A phase-field model to simulate hydraulic fracture propagation

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Abstract. The term hydraulic fracturing is used in problems where the fracture starts and propagates due to the hydraulic load of the fluid inside the fracture. In petroleum engineering, hydraulic fracturing is mainly used to recover oil and gas from reservoirs. For structural engineering, a great interest is in the analysis of concrete structures that are exposed to hydraulic environments, such as dams, offshore platforms, and bridges. Given the relevance of the subject, the aim is to study the hydraulic fracturing process using a phase-field model. Phase-field models have been intensively studied in the last decade regarding their application in hydraulic fracturing problems, with emphasis on its variational formulation that allows the detection of the initiation, propagation, and nucleation of any number of cracks without the need for additional techniques. The adopted model has been studied and extended by several researchers over the years, who have shown its robustness and efficiency. It is considered a homogeneous and impermeable reservoir and an incompressible fluid, which is not explicitly modeled, only its effect on the crack is considered, indirectly, from the phase-field model. All implementation and numerical simulations were carried out in the INSANE (INteractive Structural ANalysis Environment), an open-source software developed at the Structural Engineering Department of the Federal University of Minas Gerais.

Keywords: Hydraulic fracture; Pressurized fractures, Phase field models of fracture; INSANE software

1 Introduction

Hydraulic fracturing is a complex phenomenon with applications in different fields. Its relevance has given rise to several proposals for theoretical, experimental, and numerical models for its study in the last decades. Adachi et al. [1] highlighted the complexity of modeling hydraulic fracturing, which should encompass at least three processes: solid modeling, which includes its mechanical deformation induced by the fluid pressure; the fluid mechanics represented by the fluid flow within the fracture; and crack initiation and propagation. In addition, several phenomena can influence the cracking process, such as proppant transport, leakoff, poroleastic effects, interaction between hydraulic and natural fractures, heterogeneity of the medium, among others (Chen et al. [2]).

In this context, the present work seeks to study the hydraulic fracturing problem from a phase-field model. The use of phase field models for fracture is a relatively recent study and has shown promising results. In the model, the crack is represented by a scalar variable, the phase-field variable ϕ, which assumes values equal to 1 in the crack and 0 in distant regions. There is a smooth transition between these values in a region whose band is defined by the parameter l_0, called the length scale. Wheeler et al. [3] pointed out the main positive aspects of the phase-field model as: no need to redefine the mesh; its methodology based on the principles of energy minimization which is capable of dealing with crack appearance, nucleation and propagation automatically; the possibility of representing complex fractures with joining and branching of multiple fractures; and its easy application in heterogeneous materials. All these characteristics make it an interesting model to be applied to the hydraulic fracture problem.

The first paper that registers the application of the phase-field model for hydraulic fracturing is the one by Bourdin et al. [4] who proposed an extension of the previous classical models to account for the effects of the fluid in the fracture. It is a simplified model that disregards thermal and chemical effects, the solid is considered impermeable and without porosity, the fluid is incompressible, the leak-off phenomenon is not addressed and the pressure load is constant along the fracture. In the following years, several works brought the application of the phase-field model to the study of hydraulic fracturing under different approaches. Some authors focused on the condition of irreversibility of the crack, others on the study of the problem in three dimensions, on considering the medium as poroelastic or heterogeneous or even on the interaction between hydraulic and pre-existing fractures.

The present work employs the initial model proposed by Bourdin et al. [4] in the study of the hydraulic fracturing process. Despite being a simple model, it is an interesting starting point in modeling this complex problem, which has been validated and expanded by several authors over the years. Next, the foundations of the adopted model and its implementation will be briefly presented. Numerical examples will be discussed in order to validate the implementation and present characteristics of the model.

2 Phase-field modeling of hydraulic fracture

The phase field models are related to the *variational approach to brittle fracture*, proposed by Francfort and Marigo [5]. The authors employ the idea presented by Griffith [6] in his celebrated paper which is based on energy minimization. The total functional energy E_t of a cracked solid body depends on the displacements field *u* and on the crack surface Γ. In phase-field models, the crack set Γ is not explicitly tracked. The crack is represented by the continuous variable phase-field ϕ, which will assume values from 0 to 1, depending on the damage level of the material. A smooth transition between these two values is assumed by the model, and the width of this transition region between the unbroken and fully-broken material is called length scale (l_0). In this sense, the total energy functional in the regularized domain of the phase-field model can be written as:

