Energy Harvesting Using a Piezoelectric Nonlinear Energy Sink (PNES) to an Aeroelastic System

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Abstract. The influence of piezoelectric nonlinear energy sink (PNES) on the dynamic behaviour of an energy harvesting system applied to an aeroelastic structure in flutter condition is studied. NES is often used in aeroelastic systems to decrease the amplitude of vibration, possibly attenuating , controlling or delaying typical aeroleastic phenomena, such as flutter, galloping and VIV (vortex-induced vibrations). It is also able to harvest energy and distribute this energy to electric devices available in the system. The dynamic response of the PNES with energy harvesting was analysed for four cases: linear case, only cubic nonlinear stiffness, only quadratic nonlinear piezoelctrical coupling and both nonlinear terms combined. The inclusion of nonlinear terms increased flutter speed, and the combination of both nonlinear terms have the greatest increase. The cubic nonlinear stiffness is responsible for increasing the equivalent stiffness of the system, which causes a decrease in the amplitude of the system response, while quadratic nonlinear piezoelectrical coupling increases the energy harvest of the system, which also decrease the amplitude and increase the electric energy harvested. The influence of electric parameters in flutter speed and power were also studied. The variation of these parameters is able to maximize flutter speed and electrical power.

Keywords: Nonlinear energy sink, Energy Harvesting, Flutter, Electrical Parameters

1 Introduction

Energy harvesting is a technology that has been explored by researchers as an alternative to nonrenewable energy resources, in recent years Safaei et al. [1], due the need to create self-sustainable resources. This is a promising technology to produce sustainable energy sources, replacing fossil fuel. One of the fields of study where energy harvesting can be applied is aeronautics. It has many applications, such as low-power electricity generation, which can be applied from aircraft and helicopters to civil structures in areas with high winds Marqui Jr et al. [2]. The oscillatory movement caused by the flutter phenomenon is an interesting energy source and study for energy harvesting technology.

A device that can be used to attenuate vibration, such as flutter, due to a particular mode of the structure over a range of frequencies is known as a tuned mass damper (TMD) or dynamic vibration absorber (DVA). It is formed by another secondary system responsible for absorbing the vibration energy from the primary system to which it was coupled. The system resulting from the union of the primary absorber and secondary absorber must present the natural frequency far from the excitation frequency. The point is that if the system works in a wide range of frequencies, other amplitudes, outside resonance, can be very high, that is, the absorber is limited Rao and Yap [3]. To include nonlinear terms in systems with more than one degree of freedom has also been studied as a way to absorb energy in more than one frequency since the inclusion of quadratic and cubic nonlinear terms should change the response of the mechanical linear system Nayfeh and Jebril [4]. NES is formed by a secondary system with a nonlinear stiffness attached to the main system. These substructures are responsible for absorbing vibrational energy using target energy transfer (TET) technology Viguié et al. [5]. TET uses nonlinear modes and internal resonance to passively and irreversibly transfer vibration energy to an NES Lee et al. [6].

The work from Lee et al. [7] is represented by Figure 1, it uses the aerodynamic loads of the Dowell's model. The NES can function fully, partially, or be ineffective over a frequency range. In the best case of suppression, energy transfers from the wing to the NES are caused by nonlinear modal interactions during resonance. Plunge and pitch modes, as well as the NES exhibit responses that decay exponentially, resulting in the elimination of LCOs. This work aims to unite two NES applications, with vibration mitigation in aeronautics and energy harvesting.

