Tri-objective optimization of 3d Steel Frames Considering Columns Orientation and Bracing System Configuration as Design Variables

Cláudio H. B. Resende [1], Afonso C. C. Lemonge[2], Luiz F. Martha[1]

[1]*Department of Civil and Environmental Engineering - Pontifical Catholic University of Rio de Janeiro*
Rua Marquês de São Vicente, 225, 22451-900, Gávea, Rio de Janeiro - RJ, Brazil
claudio.horta@tecgraf.puc-rio.br
lfm@tecgraf.puc-rio.br
[2]*Department of applied and computational mechanics - Federal University of Juiz de Fora*
Rua José Lourenço Kelmer s/n, 36036-900, Juiz de Fora - MG, Brazil
afonso.lemonge@ufjf.edu.br

Abstract. In the field of steel structural design, particularly in the context of tall buildings, there is a need to minimize costs while enhancing performance with regards to dynamic behavior, and structural stability. Furthermore, determining the most suitable geometric configuration for the bracing system and the optimal orientation of the principal axes of inertia for the columns is not readily apparent. Typically, such decisions are based on the expertise of the designer. Consequently, solving this complex problem, which involves simultaneously considering three objectives, is far from straightforward. Hence, this paper focuses on the tri-objective optimization of spatial steel frames, considering both the configuration of the bracing system and the orientation of the columns as design variables. To accomplish this, four differential evolution algorithms have been employed: the third evolution step of generalized differential evolution (GDE3), the success history-based adaptive multi-objective differential evolution (SHAMODE), the SHAMODE with whale optimization (SHAMODE-WO), and the multi-objective meta-heuristic with iterative parameter distribution estimation (MM-IPDE). Additionally, a multi-criteria tournament method is utilized to extract desired solutions from the Pareto front, aligning with the preferences of the decision-maker.

Keywords: Structural optimization, multi-objective, steel frames, bracing systems, meta-heuristics

1 Introduction

Space steel frames play a significant role in civil engineering applications globally, with a rising demand for more efficient and cost-effective structures capable of supporting taller buildings. Multi-objective optimization techniques are crucial in achieving these objectives, yielding Pareto fronts (PFs) with optimal solutions from which decision-makers can choose the most suitable design based on their preferences and requirements. Wind-induced challenges, such as horizontal displacements, global stability, and dynamic behavior, become significant factors in structural design as buildings grow taller. Bracing systems address these challenges, but selecting the best geometric configuration and column orientation is complex due to numerous options available. By utilizing multi-objective optimization techniques, designers can identify a set of Pareto optimal solutions, leading to more cost-efficient and structurally optimal designs.

Real-world engineering problems necessitate optimizing both performance and cost, making it essential to include multiple objectives in optimization formulations. This paper focuses on minimizing maximum horizontal displacement, maximizing the first natural frequency of vibration, and maximizing the critical load factor for the first global buckling mode, in addition to weight minimization. The adoption of four multi-objective evolutionary algorithms based on differential evolution (Price et al. [1]) offers efficient solutions, ensuring a comprehensive understanding of the structural system's performance and resulting in more efficient and effective designs as studied by Carvalho et al. [2].

The field of structural optimization for steel frames has seen increased interest in studying bracing systems and multiple objectives. Notable studies, such as Papadrakakis et al. [3], Kicinger and Arciszewski [4], Kicinger et al. [5], explored multi-objective problems in framed structures, and evaluated various bracing configurations for tall buildings. In recent years, optimization methods have evolved, proposing multi-performance optimization approaches for designing dissipative bracing systems (Braga et al. [6]), minimizing weight and damage index in plane steel frames subjected to explosion loads (Khaledy et al. [7]), and conducting multi-objective optimization studies of controlled rocking steel braced frames (Burton et al. [8]). These diverse studies demonstrate the use of optimization algorithms to achieve practical and cost-effective designs for steel frames with various bracing configurations.