$$E_t(u, \phi) = \int_\Omega \psi(\varepsilon(u), \phi) d\Omega + \int_\Omega G_c \gamma(\phi, \nabla\phi) d\Omega - \int_\Omega b \cdot u \, d\Omega - \int_{\partial\Omega_N} t \cdot u \, d\Omega_N, \tag{1}$$

where Ω is the problem domain with external boundary $\partial\Omega$, decomposed into a part $\partial\Omega_D$ on which Dirichlet conditions are imposed and another part $\partial\Omega_N$ on which Neumann conditions are imposed. In addition, ψ is the strain energy density, ε is the linearized strain field $\varepsilon = (\nabla u + \nabla^T u)/2$, G_c is the critical energy release rate, γ is the crack surface density, *b* represents the body forces and *t* the surface forces.

When using phase-field model to study hydraulic fractures, it is necessary to consider the effects of pressure forces on the fracture surface. A schematic of the pressurized fracture problem is shown in Fig. 1 with emphasis on the presence of the pressure load due to the fluid p_f in the crack domain. In Fig. 1, $\vec{n}_\Gamma$ denote the outer unit normal to Ω on Γ. The signs of $+$ and $-$ were used as a way to indicate the opposite sides of the crack surfaces.

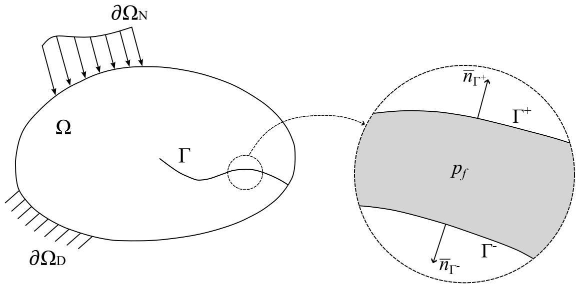


Figure 1. Representation of the fracture problem with pressure load.

This problem, studied from the variational approach, was first presented by Bourdin et al. [4] who proposed to add to the expression of the total energy functional eq. (1) a term referring to the work of the pressure load acting along each side of the crack W_{p_f}, which is approximated by:

$$W_{p_f} = \int_\Gamma p_f \, [[u]] \cdot \vec{n}_\Gamma \, d\Gamma \approx -\int_\Omega p_f u \cdot \nabla\phi \, d\Omega. \tag{2}$$

where $[[u]] = (u^+ - u^-)$ represents the displacement jump across the crack surface.

The approach used in this work considers that the fluid pressure on the fracture surface p_f is a problem datum. This type of problem is commonly referred to as pressure driven fracture propagation. Thus, the variables of interest are the displacement field and the phase-field values, which are obtained from the minimization of the total energy functional given by:

$$E_t(u,\phi) = \int_\Omega \psi(\varepsilon(u),\phi)d\Omega + \int_\Omega G_c\gamma(\phi,\nabla\phi)d\Omega - \int_\Omega b\cdot u\, d\Omega - \int_{\partial\Omega_N} t\cdot u\, d\Omega_N + \int_\Omega p_f u\cdot\nabla\phi\, d\Omega, \quad (3)$$

what can be summarized by

$$(u,\phi) = \text{ arg min } E_t \begin{cases} u \text{ kinematically admissible} \\ \phi|\phi^t \subset \phi^{t+\Delta t} \end{cases} \quad (4)$$

submitted to the following conditions

$$\nabla\sigma + b = \mathit{0} \text{ in the domain } \Omega, \quad (5a)$$

$$\sigma\cdot\vec{n} = t \text{ on the boundary } \partial\Omega_N, \quad (5b)$$

$$\sigma\cdot\vec{n}_\Gamma^\pm = p_f\vec{n}_\Gamma^\pm \text{ on the faces of the crack } \Gamma^\pm, \quad (5c)$$

with σ being the stress tensor.

3 Implementation

All implementation and numerical simulations of this work were carried out in the INSANE software (INteractive Structural ANalysis Environment). The INSANE system is a segmented and expandable software developed at the Department of Structural Engineering at the Federal University of Minas Gerais, based on Object Oriented Programming.

In solving the phase-field problem, it is expected to find the solution both for the kinematically admissible displacement field and for the phase-field variable. The phase-field solvers can be classified into two groups: the monolithic and staggered solvers. This work adopts a staggered solver, where the global convergence of the problem is obtained by alternating the solution of the displacement and phase-field equations until a global convergence is reached.