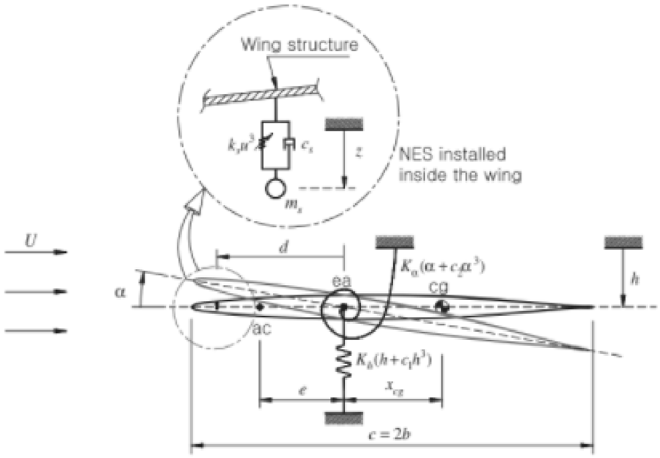

Figure 1. Aeroelastic typical section with 2 DOF with NES. Fonte: Lee et al. [7]

Designing a piezoelectric transducer that works like a NES (PNES) is a new way to reuse the energy that has been absorbed, to suppress the vibration. Silva et al. [8] experimentally validates the vibration attenuation performed by a piezoelectric based on the NES principle. The proposed model in 2 provides significant vibration attenuation.

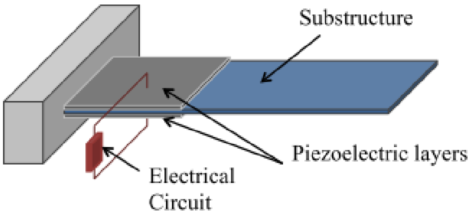

Figure 2. Electromechanically coupled beam with shunted piezo layers. Fonte: Silva et al. [8]

2 Mathematica model

In this section, the model and the dynamical equations of the aeroelastic typical section and aerodinamic loads are described. Figure 3 shows the model of the aeroelastic typical section of a system with three degrees of freedom: two mechanical degrees of freedom, plunge (h) and pitch (α), and one electrical degree of freedom, charge (q). The piezoelectric coupling is associated to plunge. The dynamical equations, based on Marqui Jr and Erturk [9], of the system presented in Fig. 3, applying the nonlinear stiffness of the cubic type associated to plunge movement and nonlinear electromechanical coupling, in dimensionless form, are given by:

$$(m + m_e)\ddot{h} + mbx_\alpha\ddot{\alpha} + d_h\dot{h} + k_h h + \delta h^3 - \frac{\Theta}{C_p}(K|h| + 1)q = -L \tag{1}$$

$$mbx_\alpha\ddot{h} + I_\alpha\ddot{\alpha} + d_\alpha\dot{\alpha} + k_\alpha\alpha = M \tag{2}$$

$$L\ddot{q} + R\dot{q} + \frac{q}{C_P} - \frac{\Theta}{C_p}(K|h| + 1)h = 0 \tag{3}$$

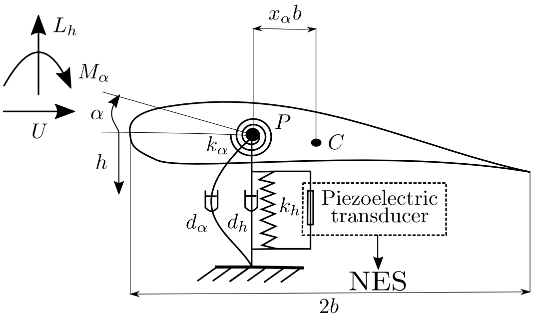

Figure 3. Aeroelastic section.

in which m is the mass of the system, m_e is the mass of fastener, bx_α is the distance between elastic axis and centroid, h is plunge, α is pitch, d_h and d_α are plunge and pitch damping coefficients, k_h and k_α are the plunge and pitch stiffness coefficients, I_α is the moment of inertia, c is the chord, q is the electrical charge, R is the electrical resistance, C_p is the equivalent capacitance, Θ is the electromechanical coupling, L is the electrical inductance, δ is cubic nonlinear stiffness coefficient, K is quadratic nonlinear electromechanical coupling coefficient, L is aerodynamical lift, M is aerodynamical moment, U is the wind speed, and . denotes differentiation by time.