2 Formulation of the optimization problem

The objective of the multi-objective optimization problem is to identify the optimal configuration for the bracing system, column orientations, and commercial steel profiles, represented by the integer index vector $\mathbf{x} = I_1, I_2, ..., I_i$ (design variables). The problem aims to achieve three objectives: (i) minimizing the overall weight of the structure ($W(\mathbf{x})$), (ii) minimizing the maximum horizontal displacement ($\delta_{max}(\mathbf{x})$), and (iii) maximizing the critical load factor for global stability ($\lambda_{cr}(\mathbf{x})$). The formulation of the multi-objective problem is presented in eq. (1), with $\mathbf{x}^L$ and $\mathbf{x}^U$ denoting the lower and upper bounds of the design variables, respectively. The total weight of the structure is mathematically expressed in eq. (2), which incorporates the specific material mass (ρ), the cross-sectional area (A_i), and length (L_i) of each element indexed by i.

$$\begin{gathered} \min \quad W(\mathbf{x}) \quad \text{and} \quad \min \quad \delta_{max}(\mathbf{x}) \quad \text{and} \quad \max \quad \lambda_{cr}(\mathbf{x}) \\ \text{s.t.} \quad \text{structural constraints} \\ \mathbf{x}^L \leq \mathbf{x} \leq \mathbf{x}^U \end{gathered} \tag{1}$$

$$W(\mathbf{x}) = \sum_{i=1}^{N} \rho A_i L_i, \tag{2}$$

The problem involves several constraints, including the inter-story drift, Load and Resistance Factor Design (LRFD) interaction equations considering combined axial force and bending moments, the LRFD shearing equation, and geometric constraints related to the beam-to-column and column-to-column connections. The maximum inter-story drift is constrained to a value of $\bar{d} = h/500$, where h represents the height between two consecutive floors (eq.(3)). This constraint aligns with both the Brazilian code ABNT [9] and the American code ANSI [10].

$$\frac{d_{max}(\mathbf{x})}{\bar{d}} - 1 \leq 0 \tag{3}$$

The frame elements must adhere to the Load and Resistance Factor Design (LRFD) equations for unsymmetrical bending (eq.(4)) and shearing (eq. (5)). The required axial strength and the required flexural strength about the major axis, and the minor axis, are denoted by P_r, M_{rx}, and M_{ry}, respectively. These strengths are compared with the available axial and flexural member strengths, denoted as P_c, M_{cx}, and M_{cy}, respectively. Additionally, the allowable shearing strength equation involves the required shearing strength, V_r, and the available shearing strength, V_c. The process of determining the allowable strengths follows a similar methodology in both ABNT [9] and ANSI [10], and this paper adopts their approach.

$$\begin{cases} \dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) - 1 \leq 0 \quad if \quad \dfrac{P_r}{P_c} \geq 0.2 \\ \\ \dfrac{P_r}{2P_c} + \left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) - 1 \leq 0 \quad if \quad \dfrac{P_r}{P_c} < 0.2 \end{cases} \tag{4}$$

$$\frac{V_r}{V_c} - 1 \leq 0 \tag{5}$$

The formulation of this problem incorporates geometric constraints, which are essential in addressing various structural aspects, especially concerning beam-to-column and column-to-column connections. The constraints on beam-column connections prevent attaching a beam with a flange wider than either the column web's height or its flange. Meanwhile, for connections between columns, constraints ensure that profiles with greater depth or mass cannot be fitted over profiles with lower values. Figure 1 illustrates the connections between structural members, where h_{wi}, b_{fi}, and d_i represent the height of the web, the width of the flange, and the depth of the i-th member, respectively. b_{fk} and b_{fj} represent the flange width of the k-th and j-th members, while d_n denotes the depth of the n-th member. The mathematical formulation of these constraints is represented by eq. (6), where m_i and m_n indicate the linear mass of the i-th and n-th profiles, respectively. Additionally, Nc refers to the total number of columns.

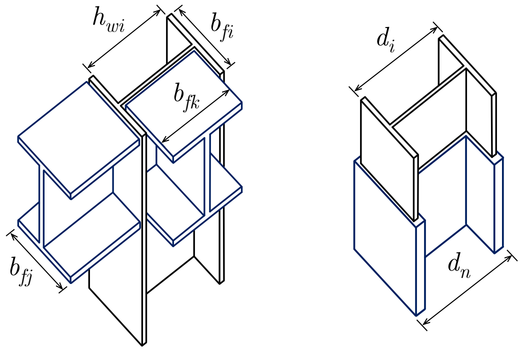


Figure 1. Column to beam and column to column geometric constraints.

