Comparative study of homogenization techniques in masonry

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Abstract. Masonry is a composite material widely used in construction, consisting of individual units, such as bricks or blocks, and mortar joints. This combination results in a highly anisotropic material, which means that its properties can vary significantly in different directions. However, adequately characterizing masonry represents a significant challenge. Due to its composite and anisotropic nature, determining the mechanical properties and structural behavior of masonry requires a careful and precise approach. One of the main difficulties in characterizing masonry is dealing with the variation in the properties of individual components, such as bricks and mortar, as well as the influence of mortar joints on the overall response of the structure. In addition, the presence of imperfections, such as cracks and discontinuities in units and joints, can significantly affect the structural performance of masonry. The analysis and homogenization of masonry have been approached through several techniques, including the Finite Element Method (FEM), which has been widely applied. However, more recent techniques, such as Mechanics of Structure Genome (MSG) and Finite-Volume Direct Averaging Micromechanics (FVDAM), have also shown promise in this field. FEM has been employed for the analysis of masonry, both for unreinforced masonry cases and for those reinforced with fiber reinforced polymer (FRP). However, MSG emerges as an interesting alternative for masonry homogenization, as it considerably reduces the computational cost compared to FEM. In addition, FVDAM, a relatively new technique, has been used to analyze composite materials and obtain their effective properties. Although it is a technique under development, FVDAM has shown excellent results compared to other numerical approaches. In a pioneering study, FVDAM was applied for the first time to the numerical study of FRP-reinforced masonry. The results obtained were compared with MSG and FEM, demonstrating the good performance of FVDAM with respect to the methods used in the comparison.

Keywords: Masonry, Homogenization, FVDAM, Constitutive modeling, FRP reinforcement

1 Introduction

Masonry is a type of structure that dates back to the beginnings of civilization, used for rustic shelter construction and still employed in various applications to this day. Generally, masonry consists of arranged blocks (or units) and binding joints (mortar). The blocks can be made of ceramic, clay, rock material, glass, or reinforced or unreinforced concrete. The mortar joints can have different compositions and proportions, typically consisting of cement, lime, sand, and water. In addition to blocks and mortar, masonry can also be reinforced with synthetic fibers (fabric reinforced cementitious matrix - FRCM) or natural fibers (natural fabric reinforced cementitious matrix - NFRCM). This type of structure becomes crucial in the context of many ancient constructions, historical cities, and monumental structures, requiring stability analyses, as well as the development of effective interventions for strengthening and repair. The careful analysis of these structures is of utmost importance, representing a significant challenge in the field of civil engineering, particularly in structural mechanics.

Despite not being considered a high-tech material, masonry is of composite/heterogeneous nature, making it complex to analyze. According to Reiki and Lebon [1], the mechanical properties of masonry constituents, i.e., blocks and joints, are challenging to obtain, especially for mortar joints. The challenges become even greater when considering ancient or historic masonry structures, where the dimensions of blocks and mortar often lack regularity. Due to the significantly lower Young's modulus of historic mortars compared to that of blocks, these structures usually exhibit significant anisotropy Cecchi and Sab [2, 3] and Cecchi et al. [4, 5]. It is also essential to note that masonry demonstrates predominantly nonlinear physical behavior and is highly susceptible to localized failures.