In hydraulic fracture simulations based on the phase-field model, a concern is to ensure that the irreversibility and boundedness conditions remain met even with the influence of a pressure load applied in the fracture domain. In this sense, a staggered bound-constrained solver was adopted. This solver was implemented in the INSANE by Bayão et al. [7] and ensures that the phase-field equation solution obeys the relation $0 \leq a_{I,n} \leq a_{I,n+1} \leq 1$, where $a_{I,n}$ is the nodal phase-field value for the node I in the step n.

The implementation is based on inserting, at the beginning of each analysis step, a pressure load vector to the external load vector. The pressure load is applied to all elements of the finite element mesh in which a phase-field gradient is identified, considering the phase-field converged in the previous step. Thus, although the model considers a constant pressure value p_f, the pressure load vector varies according to crack propagation.

4 Numerical examples

This section will present some numerical examples employing the formulation proposed by Bourdin et al. [4] in the study of hydraulic fracturing using the variational approach of the phase-field. The three simulations were carried out considering a model subject to a plane deformation state, with unitary thickness. In addition, a local tolerance of 10^{-5} and a global tolerance of 10^{-4} were adopted. In all examples, $\alpha = \phi^2$ and $g = (1-\phi)^2$, both quadratic functions proposed by Bourdin et al. [8] for geometric crack function and for energetic degradation function, respectively, were used.

4.1 Static fracture: Sneddon's 2D benchmark

The first example is based on the theoretical calculations of Sneddon and Lowengrub [9]. The problem is presented as an infinite domain with a prescribed initial crack. The geometric data from Wheeler et al. [3] was adopted, with $\Omega = (0,4)^2$, as it is presented in Fig. 2. The initial crack is defined in the region limited by $(1.8, 2.2)$

x $(2-h, 2+h)$ by assigning $\phi = 1$ to all nodes within this limit. A constant internal pressure $p_f = 10^{-3}$ was assumed. The material parameters are: Young's modulus $E = 1.0$, Poisson's ration $\nu = 0.2$, fracture toughness $G_c = 1.0$, and length scale $l_0 = 0.044$. All dimensionless parameters.

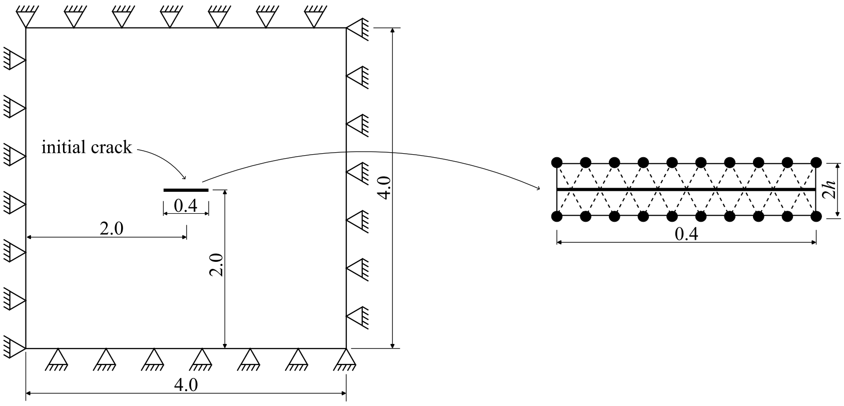


Figure 2. Problem setting of Sneddon's 2D benchmark.

In this example, the isotropic constitutive model was considered. Three different meshes were analyzed, in which the element size in the fracture region are $h = 0.011$ $(l_0/h = 4)$, $h = 0.022$ $(l_0/h = 2)$ and $h = 0.044$ $(l_0/h = 1)$. A comparison of the crack opening displacements (COD) results found in this work with the analytical values obtained by Sneddon is presented in Fig. 3 (a). In addition, Fig. 3 (b) shows a comparison of the results of this work with the numerical simulations from Wheeler et al. [3].