$$\frac{d_i}{d_n} - 1 \leq 0; \quad \frac{m_i}{m_n} - 1 \leq 0; \quad \frac{b_{fk}}{h_{wi}} - 1 \leq 0; \quad \frac{b_{fj}}{b_{fi}} - 1 \leq 0 \quad i = 1, Nc \tag{6}$$

3 Materials and methods

Multi-objective optimization involves simultaneously considering conflicting objectives, resulting in a set of non-dominated solutions forming a Pareto front. Dominance concepts described by Deb [11] are employed to rank solutions, where solution A dominates solution B if it is better or equal in all objective functions or strictly better in at least one objective function. The study employs differential evolution-based algorithms, including the third evolution step of generalized differential evolution (GDE3) intrudoced by Kukkonen and Lampinen [12], success history-based adaptive multi-objective differential evolution (SHAMODE) presented by Panagant et al. [13], success history-based adaptive multi-objective differential evolution with whale optimization (SHAMODE-WO) enhanced by Mirjalili and Lewis [14], and multi-objective meta-heuristic with iterative parameter Distribution Estimation (MMIPDE) elaborated by Wansasueb et al. [15]. Dominance and crowding distance concepts are utilized to select high-quality solutions, while constraint-based non-dominated sorting handles constraints to rank feasible solutions. Additionally, the paper employs a predefined methodology, the Multi Tournament Decision Method (MTD), introduced by Parreiras and Vasconcelos [16], to extract solutions from the Pareto front, determining weighting coefficients based on the relative importance of each objective.

4 Numerical experiment

This paper presents a numerical experiment focused on tri-objective optimization of a six-story and four-bay spatial steel frame, with each story being three meters high and each bay five meters wide as shown in Figure 2. The frame includes column groups denoted as corncer columns (CC), outer columns (OC), and inner columns (IC). The arrangement of columns and beams (outer beams (OB) and inner beams (IB)) followed a repetitive pattern, with column groups repeating every two stories and beams recurring every three levels. The optimization aims to minimize the total weight and maximum horizontal displacement of the structure on the top story while maximizing its critical load factor for global stability. Load combinations include wind pressure acting in two orthogonal directions, with lateral displacements assessed accordingly. Specifically, the inner beams are subjected to a gravity load of 22.21 kN/m, while the outer beams carried a gravity load of 7.85 kN/m. Furthermore, nodal wind loads were applied to the structure, and Table 1 provides the specific values for these nodal wind loads.

Table 1. Wind loads acting on facade nodes (Corner Nodes - CN and Middle Nodes - MN).

Story	Height (m)	C.N. (kN)	M.N.(kN)	Story	Height (m)	C.N. (kN)	M.N.(kN)
1	3	4.73	9.46	4	12	5.84	11.67
2	6	4.94	9.88	5	15	6.16	12.31
3	9	5.45	10.89	6	18	3.22	6.44

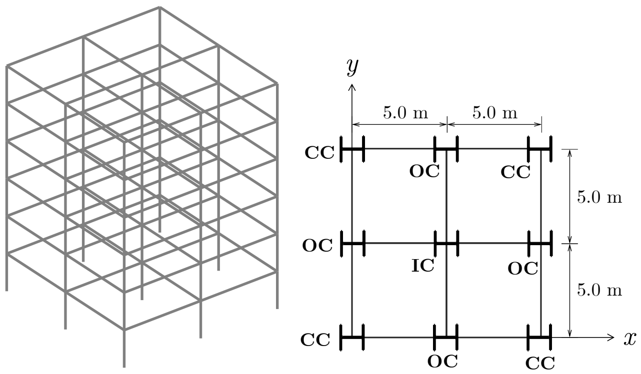


Figure 2. 6-story and 4-bay frame 3D and plain view.

To incorporate slab influence, a multi-freedom constraints approach described by Felippa [17] is utilized, assuming a rigid diaphragm behavior for the floor plane. Only the steel frames are considered in the optimization problem, excluding slab weight. The braced spatial steel frames in the experiments have pin-connected beam-to-column connections, while columns are rigidly connected by flanges and web. Shear studs are used to ensure lateral support of beams by the slab and prevent lateral torsional buckling.