According to Asteris [6], due to uncertainties in the behavior of masonry, it is often ignored in structural design and analyses of filled frames. It is concluded that there is still much to be researched experimentally and analytically to develop accurate models that predict the behavior of these structural elements. In this context, besides experimental studies for the characterization of these structures and their constituents, many researchers are conducting work in the numerical field, employing computational methods. According to Almeida and Cecchi [7], the computational modeling of masonry can be classified into three distinct approaches: micro-modeling, macro-modeling, and homogenization Milani et al. [8], Milani [9] and Milani and Lourenço [10]. Among the main numerical techniques for studying masonry, the Finite Element Method (FEM) stands out, which has been used for a long time. Using this method, Ali and Page [11] presented a plane stress model for the modeling of masonry subjected to concentrated loading. Anthoine [12] utilized an asymptotic analysis-based model, which consisted of a semi-analytical approach with FEM and periodic boundary conditions, to model masonry with real geometry. Asteris [6] developed a finite element formulation for modeling the anisotropic behavior of masonry panels under lateral loading in plane stress state. Another technique that has been used is the Mechanics of Structure Genome (MSG), which, according to Yu [13], is a unified theory for constitutive modeling of composite materials at multiple scales. This technique was developed using the concept of structure gene, defined as the smallest mathematical building block of a structure, inspired by the concept of Representative Volume Element (RVE). The first application in masonry homogenization was in the work of Almeida and Lourenço [14]. In the work of Almeida and Cecchi [7], MSG was applied to the homogenization of masonry reinforced by Fiber Reinforced Polymer (FRP) through the repointing technique, where an FRP plate is inserted at half height of the horizontal mortar bed joints. Finally, another technique applied to the homogenization of composite materials is FVDAM, which discretizes the domain and uses the direct stiffness method to assemble the stiffness matrix based on kinematic and static compatibilities, similar to the Finite Element Method. However, the local stiffness matrix of the discretized subvolume is obtained by imposing local equilibrium, rather than the energy balance between the work done by external loading and the deformation energy of the element, as in the case of the Finite Element Method. Works by Cavalcante [15], Gattu et al. [16], and Escarpini Filho and Almeida [17] can be consulted for further insights on this subject. It is worth noting that this numerical technique was applied for the first time to the problem of reinforced masonry this year. Thus, the aim of this study is to compare three numerical methods applied to reinforced masonry for homogenization, namely the Finite Element Method, the Mechanics of Structure Genome (MSG), and the Finite-Volume Direct Averaging Micromechanics (FVDAM). The remainder of this paper is organized as follows. Section 2 deals with the Mechanics of Structure Gene (MSG). Section 3 deals with the Finite-Volume Direct Averaging Micromechanics (FVDAM) homogenization method, with its considerations. Section 4 presents a masonry homogenization problem, comparing the results of FVDAM with FEM and MSG. Finally, Section 5 provides some concluding remarks.

2 Mechanics of Structure Genome (MSG)

One of the key works explaining the Mechanics of Structure Genome (MSG) is the article by Yu [13], which focuses on developing a unified theory to link the smaller scale of interest (micro) to the structural scale (macro). In this approach, the problem is modeled by considering the concept of the Structure Gene (SG), which is the smallest mathematical building block of the structure, to emphasize the fact that it contains all the constitutive information necessary for a structure in the same way that the genome contains all the genetic information for the growth and development of an organism. As shown in Figure 1, analyses of heterogeneous 3D structures can be approximated through a macroscopic 3D structural analysis with material properties provided by a constitutive modeling of the SG. For 3D structures, the SG plays a role analogous to the Representative Volume Element (RVE) in micromechanics. It's worth noting that SG and RVE are distinct, as can be observed in the following example.

For composites structures featuring 1D heterogeneity (Figure 1a), the Structure Gene (SG) is a straight line with two segments representing the phases. This line can be repeated in-plane to construct the layers of the binary composite, and out-of-plane to create the entire structure. Another application is modeling a laminate as a homogeneous solid, with the transverse normal line being the 1D SG of the laminate. Constitutive modeling over the 1D SG calculates all 3D properties and local fields. These applications of the SG are not equivalent to the Representative Volume Element (RVE). For composite materials with 2D heterogeneity (Figure 1b), the SG

