Dynamic analysis of transversal response due to moving mass in a continuous beam

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Abstract. This paper presents a computational analysis of the dynamic responses of a continuous beam subjected to a moving mass and corresponding load that travels along its entire length. It aims to understand the behavior of the structure and to determine its transversal response. Here, a discretized dynamic finite element algorithm was developed, using numerical integration by Newmark's Method for the solution of ordinary differential equations and obtain the transversal displacements of the structure in the time domain, in order to evaluate its behavior due to moving masses and loads. The analysis is made in different velocities, damping ratios, with and without the contribution of the moving mass in the inertia of the system and comparing the effects of these parameters in the responses. The main objective is to apply the results obtained with the method to obtain displacements due to moving loads in structures such as bridges and viaducts, among other applications, and to obtain the critical velocities, where the greatest deflections of the structure are found.

Keywords: Moving Loads, Structural Dynamics, Newmark's Method.

1 Introduction

Some of the loads that structures are submitted vary as function of time, like the motion of people and vehicles, wind loads, impact, earthquakes, and even projectiles or rockets. The development of trustful models that evaluate the responses of structures is essential to warrant the security of them. The displacement of loads in structures causes dynamic responses in the form of vibrations that can produce excessive displacements, which can cause the collapse of the structures.

Bajer and Dyniewicz [1] point out that Saller [2] made the first study to address the problem of mobile masses, and although these studies began in the early part of the last century and there are references such as Fryba [3] and Rao [4] present analytical solutions to the moving load problems applied in models with different boundary conditions aiming to find the responses of these structures. Even today there are few resources for analysis of mobile loads implemented in commercial software.

This paper presents a case study, in which a two-span continuous beam is subjected to a moving load and moving mass in longitudinal motion and aims to find the transverse responses of the structure for different velocities using optimization to find the velocity that causes the maximum displacement of the structure.

2 Modelling

The studied system is shown in Figure 1. It consists of a simply supported beam, with length L, modulus of elasticity E, cross section area A, moment of inertia about the y-axis I and density ρ. Along the beam, the moving mass M travels with constant velocity $v(t)$.

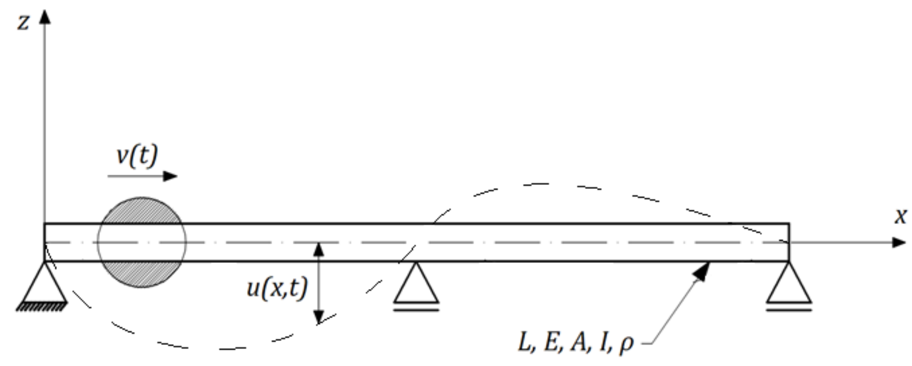

Figure 1. Simply-supported beam subjected to a moving mass system.

The model is discretized and analyzed using the finite element method where the real structure is represented as a model consisted of several elements, as shown in Figure 2, with several degrees of freedom. Each element has mass, stiffness and corresponding damping, leading to the generation of matrices of mass, stiffness and damping of each element.

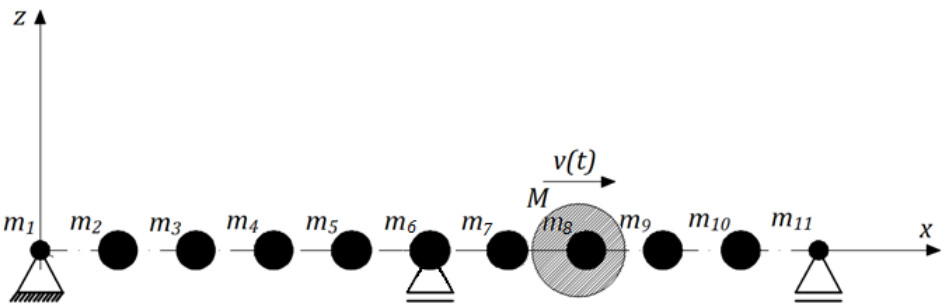

Figure 2. Discretized model

The local matrices are then converted to global matrices, and the equation of the system's motion is considered in the matrix form as

