Radial Point Interpolation Meshless Methods for applications in Mechanics and Biomechanics

Jorge Belinha[1,2]

[1]*INEGI – Institute of Science and Innovation in Industrial and Mechanical Engineering*
Campus da FEUP, R. Dr. Roberto Frias 400, 4200-465 Porto
[2]*Department of Mechanical Engineering, School of Engineering, Polytechnic University of Porto (ISEP)*
R. Dr. António Bernardino de Almeida 431, 4249-015 Porto
job@isep.ipp.pt

Abstract. Computational mechanics emerged alongside the advent of the first computers and has undergone significant development since then. Presently, the literature describes numerous advanced numerical techniques for discretization that are capable of efficiently conducting structural analyses. The finite element method (FEM) was one of the earliest discrete numerical methods to be developed and remains the most popular technique among the computational mechanics research community. FEM is known for its ease of programming, robustness, and ability to provide reasonable approximations. However, despite its efficiency and success, the last decade of the previous century witnessed the emergence of new, mature advanced discretization techniques known as meshless methods. In contrast to FEM, which discretizes the problem domain using a structured element mesh comprising a grid of nodes, meshless methods discretize the domain with an unstructured nodal distribution. Consequently, meshless methods enable the creation of discrete geometric models directly from medical images or CAD geometries. This advantage in meshing is a valuable asset in the fields of computational mechanics and biomechanics. This work presents a brief description of the evolution of advanced discretization meshless techniques in computational mechanics and biomechanics, highlighting the most significant ones and their formulations. Furthermore, it presents several demanding numerical applications in computational mechanics and biomechanics developed by the author and his research team. These applications encompass the analysis of transient behavior in bone tissue, the study of elastoplastic behavior in metallic and biological tissues, examination of blood fluid flow, and investigation of the structural response of implants and bio-structures. The results obtained using meshless methods are compared with FEM solutions to provide insights into the efficiency and accuracy of meshless techniques.

Keywords: Computational Mechanics, Computational Biomechanics, meshless methods, radial point interpolation meshless methods

1 Introduction

Within classical computational mechanics and biomechanics, three essential phases play a crucial role: modulation, simulation, and analysis. Accomplishing these phases requires the use of discretization techniques. This iterative design process is intricately linked to the chosen numerical methodology. As the research community continually strives to achieve the most accurate numerical reproduction of biological phenomena, numerous numerical methods have emerged, each capable of effectively handling the aforementioned design phases [1].

Among the various numerical approaches described in the literature, there are fundamental differences, leading to distinct numerical performances. Presently, the finite element method (FEM) stands as the most popular discretization technique [2]. The FEM replicates the physical domain through a virtual geometric model constructed with non-overlapping finite elements, ensuring a continuum within the model. However, employing the FEM requires an extensive pre-processing phase to construct a well-balanced element mesh.

The performance of the FEM heavily relies on the quality of the model's mesh, and any mesh modifications or refinements during the analysis introduce additional computational costs, which can be a significant drawback. Despite these challenges, the FEM remains a widely-used and effective method for tackling computational mechanics and biomechanics problems in the current literature.

2 Meshless methods

Recently, meshless methods have earned significant attention within the computational mechanics scientific community as viable solutions for solving partial differential equations. In contrast to the traditional FEM, meshless methods approximate field functions within an influence-domain rather than an element, enabling arbitrary node distribution [1, 3].

The influence-domain represents a crucial geometrical concept in meshless methods, facilitating nodal connectivity enforcement [1]. Unlike FEM, where elements must not overlap, in meshless methods, the influence-domains can and must overlap to ensure nodal connectivity.

Numerical methods in this domain can be classified into three fundamental parts: the field approximation (or interpolation) function, the formulation used, and the numerical integration scheme. The literature offers various options for approximation functions, including Taylor approximation, moving least-square approximation, reproducing kernel approximation, and hp-cloud approximation function. Additionally, interpolation functions like polynomial interpolation, parametric interpolation, radial interpolation, and Sibson interpolation are popular alternatives.

The concept of ”support domain” in FEM is replaced by the notion of ”element,” containing information about connectivity with neighboring elements. In meshless methods, nodal connectivity substitutes element connectivity, and the ”influence-domain” concept is employed to determine connectivity for each node within the node cloud discretizing the problem domain.