The design variables candidate vector is divided into five subsets of integer indexes, determining the bracing system configuration, orientation of column cross-sections, and commercial steel profiles for columns, beams, and bracer elements. The search space for these subsets encompasses 29 rolled profiles for columns and 56 for beams. Figure 3 illustrates the candidate vector's design variable linking.

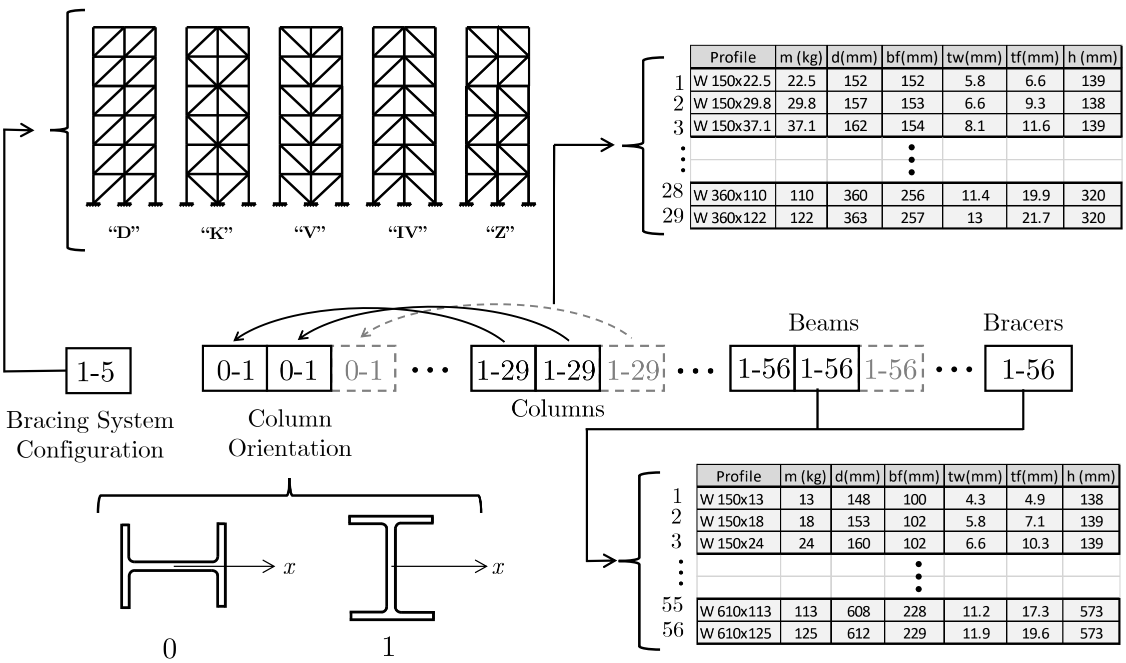


Figure 3. Candidate vector for a general problem, which includes the bracing system configuration, column orientation and commercial profiles variables.

The study utilizes four algorithms, each run independently ten times, for 500 generations, with a population of 50 candidate vectors. Additionally, six scenarios are analyzed using the MTD method, based on different combinations of importance weights for three objective functions: (i) the weight function ($W(\mathbf{x})$), (ii) the maximum horizontal displacement ($\delta_{max}(\mathbf{x})$), and (iii) the critical load factor ($\lambda_{cr}(\mathbf{x})$). The scenarios and their respective weight combinations are as follows: scenario 1: [1 0 0], scenario 2: [0 1 0], scenario 3: [0 0 1], scenario 4: [0.33 0.33 0.33], scenario 5: [0.6 0.2 0.2], and scenario 6: [0.2 0.6 0.2]. The extracted solutions, along with the commercial profiles assigned to each group, cross-sectional area orientations, bracing system configuration, and values for constraints and objective functions, are depicted in Table 2. Furthermore, the 3D Pareto Front with the extracted solutions is shown in Figure 4. To visually represent the extracted solutions, Figure 5 is provided.