Dowell et al. [10] presents a mathematical model with lift (L) and moment (M), for an aerodynamic sistem quasi-steady in a incompressible flow, it includes the contribution of the effective angle of attack. The expressions are reproduced here:

$$L = qS\frac{\partial C_L}{\partial \alpha}\left[\alpha + \frac{\dot{h}}{U}\right] \tag{4}$$

$$M = \rho\frac{U^2}{2}Se\frac{\partial C_L}{\partial \alpha}\left[\alpha + \frac{\dot{h}}{U}\right]$$

in which ρ é the air density, e is the distance between the center and the aeroelastic axis. For a flat plate with incompressible flow: $\partial C_L/\partial\alpha = 2\pi$, $C_{L0} = C_{MAC} = 0$ and $q = \rho U^2/2$.

The values of parameters used here are: $m = 0.804$ kg, $m_e = 1.060$ kg, $b = 0.125$ m, $x_\alpha = 0.2064$, $d_h = 1.8770$ Ns/m, $d_\alpha = 0.0199$ Ns/m, $k_h = 2100$ N/m, $k_\alpha = 2.65$ N/m, $\rho = 1.07$kg/m^3, $\theta = 0.00155$ N/V, $C_{res} = 0.00005$ F, $I_\alpha = 0.0028$ kg m^2, $L = 22$ H, $R = 330000\ \Omega$, $U = 7.45$ m/s, $\delta = 0$ and $K = 0$. The initial conditions used are: $h = 0.003$ m, $\alpha = 0$, $h' = 0$, $\alpha' = 0$ and $q = 0$.

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3 Methodology

In this section the methodology of the project is presented. The linear system is studied through eigenvalues. The nonlinear system are described through 4th order Runge-Kutta method for numerical analysis with a simplified model; The 4th order Runge-Kutta method, with an optimisation procedure based on the interval halving method (or bissection method) is used to describe the movement of the aeroelastic system. In this method, one-half of the current interval of uncertainty is discarded in every stage, until the right solution is found, for the middle point of the final interval Rao [11]. With the otimization, it is possible to determine the flutter speed and its respective amplitude in the movement. Flutter speed must be determined for each combination of different parameters.

4 Influence of nonlinear terms

Figure 4 shows flutter speed as function of nonlinear terms ($\delta = 700000$ and $K = 700.$). The cubic nonlinear stiffness coeficient (δ) is responsible for increasing the equivalent stiffness in the system, which causes a decrease in the amplitude of the system response. On the other hand, quadratic nonlinear piezoelctrical coupling (K) increases the energy harvest of the system, which also decrease the amplitude of the system response. Decreasing the amplitude of the system response leads to an increase in the flutter speed.

Figure 5 shows power as function of nonlinear terms. We have mechanical power related to plunge (P_h) and mechanical power related to pitch (P_α). P_h does not have much influence from the variation of cubic nonlinear