Regarding formulation, numerical methods fall into two categories: strong formulation and weak formulation. The strong formulation directly employs partial differential equations to obtain solutions, while the weak formulation uses variational principles to minimize the residual weight of the governing differential equations [1]. Both FEM and most meshless methods utilize weak formulations, as they accommodate differential equations with non-differentiable solutions common in biomechanical phenomena.

The third fundamental part is numerical integration, necessary for obtaining the integral of the residual weight of differential equations. Common schemes involve background meshes with integration points covering the entire problem domain. Integration points possess an influence area (or volume) and weight, representing the theoretical infinitesimal mass portion in the integral expression. Meshless methods can be considered ”truly meshless” if the integration points’ locations and weights are obtained directly from the nodal distribution. Point collocation and nodal integration techniques enable truly meshless methods [1, 4–9], allowing for nodal clouds directly obtained from CAD parts in structural mechanical analyses or medical CAT scans and MRI in biomechanics, where nodal connectivity, integration points, and shape functions are determined solely based on nodal spatial information. Notably, truly meshless methods can even identify distinct biomaterials from medical images and directly affect nodes with corresponding material properties.

Due to their interpolating properties, truly meshless methods hold substantial appeal in computational mechanics. Subsequent subsections explore relevant works demonstrating the efficiency of these techniques.

3 Meshless methods in structural mechanics

Subsequent to an initial foray into programming and the development of the Element-Free Galerkin Method (EFGM) [10], designed for the analysis of elastostatic and nonlinear elastoplastic behaviors within thick plates and laminates employing equivalent single-layer theories [4, 11, 12], the author embarked on the exploration of novel meshless methodologies. The overarching objective was the creation of genuinely interpolation-based meshless methods, underpinned by the ability to leverage the boundary imposition techniques previously harnessed by the Finite Element Method (FEM). This initiative resulted in the formulation of two such interpolation meshless methods: the Natural Neighbour Radial Point Interpolation Method (NNRPIM) [4] and the Natural Radial Element Method (NREM) [5]. The NNRPIM seamlessly integrates the shape function methodology of Radial Point Interpolators (RPI) [1, 13, 14] with the natural neighbour concept [6, 7]. Importantly, the NNRPIM exhibits exceptional accuracy, enabling the acquisition of numerical solutions that closely align with their analytically exact counterparts. The NREM represents an enhanced iteration of the NNRPIM, where shape functions are constructed using a notably reduced number of nodes. The formulation of nodal connectivity and integration schemes again draws from the concept of natural neighbours. Notably, the NREM can be seamlessly coupled with FEM [5].

Subsequent to this foundational work, both NNRPIM and NREM underwent expansion to accommodate elastostatic analyses of thick plates and laminates, embracing the First Order Shear Deformation Theory (FOSDT) [15–17], as well as other high-order shear deformation theories [18]. Specifically concerning thick laminates within the FOSDT framework, a performance comparison of these two interpolation meshless methods was conducted against high-order finite element formulations and the EFGM [6]. Additionally, for beam structures, the NNRPIM was applied to scrutinize the behavior of thick laminated beams under various loading conditions [19, 20].

Subsequently, an innovative shell-like 3D formulation was conceived for NNRPIM, enabling the simulation of thin structures subject to transverse loads [21, 22]. This formulation found application in resolving the mechanical characteristics of thin isotropic and laminated shells.

The ambit of nonlinear dynamic analysis for structures was also a salient focus of NNRPIM's application. Preliminary undertakings encompassed the dynamic analysis of both 2D and 3D structures [23, 24]. This trajectory led to the extension of NNRPIM to encompass the dynamic analysis of laminated plates under transverse loads, facilitated by the utilization of an unconstrained high-order shear deformation theory [25].

Leveraging the inherent adaptability of NNRPIM, a nonlinear geometric algorithm was implemented to facilitate large strain analysis [26], thereby validating NNRPIM's heightened performance in addressing demanding nonlinear scenarios. Beyond accommodating substantial deformations, NNRPIM's domain expanded to encompass the nonlinear analysis of elastoplastic anisotropic materials demonstrating both elastic and hardening anisotropy [27], as well as continuum damage mechanics for brittle materials such as concrete and ceramics [9, 28–30]. Noteworthy research endeavors on NNRPIM effectively validated its efficacy via experimental data, thereby substantiating its capacity to accurately simulate the elastoplastic response of materials like steel alloys [31] and aluminum alloys [31].