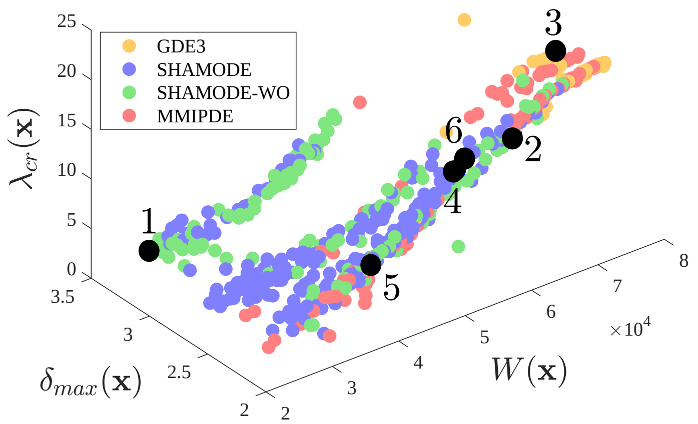
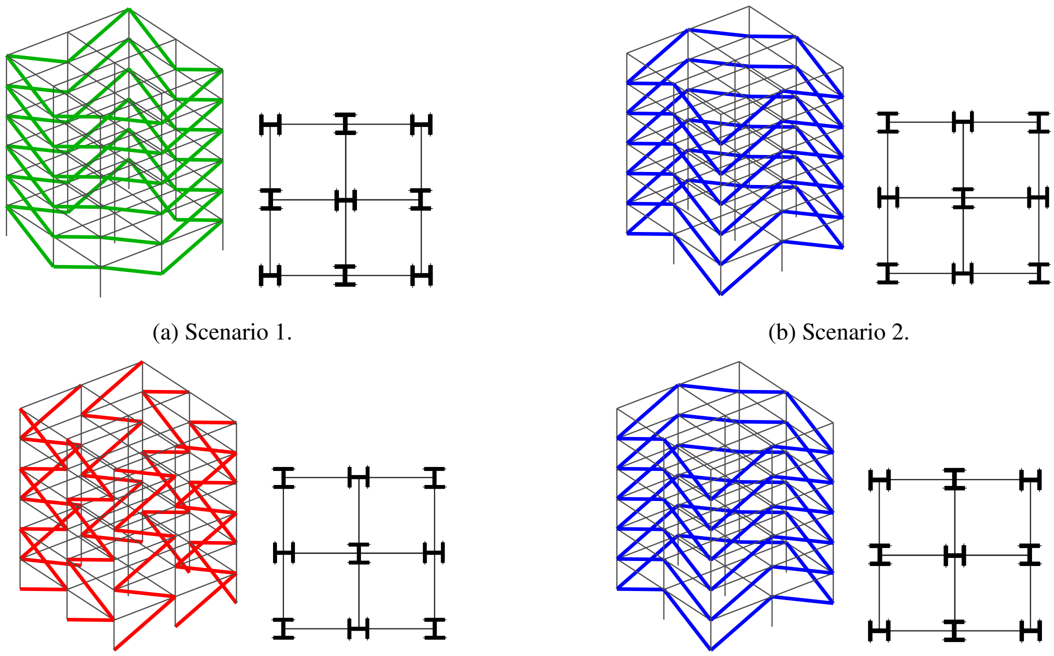


Figure 4. 3D Pareto front with the extracted solutions.

In the analysis of Table 2, it is essential to first note the bracing systems corresponding to each solution. For scenario 1, representing the solution with the lowest weight, the configuration denoted as 'V' was adopted, while 'IV' and 'Z' were preferred in cases where minimal horizontal displacement and maximum critical load were desired, respectively. Moreover, the 'IV' configuration was consistently employed in scenarios 4, 5, and 5. Another noteworthy observation pertains to the upper and lower bounds of each objective function within the set of non-dominated solutions from the PF. The solution with the least weight was characterized by $W(\mathbf{x}) = 24,285$ kg, the one with the smallest displacement was measured at $\delta(\mathbf{x}) = 2.0$ mm, and the highest critical load was attained with $\lambda_{cr}(\mathbf{x})$ = 20.57. Among the solutions extracted, the SHAMODE-WO algorithm accounted for two, GDE3 for one, and SHAMODE for three, with no solutions derived from MMIPDE.

Table 2. Best results found for F4_4, presenting details of the profiles assigned to each member group, constraints, and objective function values.