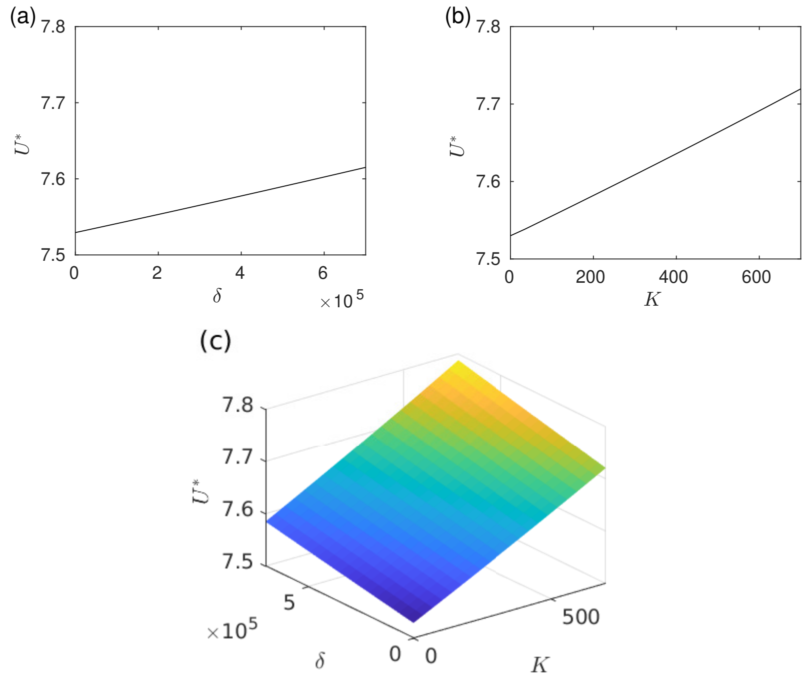

Figure 4. Flutter as function of a) cubic nonlinear stiffness coeficient, b) quadratic nonlinear piezoelctrical coupling coeficient, c) both nonlinear terms.

stiffness coefficient (δ) and of quadratic nonlinear electromechanical coupling coefficient (K). P_α decreases when nonlinear terms increase. Electrical power (P_e) increases when nonlinear terms increase. It indicates a strong relation between (P_α) and (P_e).

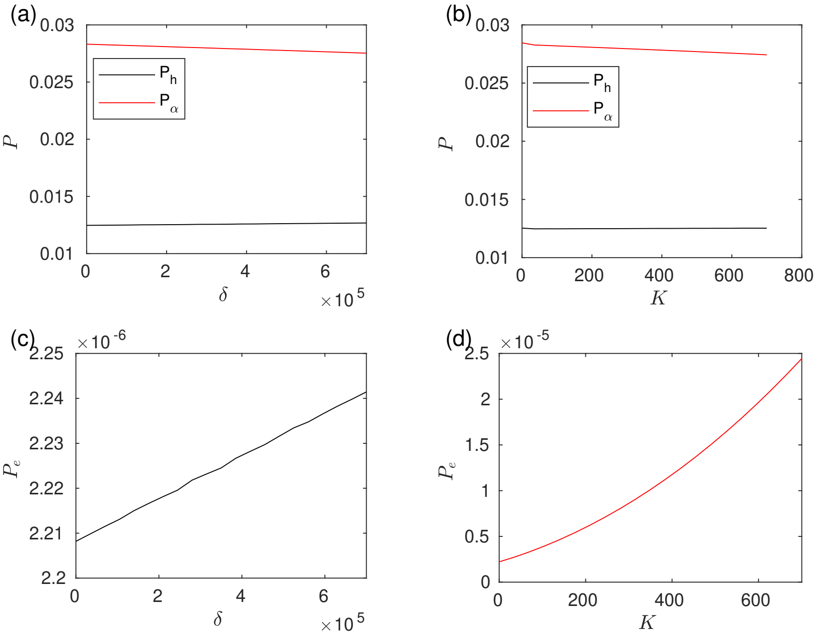

Figure 5. Power as function of nonlinear terms.

5 System behaviour as a function of electrical parameters

In this section, the behaviour of the system with respect to the parameters Θ, R, C_p and L is analysed. When we compare the matrices of a purely mechanical NES and an electromechanical PNES, Θ is embedded in the equivalent stiffness matrix of the secondary system, so it increases the equivalent stiffness of the system. Again, we can notice a strong relation between (P_α) and (P_e). The parameter R is embedded in the equivalent damping matrix of the secondary system, it increases flutter speed and mechanical power related to plunge, but decreases mechanical power related to pitch. For electrical power it has a maximum near $R = 3 \times 10^5$. R is equivalent to damping of the secondary system, so it can reduces the amplitude of the system, what can increases flutter speed and electrical power, until an optimal value. The parameter C_p is the opposite of Θ, so it decreases the equivalent stiffness of the system. The increase of C_p decreases flutter speed and electrical power, but increases mechanical power related to plunge and mechanical power related to pitch. The increase of L does not have a big influence in flutter speed and power for this range. L is embedded in the equivalent mass matrix of the secondary system, but it does not change the behaviour for this case.