Furthermore, NNRPIM exhibited notable applicability across diverse domains within computational mechanics, including fracture mechanics [32–36]. In these contexts, NNRPIM's capability to achieve solutions closely approximating experimental data or well-established techniques, including the eXtended-FEM (XFEM) [37], was demonstrated. Notably intricate examples were investigated, involving mixed crack modes, double fractures [32, 33], and the presence of material discontinuities such as small and large holes [34, 35]. A more recent development saw NNRPIM's application in the multi-scale analysis of composite materials, enabling the derivation of homogenized material properties at the microscale for macroscopic utilization [38, 39].

4 Meshless methods in biomechanics

Meshless methods offer a multitude of advantages over the Finite Element Method (FEM), a prime example being their proficiency in handling remeshing. This characteristic allows for explicit simulation of fluid flow phenomena such as hemodynamics, swallowing, and respiration, as well as accommodating significant distortions in soft materials like internal organs, muscles, tendons, and skin. Furthermore, the inherent smoothness and precision exhibited by solution fields obtained through meshless methods, encompassing displacements, stresses, and strains, prove invaluable in forecasting the remodeling of biological tissues and predicting the rupture or damage of biomaterials. Recent investigations highlight the enhanced efficacy of meshless methods in conjunction with medical imaging modalities like CAT scans and MRI, surpassing the capabilities of FEM [40, 41]. This observation has been further substantiated via the application of the NNRPIM [42]. Given their benefits, meshless methods emerge as a compelling alternative to FEM within the realm of biomechanics [43].

Doweidar et al.'s work [44] substantiates the superiority of meshless methods in biomechanical scenarios characterized by substantial strains, exemplified by the simulation of human lateral collateral ligaments and knee joints. A notable extension by Zhang et al. [45] involves the adaptation of meshless methods to nonlinear explicit dynamic analysis of brain tissue response. Subsequent use of NNRPIM in studying the brain's reaction to abrupt impacts [42, 46] reaffirms the meshless approach's accuracy in addressing intricate nonlinear hyperelastic biomaterial dynamics.

The application of meshless methods in hemodynamics is widespread. Literature reveals instances where these techniques model deformable red blood cells in flowing plasma [47] and investigate the influence of red blood cells on primary thrombus formation [48]. Particle-based meshless methodologies extend their prowess to studying other biofluids, such as endolymph within the inner ear [49], enabling insights into vestibular system disorders [50, 51]. The NNRPIM complements these studies by offering insight into solid structure dynamics, particularly in contexts like the cupula encompassed by endolymph [52].

Within computational biomechanics, meshless methods have garnered acclaim in predicting bone tissue remodeling [53]. An early milestone in this domain was set by Liew et al. [54], followed by successful applications by others in simulating bone tissue remodeling [55, 56]. Belinha et al. [57, 58] presented a novel bone tissue remodeling algorithm founded upon the accuracy of meshless methods. This methodology closely approximated clinical X-ray images of natural and implanted bones [33, 59, 60], even extending its purview to predicting dental biomechanical behavior with and without implants [60–64].

In the sphere of bone tissue simulation, a harmonious integration of the NNRPIM and RPIM with the fabric tensor concept gave rise to a bone tissue homogenization procedure [65, 66]. This technique facilitates multiscale analyses with remarkable efficiency, outperforming conventional homogenization approaches in terms of computational time. This efficiency is further accentuated by the inherent accuracy of both meshless methods, enabling precise results with a reduced node count.

The simulation of nonlinear biological material behavior is another domain where meshless methods excel. Given the iterative nature of such problems, the accuracy and smoothness of stress and strain fields assume paramount importance for achieving stable and robust solutions. Belinha and colleagues have advanced nonlinear elasto-plastic constitutive models for emulating the biomechanical responses of bone structures [64] and atherosclerotic plaque tissue [67], seamlessly integrating these models with meshless methods. The outcomes of these simulations offer precise predictions of failure in these biological structures. Notably, meshless methods also find application in simulating the behavior of endolymph, a pivotal component of the vestibular system that plays a crucial role in vertigo.