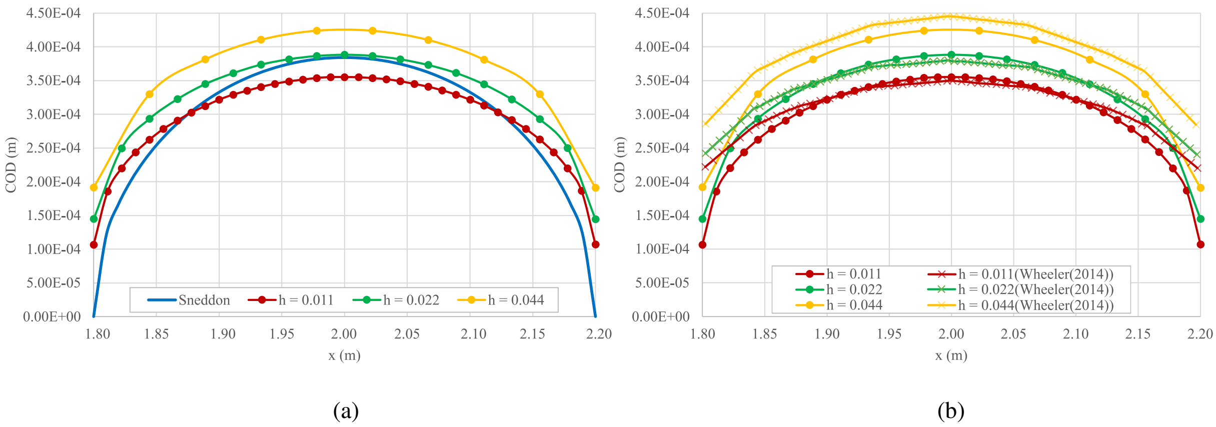


Figure 3. (a) Crack opening displacement for different mesh sizes. Sneddon's blue line corresponds to his analytical solution. (b) Crack opening displacement (Comparison with the results presented by Wheeler et al. [3]).

As the cracked region depends on the value of h, the COD decreases with mesh refinement, as expected. The approximation of the opening displacement is reasonable for the three meshes studied, reaching values closer to the analytical ones for the mesh with $l_0/h = 2$. It is possible to notice a convergence of the displacements in the regions close to the crack tips with mesh refinement. Furthermore, a great agreement between the result obtained by this work and that presented by Wheeler et al. [3] can be noticed.

4.2 Single propagating fracture

The second example is inspired by the one presented by Mikelić et al. [10] and has the same geometry and material parameters as the previous one, with the aim of evaluating the fracture propagation due to a step-wise increasing pressure. The constitutive model by Amor et al. [11] is adopted, and pressure is defined by $p_f = \Delta t\, p_0$, with time step size $\Delta t = [1, 40]$, which represents the analysis steps, and $p_0 = 10^{-1}$.

A comparison of the total crack length at each analysis step with those presented by Mikelić et al. [10] is shown in Fig. 4. Despite the differences, it can be seen that the pattern is the same. The crack length only begins to change at the end of the analysis and has a sharp increase in the final few steps. In relation to the meshes, it is noticed that the crack of the finer meshes propagates in advance in relation to the coarser meshes.

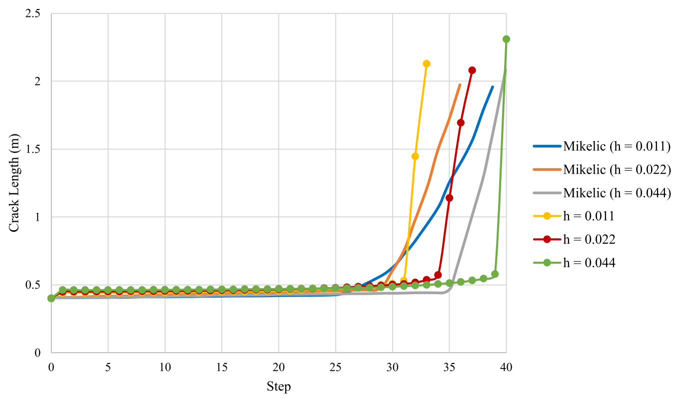
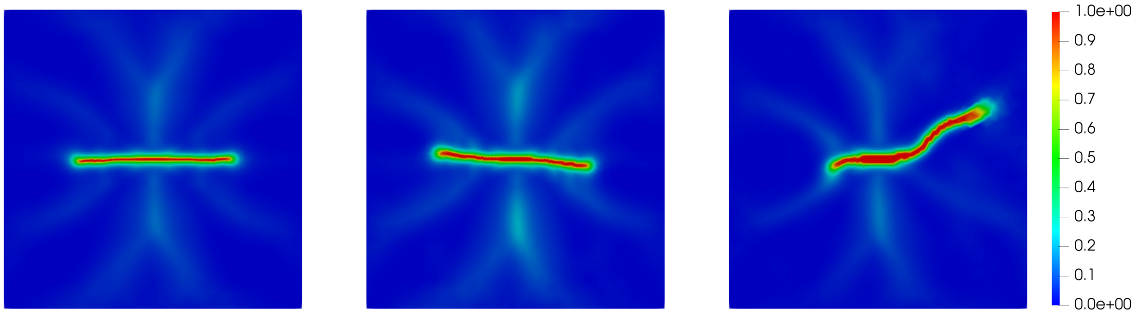