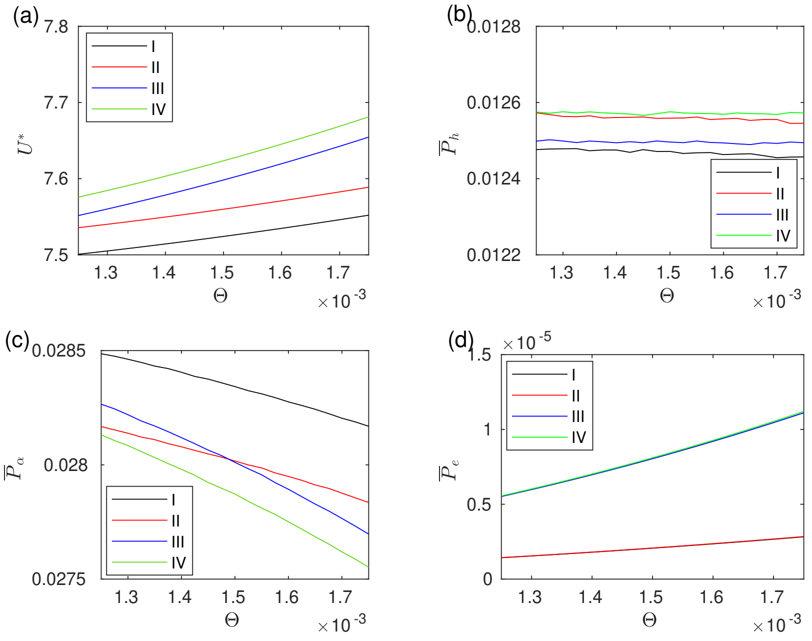

Figure 6. Flutter speed and power as function of electromechanical coupling.

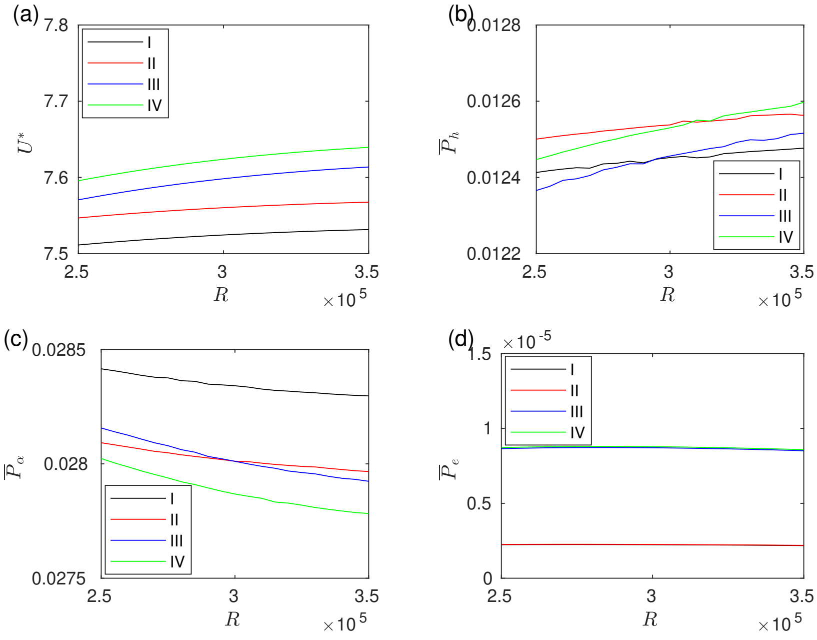

Figure 7. Flutter speed and power as function of electrical resistance.