5 FEMAS

The Finite Element Method (FEM) stands as the preeminent discretization technique within the realm of computational mechanics. The widespread success of FEM is attributed to its robustness, accuracy, and inherent programming simplicity. Such attributes are such that even undergraduate engineering students can adeptly develop FEM programs. Furthermore, the availability of a plethora of FEM software, many of which are freely accessible, and the prevalence of student versions within commercial offerings extend opportunities for both students and researchers to harness its capabilities. In stark contrast, the landscape for meshless methods assumes a distinct complexion. Despite their high accuracy, meshless methods are considerably more intricate than FEM and pose greater programming challenges. Moreover, the availability of software catering to meshless methods remains scarce, often beyond the reach of the broader computational mechanics community.

In response to this challenge, the author has dedicated the past five years to crafting a comprehensive and user-friendly meshless software solution: the Finite Element method and Meshless Analysis Software (FEMAS – cmech.webs.com). Distinctive to FEMAS is its capacity to empower students and researchers through an intuitive graphical user interface (GUI). The software has been endowed with a diverse array of capabilities, encompassing static linear-elastic and elasto-plastic material analyses, crack opening path prognostication, bone tissue remodeling simulations, free vibration and buckling analyses, analyses of viscoelastic and viscoplastic fluid flows at low velocities, thermo-mechanical analyses, and structural optimization studies.

Furthermore, the software encompasses a spectrum of finite element formulations for both 2D and 3D analyses. Additionally, the inclusion of three meshless methods – the Radial Point Interpolators Method (RPIM), the Natural Neighbour Radial Point Interpolation Method (NNRPIM) and the Smoothed Particle Hydrodynamics (SPH) – reinforces the versatility of the toolset. By harnessing this software, users are afforded the ability to employ various numerical approaches and undertake comparative studies of diverse analyses/formulations. Importantly, the platform permits users to seamlessly incorporate their own routines, thereby reducing the learning curve and fostering accelerated scientific progress.

6 Conclusion

A substantial body of research literature underscores the compelling numerical efficacy of meshless methods [1]. Drawing from these scholarly contributions and informed by the author's personal insights, a near-term horizon portends the substitution of conventional numerical methodologies, such as Finite Element Method (FEM), with meshless methods or their advanced counterparts in both computational mechanics and biomechanical analyses. A wealth of unexplored mechanical computational domains exists, encompassing diverse physical behaviors encompassing electrical, magnetic, chemical, thermal, biological, and fluid-solid interactions. Significantly, meshless methods have thus far demonstrated their adeptness in offering precise solutions to all these physics-laden conundrums. The amalgamation of the discretization versatility intrinsic to this innovative methodology with its exceptional accuracy holds the promise of shattering existing scientific barriers. It has the potential to unveil novel therapeutic avenues within biomechanics and facilitate the prognostication of pathological states.

In the realm of biomechanics, a future outlook anticipates meshless methods contributing to: (1) Guiding surgical interventions through the governance of real-time virtual numerical models, thereby aiding surgeons; (2) Enabling a myriad of in-silico experiments that assess the impacts of new pharmaceutical agents at both the micro-scale (cellular level) and macro-scale (muscles, bones, tendons, etc.); (3) Prognosticating the regeneration trajectories of soft and hard tissues, thus facilitating the optimal selection of physical or chemical therapies; (4) Predicting the post-scan health status or failure susceptibility of biological structures post a comprehensive CAT scan; (5) Designing patient-specific instrumentation or prosthetics, intrinsically tailored to the unique physiognomy of each patient.

Within the realm of meshless methods, the boundaries of computational mechanics and biomechanics remain boundlessly expansive. The constraints are delimited solely by our creativity and the necessities that underpin them.

Acknowledgements. The author truly acknowledges the funding provided by Ministério da Ciência, Tecnologia e Ensino Superior - Fundação para a Ciência e a Tecnologia (Portugal), under project UIDB/50022/2020

Authorship statement. The author hereby confirm that he is the sole liable person responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the author, or has the permission of the owners to be included here.