$$[\boldsymbol{M}]\{\ddot{\boldsymbol{u}}\} + [\boldsymbol{C}]\{\dot{\boldsymbol{u}}\} + [\boldsymbol{K}]\{\boldsymbol{u}\} = p(t) \tag{1}$$

where $[M]$ is the mass matrix, $[C]$ the damping matrix and $[K]$ the stiffness matrix of the system. Mazzilli et al. [5] report that it is a sufficient condition for the damping to be of the proportional type, the damping matrix $[C]$ to be a linear combination of the mass and stiffness matrices, expressed by

$$[C] = \sum_b a_b [M]([M]^{-1}[K])^b \tag{2}$$

where the particular case of the Rayleigh damping can be considered

$$[C] = a_0[M] + a_1[K] \tag{3}$$

where the factors a_0 e a_1 are obtained imposing arbitrary damping ratios ξ for two chosen modes frequencies, finding the solution for the system

$$\begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \frac{1}{\omega_i} & \omega_i \\ \frac{1}{\omega_j} & \omega_j \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \tag{4}$$

To solve the problem of moving loads, the moving load P is considered changing position between the nodes of each element, changing the load vector $[P]$ at each time step, depending on the constant speed $v(t)$, solving the nonlinear problem in a linearized form.

In order to obtain system responses, it is necessary to integrate the equations of motion, so the Newmark method is used for direct integration, Toledo [6] reports that displacements and velocities are developed in Taylor series, with the rest being calculated approximately according to free parameters that are fixed later. Brasil and Silva [7] report that, given the vectors of displacements, velocities and accelerations in a given time *t*, their values are determined in an instant $t + \Delta t$.

$$\dot{u}_{t+\Delta t} = \dot{u}_t + b_6 \ddot{u}_t + b_7 \ddot{u}_{t+\Delta t} \tag{1}$$

$$\ddot{u}_{t+\Delta t} = b_0 (u_{t+\Delta t} - u_t) - b_2 \dot{u}_t - b_3 \ddot{u}_t \tag{2}$$

The coefficients b_0, b_1, b_2, b_3, b_4, b_5 are chosen in order to approximate the variation of the vectors,

$$b_0 = \frac{1}{\beta \Delta t^2}; \quad b_1 = \frac{\alpha}{\beta \Delta t}; \quad b_2 = \frac{1}{\beta \Delta t}; \quad b_3 = \frac{1}{2\beta} - 1; \quad b_4 = \frac{\alpha}{\beta} - 1; \quad b_5 \frac{\Delta t}{2}\left(\frac{\alpha}{\beta} - 2\right);$$

$$b_6 = \Delta t(1 - \alpha), b_7 = \Delta t \alpha \tag{3}$$

obtaining a system of algebraic equations that allows to find the increments of deflections in the step,

$$\hat{K} u_{t+\Delta t} = \hat{p}_{t+\Delta t} \tag{4}$$

with equivalent stiffness

$$\hat{K} = b_0 M + b_1 C + K \tag{5}$$

and equivalent step load

$$\hat{p}_{t+\Delta t} = p_{t+\Delta t} + M(b_0 u_t + b_2 \dot{u}_t + b_3 \ddot{u}_t) + C(b_1 u_t + b_4 \dot{u}_t + b_5 \ddot{u}_t) \tag{6}$$

from which the deflections, velocities and accelerations of the next step are determined.

Assan [8] reports that the constants α and β can be considered respectively $\frac{1}{2}$ and $\frac{1}{4}$ for constant acceleration, and for linear variation of acceleration the values are $\frac{1}{2}$ and $\frac{1}{6}$.

For the model considering moving mass, its contribution is also considered in the mass matrix [M] at each time step, in function of the constant speed v(t).