Bracing System	**V**	**IV**	**Z**	**IV**	**IV**	**IV**
Scenario	[1 0 0]	[0 1 0]	[0 0 1]	[.33 .33 .33]	[0.6 0.2 0.2]	[0.2 0.6 0.2]
Group (Stories)	W Profiles (Orientations for columns)					
CC (1-2)	310x107 (⟝⟞)	310x125 (**I**)	310x79 (**I**)	310x125 (⟝⟞)	310x125 (⟝⟞)	310x125 (⟝⟞)
CC (3-4)	150x22.5 (⟝⟞)	310x125 (**I**)	150x22.5 (**I**)	310x125 (⟝⟞)	310x125 (⟝⟞)	310x125 (⟝⟞)
CC (5-6)	150x22.5 (⟝⟞)	250x115 (**I**)	150x22.5 (**I**)	250x85 (⟝⟞)	200x46.1 (⟝⟞)	250x85 (⟝⟞)
OC (1-2)	360x91 (**I**)	360x122 (⟝⟞)	200x35.9 (⟝⟞)	360x122 (**I**)	360x122 (**I**)	360x122 (**I**)
OC (3-4)	200x52 (**I**)	360x122 (⟝⟞)	150x22.5 (⟝⟞)	310x117 (**I**)	310x117 (**I**)	310x117 (**I**)
OC (5-6)	200x52 (**I**)	250x115 (⟝⟞)	150x22.5 (⟝⟞)	250x62 (**I**)	250x89 (**I**)	250x89 (**I**)
IC (1-2)	360x122 (⟝⟞)	360x122 (**I**)	310x79 (**I**)	360x122 (⟝⟞)	360x122 (⟝⟞)	360x122 (⟝⟞)
IC (3-4)	360x91 (⟝⟞)	360x122 (**I**)	200x35.9 (**I**)	360x122 (⟝⟞)	360x122 (⟝⟞)	360x122 (⟝⟞)
IC (5-6)	310x79 (⟝⟞)	310x79 (**I**)	150x22.5 (**I**)	310x117 (⟝⟞)	310x117 (⟝⟞)	360x91 (⟝⟞)
OB (1-3)	250x17.9	460x106	250x17.9	360x79	200x26.6	360x79
OB (4-6)	250x17.9	360x79	250x17.9	360x64	200x31.3	360x64
IB (1-3)	360x32.9	360x79	360x32.9	310x52	310x44.5	360x64
IB (4-6)	310x44.5	360x79	360x32.9	360x79	360x44	360x79
BC (1-6)	150x24	150x24	150x24	150x24	150x24	150x24
Constraints and objective functions values						
$LRFD_{max}(\mathbf{x})$	0.98	0.85	0.95	0.83	0.83	0.84
$V_{max}(\mathbf{x})$	0.20	0.12	0.10	0.17	0.19	0.15
$d_{max}(\mathbf{x})$ (mm)	0.7	0.5	0.5	0.5	0.5	0.5
$\delta_{max}(\mathbf{x})$ (mm)	3.2	2.0	2.2	2.1	2.1	2.1
$\lambda_{cr}(\mathbf{x})$	3.76	15.72	20.57	14.39	8.18	15.13
$W(\mathbf{x})$ (kg)	24285	57995	67343	49526	36511	50705
Algorithm	SHAMODE-WO	SHAMODE-WO	GDE3	SHAMODE	SHAMODE	SHAMODE



(a) Scenario 1. (b) Scenario 2.

(c) Scenario 3. (d) Scenarios 4, 5 and 6.

Figure 5. Non-dominated extracted solutions 3D and plain view, detailing the bracing system configuration and column orientation.

5 Conclusions

In conclusion, this paper demonstrated the application of multi-objective optimization techniques to design efficient and cost-effective bracing systems for spatial steel frames. By considering conflicting objectives, such as minimizing maximum horizontal displacement, maximizing the critical load factor for global stability, and minimizing weight, the study provided a comprehensive understanding of the structural system's performance. The implementation of four multi-objective evolutionary algorithms based on differential evolution ensured efficient solutions, as validated in previous works. The utilization of the Multi Tournament Decision Method (MTD) allowed for the selection of the most suitable design based on the decision-maker's preferences and objective priorities. The results presented a set of non-dominated solutions forming a Pareto front, and each scenario showcased specific configurations and bracing system preferences. The SHAMODE-WO algorithm, GDE3, and SHAMODE were successful in yielding high-quality solutions, while MMIPDE did not contribute to the extracted solutions. Overall, this work contributes to the advancement of structural optimization in civil engineering, providing practical and cost-effective designs for tall buildings with various bracing configurations, and addressing the challenges posed by wind-induced horizontal displacements and global stability concerns in spatial steel frames. The findings provide valuable insights and opportunities for advancing research and employing multi-objective optimization techniques in an efficient manner to address intricate challenges in structural engineering.