References

[1] J. Belinha. *Meshless Methods in Biomechanics - Bone Tissue Remodelling Analysis, Vol.16. Lecture Notes in Computational Vision and Biomechanics.* Springer Netherlands, 2014.
[2] O. C. Zienkiewicz and R. L. Taylor. *The Finite Element Method, 4th ed.* McGraw-Hill London, 1994.
[3] T. Belytschko, Y. Krongauz, D. Organ, M. Fleming, and P. Krysl. Meshless methods: An overview and recent developments. *Comput. Methods Appl. Mech.*, vol. 139, n. 1-4, pp. 3–47, 1996.
[4] L. Dinis, R. N. Jorge, and J. Belinha. Analysis of 3d solids using the natural neighbour radial point interpolation method. *Comput. Methods Appl. Mech. Eng.*, vol. 196, n. 13-16, pp. 2009–2028, 2007.
[5] J. Belinha, L. Dinis, and R. N. Jorge. The natural radial element method. *Int. J. Numer. Methods Eng.*, vol. 93, n. 12, pp. 1286–1313, 2012a.
[6] N. Sukumar, B. Moran, and T. Belytschko. The natural element method in solid mechanics. *Int. J. Numer. Methods Eng.*, vol. 43, n. 5, pp. 839–887, 1998.
[7] N. Sukumar, B. Moran, A. Y. Semenov, and V. V. Belikov. Natural neighbour galerkin methods. *Int. J. Numer. Methods Eng.*, vol. 50, n. 1, pp. 1–27, 2001.
[8] A. Ferreira. A formulation of the multiquadric radial basis function method for the analysis of laminated composite plates. *Compos. Struct.*, vol. 59, n. 3, pp. 385–392, 2003.
[9] B. Farahani, J. Belinha, F. A. Pires, and P. Moreira. A meshless approach to non-local damage modelling of concrete. *Eng. Anal. Bound. Elem.*, vol. 79, n. -, pp. 62–74, 2017a.
[10] T. Belytschko, Y. Lu, and L. Gu. Element-free galerkin methods. *Int. J. Numer. Methods Eng.*, vol. 37, n. 2, pp. 229–256, 1994.
[11] J. Belinha and L. Dinis. Elasto-plastic analysis of plates by the element free galerkin method. *Eng. Comput.*, vol. 23, n. 5, pp. 525–551, 2006a.
[12] J. Belinha and L. Dinis. Nonlinear analysis of plates and laminates using the element free galerkin method. *Compos. Struct.*, vol. 78, n. 3, pp. 337–350, 2007.
[13] J. Wang and G. Liu. A point interpolation meshless method based on radial basis functions. *Int. J. Numer. Methods Eng.*, vol. 54, n. 11, pp. 1623–1648, 2002a.
[14] J. Wang and G. Liu. On the optimal shape parameters of radial basis functions used for 2-d meshless methods. *Comput. Methods Appl. Mech. Eng.*, vol. 191, n. 23-24, pp. 2611–2630, 2002b.
[15] J. Belinha and L. Dinis. Analysis of plates and laminates using the element-free galerkin method. *Comput. Struct.*, vol. 84, n. 22–23, pp. 1547–1559, 2006b.
[16] J. Belinha, L. Dinis, and R. Jorge. Analysis of plates and laminates using the natural neighbour radial point interpolation method. *Compos. Struct.*, vol. 103,, n. -, pp. 50–67, 2013a.
[17] J. Belinha, L. Dinis, and R. Jorge. Analysis of thick plates by the natural radial element method. *Int. J. Mech. Sci.*, vol. 76,, n. -, pp. 33–48, 2013b.
[18] L. Dinis, R. Jorge, and J. Belinha. An unconstrained third-order plate theory applied to functionally graded plates using a meshless method. *Mech. Adv. Mater. Struct.*, vol. 17,, n. 2, pp. 108–133, 2010a.
[19] S. Moreira, J. Belinha, L. Dinis, and R. N. Jorge. Analysis of laminated beams using the natural neighbour radial point interpolation method. *Rev. Int. Metod. Numer. para Calc. Y Disen. en Ing.*, vol. 30,, n. -, pp. 108–120, 2014a.
[20] S. Sadek, J. Belinha, M. Parente, R. N. Jorge, de J. Sá, and A. Ferreira. The analysis of composite laminated beams using a 2d interpolating meshless technique. *Acta Mech. Sin.*, vol. 34,, n. -, pp. 99–116, 2018.
[21] L. Dinis, R. Jorge, and J. Belinha. Composite laminated plates: A 3d natural neighbor radial point interpolation method approach. *J. Sandw. Struct. Mater.*, vol. 12,, n. 2, pp. 119–138, 2010b.

