre over the LQR method has been demonstrated in [10]. Here, the simulation is to demonstrate the ability of the SDRE techniques to control a nonlinear plant based on switching control algorithm using the previously criterion of energy to change from the gas jets to reaction wheel action. The simulator platform can accommodate various satellites components, like sensors, actuators, computers, and its respective interface and electronic. Therefore, the inertia moments of the simulator depend on the equipment's distribution over it. Here, one assumes and uses the following typical inertia moment for the simulator: $I_{11} = I_{22} = 1.17$ Kg·m^2 and $I_{33} = 1.13$ Kg·m^2; and for the reaction wheel $I_x = I_y = I_z = 0.0018$ Kg·m^2. The maximum and minimum gas jet torque used is 10 Nm and the total amount of system energy to change from gas jets to reaction wheel is 0.5J. In the fine pointing mode, the typical sensor noises used are theta = 0.2 (deg) and rate theta = 0.1 (deg/s).

To demonstrate the performance of the SRRE controller one imposes a severe large angle maneuver which begins on 0º and in the end it has to tracking a angular reference of (100, 50, 70) deg. The controller performance requirements are small overshoot and quick time of response. The controller robustness is associated with its ability to perform big tracking maneuver apart from the perturbations due to sensor noise and plant nonlinear terms. Besides, it is important to say that this performance is a function of the weighting matrices of the SDRE controllers. After some trial and error, one gets the following values for matrices R= diag [0.0001 0 0;0 0.0001 0;0 0 0.0001] and Q= diag [1 0 0 0 0 0;0 1 0 0 0 0;0 0 1 0 0 0;0 0 0 100 0 0;0 0 0 0 100 0;0 0 0 0 0 100].

Figures 2 and 3 show the angular displacement and angular velocity when the SDRE controller performs the simulator large maneuver from 0º and it has to tracking the previously angular reference. One observes that at the maneuver end the nonlinear terms of the plant are more relevant. The SDRE controller can get the reference in about 250s.

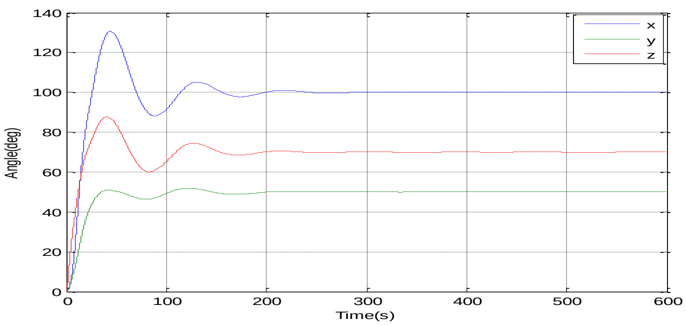

Fig. 2 - Simulator angular displacement in x, y and z.

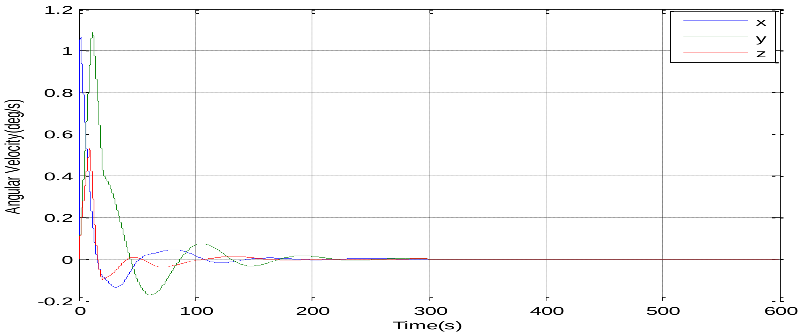

Fig. 3 – Simulator angular velocities in x, y, and z

Figures 4 and 5 show the SDRE controller action during the transition phase of the previously manoeuvre where the torque due to the gas jets is in the first instants and the torque due to the reaction wheel is the rest of the time, respectively.

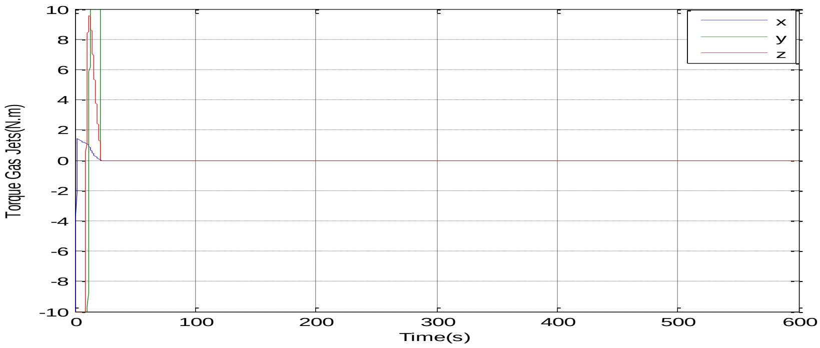

Fig. 4 - SDRE controller using torque due to the gas jets.