The concept of optimization is then applied to find the maximum displacements in the midspan considering different constant speeds of displacement of the mass over the bar element to obtain the solution of the problem. Brasil and Silva [9] present the concept of the golden ratio optimization, from which, in an iterative way, the solution of the problem can be found. This method uses the golden ratio, defined by Euclid as

$$\varphi = \frac{1+\sqrt{5}}{2} \tag{11}$$

to find the minimum of a function, an interval is defined so that the minimum value is sought, where two points chosen considering the golden ratio are compared. The functions are applied to these points, and the minimum between them is verified. After that, one of the points becomes the new limit of the interval, excluding the left or right domain, and the analysis is repeated. The interval is then reduced by 61.8% at each iteration and is repeated until the minimum of the function is found.

3 Discussion and Results

Type your conclusions or closing remarks here. Please be as concise and objective as possible. Do not make a summary of the paper, but instead comment on the main findings and results, even if these are only partial conclusions so far.

For the study, a model was analyzed using Mathworks Matlab® to find the solutions, consisting of a continuous concrete beam, with two spans of 100.00m each, modulus of Young $E = 3.0105x10^{10}$, cross section area $A = 6.534. m^2$, moment of inertia about y-axis $I = 3.6004\ m^4$ and density $\rho = 2500\ kg/m^3$. The moving mass $M = 45000\ kg$ moves at a constant speed. We analyzed the structure's response to different speeds between $v(t) = 1{,}00\ m/s$ and $(t) = 200{,}00\ m/s$. The studied model is shown in Figure 3.

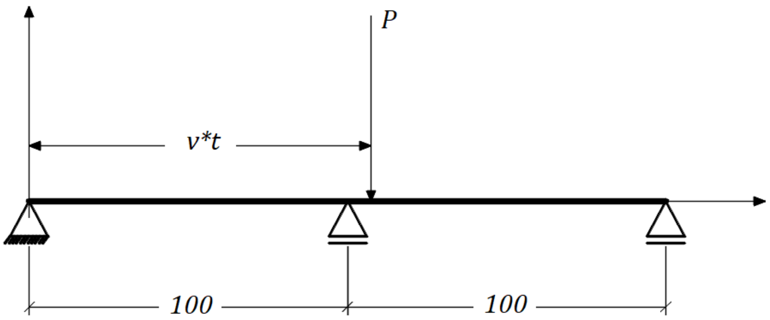

Figure 3. Discretized model

The model is then discretized to perform computational analysis using the finite element method in a model with 16 elements and 17 nodes, each node with two degrees of freedom, relative to vertical displacement and rotation, and axial efforts can be disregarded.

Stiffness matrices and mass matrices are then generated for each moving load position.

Using the Newmark Method, the responses of the motion equation for each position of the mobile load are then calculated, as a function of the displacement speed, for different constant speeds between $1{,}00\ m/s$ and $200{,}00\ m/s$.

Then we apply the golden ratio method of optimization to find the velocity that causes the maximum deflections in midspan for different ratios of damping between 0 and 0.02, and its respective maximum transversal deflection in the midspan, the results are shown in Table 1.

Table 1. Maximum deflection speeds and maximum transversal deflections without contribution of the mobile mass in the global mass matrix, for different damping ratios

ξ	Maximum deflection speeds (m/s)	Maximum transversal deflections (mm)	Dynamic amplification coefficient
0	72.3748	194.7455	3.1325
0.01	73.2117	173.6479	2.7931
0.02	72.9235	157.1278	2.5274

The dynamic amplification coefficients obtained comparing the static deflection and the dynamic deflection are shown in Figure 4.