[22] L. Dinis, R. Jorge, and J. Belinha. A 3d shell-like approach using a natural neighbour meshless method: Isotropic and orthotropic thin structures. *Compos. Struct.*, vol. 92,, n. 5, pp. 1132–1142, 2010c.
[23] L. Dinis, R. Jorge, and J. Belinha. The natural neighbour radial point interpolation method: dynamic applications. *Eng. Comput.*, vol. 26,, n. 8, pp. 911–949, 2009a.
[24] L. Dinis, R. Jorge, and J. Belinha. A natural neighbour meshless method with a 3d shell-like approach in the dynamic analysis of thin 3d structures. *Thin-Walled Struct.*, vol. 49,, n. 1, pp. 185–196, 2011a.
[25] L. Dinis, R. Jorge, and J. Belinha. Static and dynamic analysis of laminated plates based on an unconstrained third order theory and using a radial point interpolator meshless method. *Comput. Struct.*, vol. 89,, n. 19–20, pp. 1771–1784, 2011b.
[26] L. Dinis, R. Jorge, and J. Belinha. Large deformation applications with the radial natural neighbours interpolators. *Comput. Model. Eng. Sci.*, vol. 44,, n. 1, pp. 1–34, 2009b.
[27] S. Moreira, J. Belinha, L. Dinis, and R. Jorge. The anisotropic elasto-plastic analysis using a natural neighbour rpim version. *J. Brazilian Soc. Mech. Sci. Eng.*, vol. 39,, n. -, pp. 1773–1795, 2016.
[28] B. Farahani, F. A. Pires, P. Moreira, and J. Belinha. A meshless method in the non-local constitutive damage models. *Procedia Struct. Integr.*, vol. 1,, n. -, pp. 226–233, 2016a.
[29] B. Farahani, F. A. Pires, P. Moreira, and J. Belinha. Extending a radial point interpolation meshless method to non-local constitutive damage models. *Theor. Appl. Fract. Mech.*, vol. 85, n. -, pp. 84–98, 2016b.
[30] B. Farahani, J. Belinha, F. A. Pires, and A. Ferreira. A radial point interpolation meshless method extended with an elastic rate-independent continuum damage model for concrete materials. *Mech. Adv. Mater. Struct.*, vol. 25, n. 10, pp. 855–867, 2017b.
[31] B. Farahani, J. Belinha, P. Tavares, and P. Moreira. Elastoplastic response and failure assessment of steel alloys: Empirical and computational analyses. *Fatigue Fract. Eng. Mater. Struct.*, vol. 42, n. 6, pp. 1247–1261, 2018.
[32] J. M. C. Azevedo, J. Belinha, L. M. J. S. Dinis, and R. M. N. Jorge. Crack path prediction using the natural neighbour radial point interpolation method. *Eng. Anal. Bound. Elem.*, vol. 59, pp. 144–158, 2015.
[33] J. Belinha, J. M. C. Azevedo, L. M. J. S. Dinis, and R. M. N. Jorge. The natural neighbor radial point interpolation method extended to the crack growth simulation. *Int. J. Appl. Mech.*, vol. 8, n. 1, pp. 1650006, 2016a.