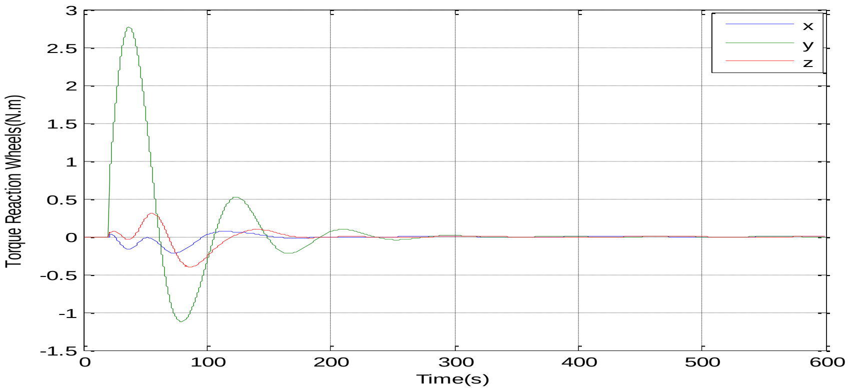

Fig. 5 - SDRE controller using torque due to the reaction wheel.

Figure 6 shows how the switching control algorithm works. That is, the gas jets stop acting and the reaction wheel started acting when the criterion for changing actuators is achieves, system total energy equal to 0.5J.

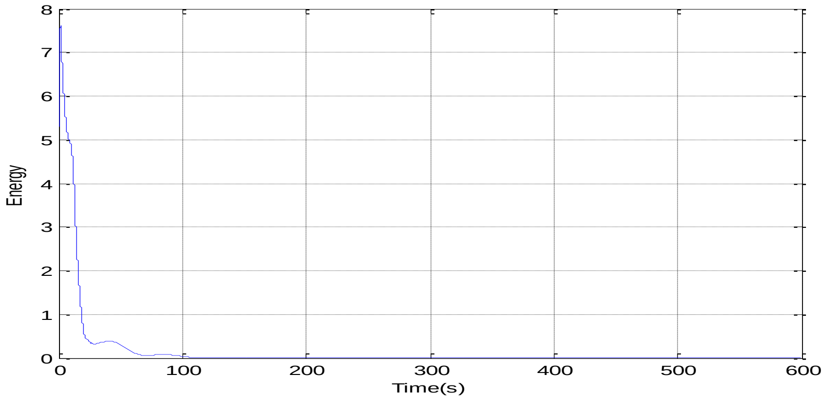

Fig. 6 – Criterion for changing actuators is achieves - 0.5J.

From simulations one observes that at the beginning of the manoeuvre the level of energy is high because the simulator is far from the final attitude to be follow. As a result, the switching control algorithm selects as actuator the thrusters to deal with high angular velocities reduction. On the other hands, when the simulator reaches the small reference attitude the total energy decreases rapidly. Therefore, the switching control algorithm selects as actuators the reaction wheels, to perform fine pointing adjustment of the simulator.

Finally, it is important to say that the criterion for changing the actuated based on the energy value defined in the program was established just to provide a good visualization of the torques from the two actuators during the simulation. However, further study of this actuated change can be done based in other criterion of optimization like minimum time maneuver or fuel, reaction wheel speed and pointing accuracy.

6 Conclusions

In this paper one develops a general 3-D simulator nonlinear model since it only depends on the inertia moment of the system. The model is used to investigate big angle tracking manoeuvre to design a control algorithm based on the gas jet and reaction wheel, where the first actuator is used to reduce high angular velocity and the second one to perform fine pointing control. The switching control algorithm used to chance from gas jet to reaction wheel action is based on the potential plus the kinetic energy of the system. Therefore, the transition between modes of operation occurs when the system reaches a certain level of energy. The nonlinear controller design uses the State Dependent Riccati Equation - SDRE method to deal with high nonlinear simulator plant, system noise and perturbation due to nonlinear terns in the dynamics of the satellite. Simulations have demonstrated the good performance and robustness of the SDRE controller to perform large angle tracking manoeuvre. The investigation has also shown that SDRE control algorithm can be implemented in satellite onboard computer is very promising, since the gains of the control laws can be obtained in the laboratory through computer simulations as in this work was carried out. The next step of this work is to implement and tests the control algorithm developed here in an experimental simulator with reaction wheel and gas jets.

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