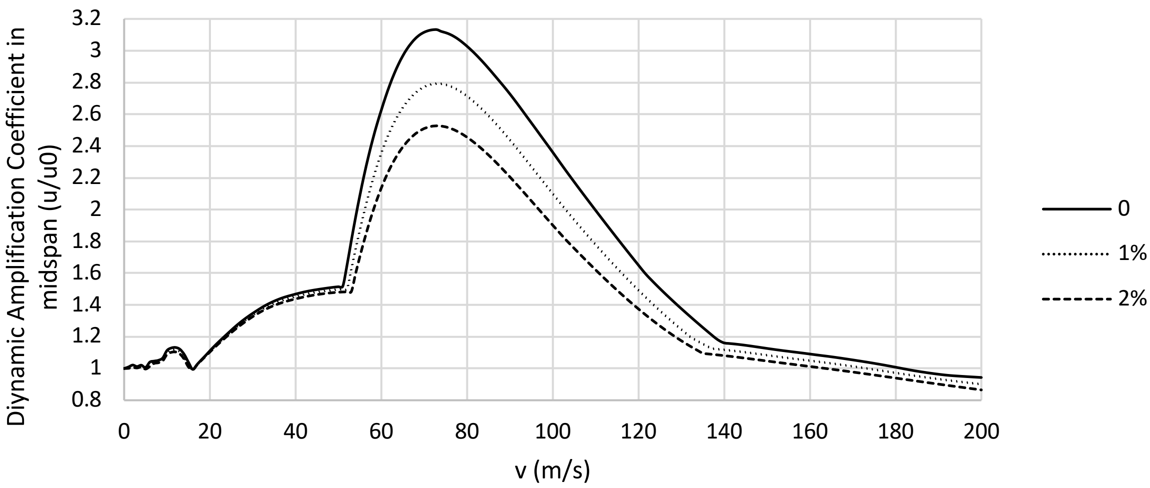

Figure 4. Relation between the displacement speed of the mobile mass and the dynamic amplification coefficient for two span continuous beam, for different damping ratios

Then the effect of the moving mass is considered in the global mass matrix, the responses of the system obtained considering the contribution of the moving mass in the global matrixes are shown in Table 2.

Table 2. Maximum deflection speeds and maximum transversal deflections with contribution of the mobile mass in the global mass matrix, for different damping ratios

ξ	Maximum deflection speeds (m/s)	Maximum transversal deflections (mm)	Dynamic amplification coefficient
0	71.7455	196.8463	3.1663
0.01	71.7587	175.3424	2.8204
0.02	71.6681	158.6384	2.5517

The dynamic amplification coefficients obtained comparing the static deflection and the dynamic deflection for the system considering contribution of the mobile mass are shown in Figure 5.

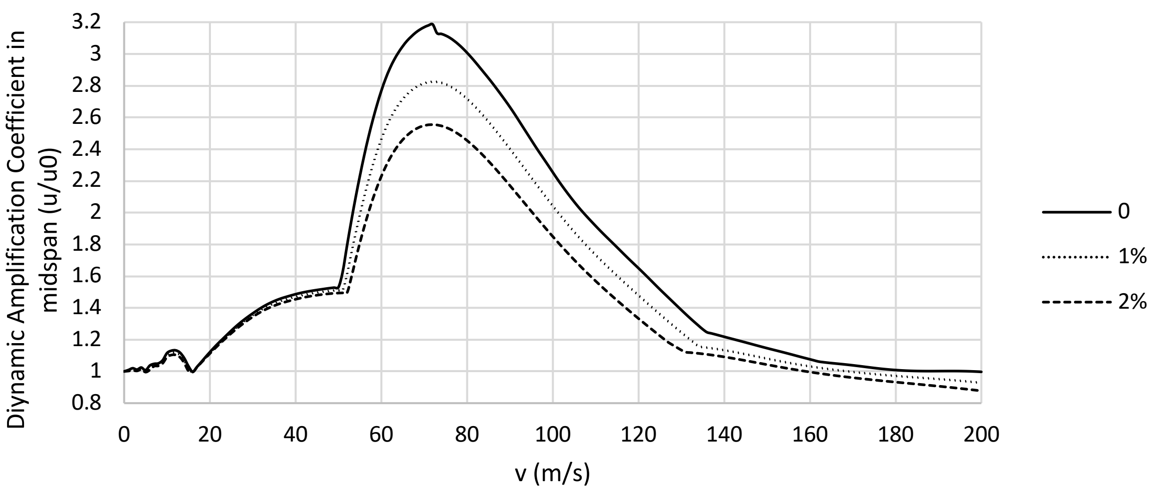

Figure 5. Relation between the displacement speed of the mobile mass and the dynamic amplification coefficient for two span continuous beam, for different damping ratios

4 Conclusions

It is possible to conclude that the application of a moving load in a continuous beam has a significant dynamic amplification of the responses of the structure. It is also possible observe the effect of the damping in the reduction of the responses.

The consideration of the moving mass in the global mass matrix promotes the increase of the responses when compared to the analysis only considering the moving load, in this study, due to the relation of the vehicle and bridge masses, the difference is small when comparing the models with and without consideration of the mass influence.

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