[34] J. Belinha, J. M. C. Azevedo, L. M. J. S. Dinis, and R. M. N. Jorge. The natural neighbor radial point interpolation method in computational fracture mechanics: A 2d preliminary study. *Int. J. Comput. Methods*, vol. 14, n. 4, pp. 1750045, 2017a.
[35] J. Belinha, J. M. C. Azevedo, L. M. J. S. Dinis, and R. M. N. Jorge. Simulating fracture propagation in brittle materials using a meshless approach. *Eng. With Comput.*, vol. 34, n. -, pp. 503–522, 2017b.
[36] B. Farahani, J. Berardo, A. F. J. Belinha, P.J.Tavares, and P. Moreira. On the optimal shape parameters of distinct versions of rbf meshless methods for the bending analysis of plates. *Eng. Anal. Bound. Elem.*, vol. 84, n. -, pp. 77–86, 2017c.
[37] N. Moes, J. Dolbow, and T. Belytschko. A finite element method for crack growth without remeshing. *Int. J. Numer. Methods Eng.*, vol. 46, n. 1, pp. 131–150, 1999.
[38] D. E. S. Rodrigues, J. Belinha, F. M. A. Pires, L. M. J. S. Dinis, and R. M. N. Jorge. Homogenization technique for heterogeneous composite materials using meshless methods. *Eng. Anal. Bound. Elem.*, vol. 92, pp. 73–89, 2018a.
[39] D. E. S. Rodrigues, J. Belinha, F. M. A. Pires, L. M. J. S. Dinis, and R. M. N. Jorge. Material homogenization technique for composites: A meshless formulation. *Sci. Technol. Mater.*, vol. 30, n. 1, pp. 50–59, 2018b.
[40] K. C. L. Wong, L. Wang, H. Zhang, H. Liu, and P. Shi. Meshfree implementation of individualized active cardiac dynamics. *Comput. Med. Imaging Graph.*, vol. 34, n. 1, pp. 91–103, 2010.
[41] G. Chen, B. Schmutz, D. Epari, K. Rathnayaka, S. Ibrahim, M. A. Schuetz, and M. J. Pearcy. A new approach for assigning bone material properties from ct images into finite element models. *J. Biomech.*, vol. 43, n. 5, pp. 1011–5, 2010.
[42] M. Marques, J. Belinha, L. M. J. S. Dinis, and R. M. N. Jorge. A new numerical approach to mechanically analyse biological structures. *Comput. Methods Biomech. Biomed. Engin.*, vol. 22, n. 1, pp. 100–111, 2018a.
[43] J. Belinha. Meshless methods: The future of computational biomechanical simulation. *J. Biom. Biostat.*, vol. 7, n. 4, pp. 1–3, 2016.
[44] M. H. Doweidar, B. Calvo, I. Alfaro, P. Groenenboom, and M. Doblaré. A comparison of implicit and explicit natural element methods in large strains problems: Application to soft biological tissues modeling. *Comput. Methods Appl. Mech. Eng.*, vol. 199, n. 25–28, pp. 1691–1700, 2010.
[45] G. Y. Zhang, A. Wittek, G. R. Joldes, X. Jin, and K. Miller. A three-dimensional nonlinear meshfree algorithm for simulating mechanical responses of soft tissue. *Eng. Anal. Bound. Elem.*, vol. 42, pp. 60–66, 2014.
[46] M. Marques, J. Belinha, L. M. J. S. Dinis, and R. N. Jorge. A brain impact stress analysis using advanced