Figure 4. Total crack length for each analysis step for different mesh sizes. Curves flagged as 'Mikelic' are presented by the authors in Mikelić et al. [10].

Figure 5 shows the phase-field distribution in the final step for the three meshes analyzed. It is observed that the most linear and regular crack is obtained for the most refined mesh, although the intermediate mesh, with $l_0/h = 2$, still presents good results. The coarse mesh, which has $h = l_0$, presents a crack pattern notably different from the others, with tortuosity and loss of symmetry.



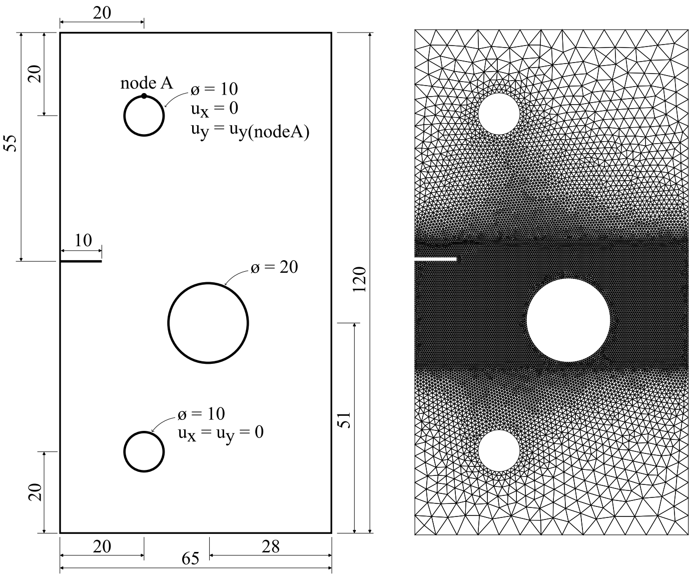
(a) (b) (c)

Figure 5. Phase-field contour plot. (a) $h = 0.011$. (b) $h = 0.022$. (c) $h = 0.044$.

4.3 Stretching of notched plate with a hole

The last example seeks to qualitatively analyze the influence of the presence of pressure load on fracture propagation. The model consists of a plate with an off-center hole and two smaller holes where initial conditions are imposed. The geometry and boundary conditions are presented in Fig. 6, as well as the mesh used in the analysis. This example is based on the one provided by Sargado et al. [12]. The vertical displacement of the node A is controlled during the analysis, ranging from 0.0025 mm to 1.5 mm. The material parameters are: Young's modulus $E = 5983$ MPa, Poisson's ration $\nu = 0.22$, fracture toughness $G_c = 2.28$ N/mm, and length scale $l_0 = 1$ mm. The constitutive model by Wu [13] was considered. Two scenarios will be analyzed: one that does not consider the presence of pressure load on the crack surface; and another that considers the presence of this stimulus, being $p_f = 0.1$ MPa.

The phase-field profile for different moments of the analysis are presented in Fig. 7. It is possible to notice that the propagation of the fracture always occurs in advance in the model that considers a hydraulic pressure load,



(a) (b)

Figure 6. Notched plate with a hole. (a) Problem setting (dimensions in mm). (b) T3 Mesh.

showing the influence of this pressure. Furthermore, when the pressure load is considered, the crack propagates more abruptly and in a more unstable manner.

5 Conclusions

The purpose of this work was to present and discuss a phase-field model for hydraulic fractures presented by Bourdin et al. [4]. The numerical examples were able to validate the implementation of the model through comparison with analytical and numerical results from other authors. It is valid to say that the model was able to efficiently produce results regarding problems with pressurized internal fractures as well as reproduce problems of classical elasticity considering in addition to external loads, also the stimulus due to the fluid in the fracture domain. From the numerical simulations presented, it can be concluded that the studied model is characterized by a fast crack propagation that, many times, can occur in an unstable way. In addition, the presence of fluid pressure, despite not having a significant impact on the path of the crack, significantly accelerates its propagation, making it essential to consider it in cases where the presence of fluid in the crack region may occur.