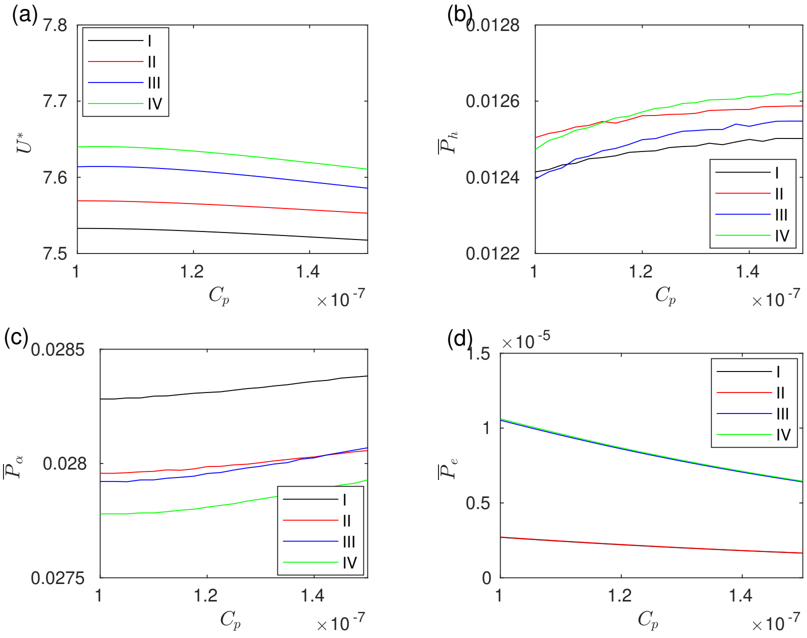

Figure 8. Flutter speed and power as function of equivalent capacitance.

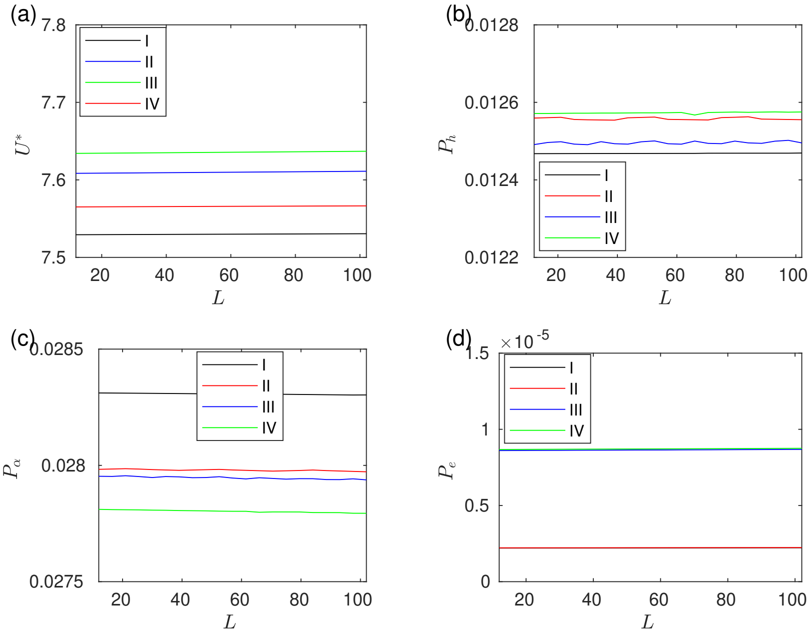

Figure 9. Flutter speed and power as function of inductance.

6 Conclusions

The influence of piezoelectric nonlinear energy sink (PNES) on the dynamic behaviour of an energy harvesting system applied to an aeroelastic structure in flutter condition is analysed. The influence of cubic nonlinear stiffness and quadratic nonlinear piezoelectrical coupling in the aeroelastic typical section were studied. The inclusion of nonlinear terms increased flutter speed, and the combination of both nonlinear terms have the greatest increase. The cubic nonlinear stiffness is responsible for increasing the equivalent stiffness of the system, which causes a decrease in the amplitude of the system response, while quadratic nonlinear piezolectrical coupling increases the energy harvest of the system, which also decrease the amplitude and increase the electric energy harvested. It is possible to notice a strong relation between (P_α) and (P_e). The influence of electric parameters in flutter speed and power were also studied. The variation of these parameters is able to maximize flutter speed and electrical power. Increasing the amount of energy harvested using nonlinear elements has the potential to increase the applicability of energy harvesting to engineering systems in general. The next step is to evaluate the section through the multiphysics software for a continuous model.

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