discretization meshless techniques. *Proc. Inst. Mech. Eng. Part H J. Eng. Med.*, vol. 232, n. 3, pp. 257–270, 2018b.
[47] K. Tsubota, S. Wada, and T. Yamaguchi. Particle method for computer simulation of red blood cell motion in blood flow. *Comput. Methods Programs Biomed.*, vol. 83, n. 2, pp. 139–146, 2006.
[48] D. Mori, K. Yano, K. Tsubota, T. Ishikawa, S. Wada, and T. Yamaguchi. Computational study on effect of red blood cells on primary thrombus formation. *Thromb. Res.*, vol. 123, n. 1, pp. 114–121, 2008.
[49] C. F. Santos, J. Belinha, F. Gentil, M. Parente, R. N. Jorge, and R. R. Frias. An alternative 3d numerical method to study the biomechanical behaviour of the human inner ear semicircular canal. *Acta Bioeng. Biomech.*, vol. 19, n. 1, pp. 1–26, 2017a.
[50] C. F. Santos, J. Belinha, F. Gentil, M. Parente, B. Areias, and R. N. Jorge. Biomechanical study of the vestibular system of the inner ear using a numerical method. *Procedia IUTAM*, vol. 24, pp. 30–37, 2017b.
[51] C. F. Santos, J. Belinha, F. Gentil, M. Parente, B. Areias, and R. N. Jorge. A computational framework to simulate the endolymph flow due to vestibular rehabilitation maneuvers assessed from accelerometer data. *Comput. Methods Biomech. Biomed. Engin.*, vol. 21, n. 6, pp. 461–469, 2018a.
[52] C. F. Santos, J. Belinha, F. Gentil, M. Parente, and R. N. Jorge. The free vibrations analysis of the cupula in the inner ear using a natural neighbor meshless method. *Eng. Anal. Bound. Elem.*, vol. 92, pp. 50–63, 2018b.
[53] M. Doblaré, E. Cueto, B. Calvo, M. A. Martínez, J. M. Garcia, and J. Cegoñino. On the employ of meshless methods in biomechanics. *Comput. Methods Appl. Mech. Eng.*, vol. 194, n. 6–8, pp. 801–821, 2005.
[54] K. M. Liew, H. Y. Wu, and T. Y. Ng. Meshless method for modeling of human proximal femur: treatment of nonconvex boundaries and stress analysis. *Comput. Mech.*, vol. 28, n. 5, pp. 390–400, 2002.
[55] J. D. Lee, Y. Chen, X. Zeng, A. Eskandarian, and M. Oskard. Modeling and simulation of osteoporosis and fracture of trabecular bone by meshless method. *Int. J. Eng. Sci.*, vol. 45, n. 2–8, pp. 329–338, 2007.
[56] F. Taddei, M. Pani, L. Zovatto, E. Tonti, and M. Viceconti. A new meshless approach for subject-specific strain prediction in long bones: Evaluation of accuracy. *Clin. Biomech. (Bristol, Avon)*, vol. 23, n. 9, pp. 1192–9, 2008.
[57] J. Belinha, R. M. N. Jorge, and L. M. J. S. Dinis. Bone tissue remodelling analysis considering a radial point interpolator meshless method. *Eng. Anal. Bound. Elem.*, vol. 36, n. 11, pp. 1660–1670, 2012b.
[58] J. Belinha, L. M. J. S. Dinis, and R. M. N. Jorge. A meshless microscale bone tissue trabecular remodelling analysis considering a new anisotropic bone tissue material law. *Comput. Methods Biomech. Biomed. Engin.*, vol. 16, n. 11, pp. 1170–1184, 2013c.
[59] J. Belinha, L. M. J. S. Dinis, and R. M. N. Jorge. The meshless methods in the bone tissue remodelling analysis. *Procedia Eng.*, vol. 110, pp. 51–58, 2015a.
[60] J. Belinha, L. M. J. S. Dinis, and R. M. N. Jorge. The mandible remodeling induced by dental implants: a meshless approach. *J. Mech. Med. Biol.*, vol. 15, n. 4, pp. 1550059, 2015b.
[61] J. Belinha, L. M. J. S. Dinis, and R. M. N. Jorge. The analysis of the bone remodelling around femoral stems: A meshless approach. *Math. Comput. Simul.*, vol. 121, pp. 64–94, 2016b.
[62] S. F. Moreira, J. Belinha, L. M. J. S. Dinis, and R. M. N. Jorge. A global numerical analysis of the 'central incisor / local maxillary bone' system using a meshless method. *MCB Mol. Cell. Biomech.*, vol. 11, n. 3, pp. 151–184, 2014b.
[63] H. M. S. Duarte, J. R. Andrade, L. M. J. S. Dinis, R. M. N. Jorge, and J. Belinha. Numerical analysis of dental implants using a new advanced discretization technique. *Mech. Adv. Mater. Struct.*, vol. 23, n. 4, pp. 467–479, 2015.
[64] C. S. S. Tavares, J. Belinha, L. M. J. S. Dinis, and R. M. N. Jorge. The elasto-plastic response of the bone tissue due to the insertion of dental implants. *Procedia Eng.*, vol. 110, pp. 37–44, 2015.
[65] C. S. Tavares, J. Belinha, L. Dinis, and R. Natal. The biomechanical response of a restored tooth due to bruxism: a meshless approach. In T. D. R. I. Ramos, J. M. R. Tavares, J. A. Duarte, M. E. Costa, O. Pinho, M. H. Figueiral, R. N. Jorge, S. Brandão, and T. Mascarenhas, eds, *BioMedWomen*, pp. 7. CRC Press - Taylor & Francis Group, 2014.
[66] M. M. A. Peyroteo, J. Belinha, S. Vinga, L. M. J. S. Dinis, and R. M. N. Jorge. Mechanical bone remodelling: Comparative study of distinct numerical approaches. *Eng. Anal. Bound. Elem.*, vol. 100, pp. 125–139, 2019.
[67] J. Belinha, L. Dinis, and R. Natal. *The meshless simulation of the atherosclerotic plaque tissue using an elasto-plastic model*, pp. 10. CRC Press - Taylor & Francis Group, 1st edition, 2014.