Acknowledgements. The authors gratefully acknowledge the support from the Brazilian research agencies CAPES (*Coordenação de Aperfeiçoamento de Pessoal de Nível Superior*), FAPEMIG (*Fundação de Amparo à Pesquisa do Estado de Minas Gerais*; Grant PPM-00747-18) and CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico; Grant 316240/2021-4).

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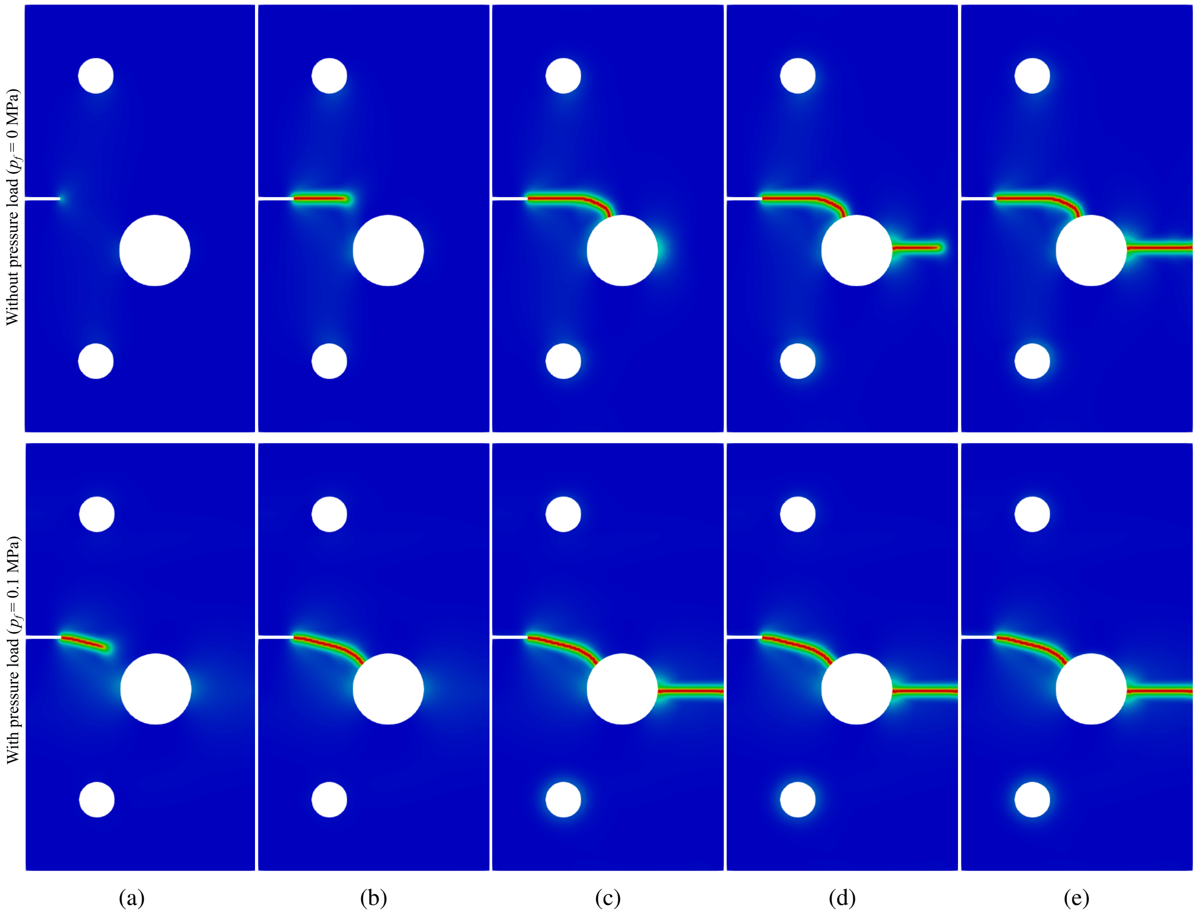

Figure 7. Crack phase-field at displacements: (a) 0.2 mm. (b) 0.2675 mm. (c) 0.655 mm. (d) 0.765 mm. (e) 1.5 mm.

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