Acknowledgements. The authors thank the Postgraduate Program in Civil and Environmental Engineering of PUC-Rio, Brazilian Agencies CNPq (grant 306186/2017-9) and CAPES for the financial support.

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] K. Price and others. *Differential evolution: a practical approach to global optimization*. Springer Science & Business Media, Berlin, 2006.

[2] J. Carvalho, E. Carvalho, D. Vargas, P. Hallak, B. Lima, and A. Lemonge. Multi-objective optimum design of truss structures using differential evolution algorithms. *Computers & Structures*, vol. 252, pp. 106544, 2021.

[3] Papadrakakis and others. Multi-objective optimization of skeletal structures under static and seismic loading conditions. *Engineering Optimization*, vol. 34, n. 6, pp. 645–669, 2002.

[4] R. Kicinger and T. Arciszewski. Multiobjective evolutionary design of steel structures in tall buildings. In *AIAA 1st Intelligent Systems Technical Conference*, Chicago. American Institute of Aeronautics and Astronautics, Inc., 2004.

[5] Kicinger and others. Evolutionary multiobjective optimization of steel structural systems in tall buildings. In S. Obayashi and others, eds, *Evolutionary Multi-Criterion Optimization*, pp. 604–618, Matsushima. Springer, 2007.

[6] Braga and others. Intervention cost optimization of bracing systems with multiperformance criteria. *Engineering Structures*, vol. 182, pp. 185–197, 2019.

[7] Khaledy and others. Multi-objective optimisation of steel moment frames subjected to blast. *International Journal of Structural Engineering*, vol. 10, n. 1, pp. 77–94, 2019.

[8] H. V. Burton, J. Y. Lee, S. Moradi, and S. Dastmalchi. Multi-objective performance-based design optimization of a controlled rocking steel braced frame system. In E. N. Farsangi and others, eds, *Resilient Structures and Infrastructure*, pp. 243–268. Springer, Singapore, 2019.

[9] ABNT. *NBR 8800: Projeto de estruturas de aço e de estruturas mistas de aço e concreto de edifícios.* ABNT Editora, Rio de Janeiro, Brasil, 2008.

[10] ANSI. *AISC 360-16 Specification for Structural Steel Buildings*. AISC, Chicago, USA, 2016.

[11] K. Deb. *Multi-objective Optimization using Evolutionary Algorithms*. John wiley & sons, Kanpur, 2001.

[12] S. Kukkonen and J. Lampinen. Gde3: The third evolution step of generalized differential evolution. In *2005 IEEE congress on evolutionary computation*, volume 1, pp. 443–450. IEEE, 2005.

[13] N. Panagant, S. Bureerat, and K. Tai. A novel self-adaptive hybrid multi-objective meta-heuristic for reliability design of trusses with simultaneous topology, shape and sizing optimisation design variables. *Structural and Multidisciplinary Optimization*, vol. 60, n. 5, pp. 1937–1955, 2019.

[14] S. Mirjalili and A. Lewis. The whale optimization algorithm. *Advances in engineering software*, vol. 95, pp. 51–67, 2016.

[15] K. Wansasueb, N. Pholdee, N. Panagant, and S. Bureerat. Multiobjective meta-heuristic with iterative parameter distribution estimation for aeroelastic design of an aircraft wing. *Engineering with Computers*, vol. , pp. 1–19, 2020.

[16] R. Parreiras and J. Vasconcelos. Decision making in multiobjective optimization aided by the multicriteria tournament decision method. *Nonlinear Analysis: Theory, Methods & Applications*, vol. 71, n. 12, pp. e191–e198, 2009.

[17] C. A. Felippa. Introduction to finite element methods. *University of Colorado*, vol. 885, 2004.