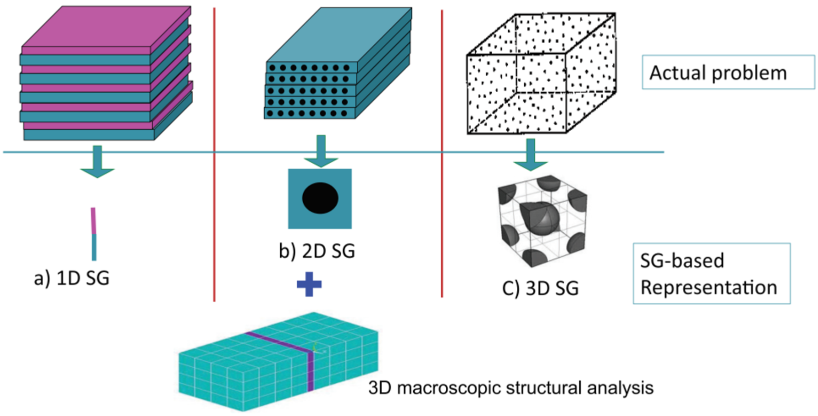

Figure 1. Analysis of 3D heterogeneous structures approximated by a constitutive modeling over SG and a corresponding 3D macroscopic structural analysis.

will be 2D. While 2D RVEs are utilized in micromechanics, they only provide properties and local fields in-plane. For comprehensive 3D structural analyses, a 3D RVE is typically required Sun and Vaidya [18] and Fish [19], but for 2D domains, SG-based models (Figure 1b) or semi-analytical models like GMC/HFGMC, Aboudi et al. [20], are sufficient. In the case of 3D heterogeneity (Figure 1c), the SG extends into a 3D volume. Although a 3D SG for 3D structures is akin to an RVE, SG-based models do not necessitate the essential boundary conditions of displacement and traction used in RVE models, Yu [13].

In order for the SG not to remain merely a concept, it must be governed by a physics-based theory known as the Mechanics of Structure Genome (MSG), so that there exists a two-way communication between microstructural details and structural analysis: microstructural information can be rigorously passed to structural analysis for predicting structural performance, and structural performance can be fed back to predict local fields within the microstructure for failure prediction and other detailed analyses.

Due to the mathematical complexity of the MSG formulation and the limited space in this article, the authors recommend referring to the work by Yu [13] for a more in-depth understanding of the SG and the methodology.

3 Finite-Volume Direct Averaging Micromechanics (FVDAM)

The Finite Volume Theory is an alternative to the Finite Element Method. This method was developed by Bansal and Pindera [21, 22]. This numerical technique is geared towards elastic, plastic, viscoelastic, and thermal structural analysis. The Finite Volume Theory, as explained, is focused on macroscopic analysis or at the structural level. According to Gattu [16], the development of this method is rooted in the so-called higher-order theory for materials, originally devised by Aboudi, Pindera, and Arnold in a series of papers published in the 1990s and summarized in a review article by Aboudi et al. [23]. Furthermore, the incorporation of periodic boundary conditions into this higher-order theory within the framework of homogenization theory led to a micromechanical model for periodic materials named the high-fidelity generalized method of cells (HFGMC), Aboudi et al. [24, 25]. Still, following Gattu et al. [16], the subsequent reconstruction of this micromechanical model based on a simplified Representative Unit Cell (RUC) volume discretization and the utilization of the local/global stiffness matrix approach. Bufler [26] and Pindera [27] demonstrated that this theory constitutes a finite-volume direct averaging micromechanical model, thus originating the FVDAM. For structural problems, the convergence of the method, known as Finite Volume Theory (FVT), was evaluated in Cavalcante and Pindera [28] and Araújo et al. [29]. In the case of homogenization problems, the convergence of the FVDAM was studied in the work of Escarpini Filho and Almeida [17], where the technique was applied in the context of masonry, thus demonstrating its robustness, simplicity, and quality compared to the other methods studied.

4 Numerical Results

In this section, an example of reinforced masonry homogenization will be presented. As the focus of this study is the comparison between numerical techniques applied to homogenization, the results from the Finite Element Method (FEM), Mechanics of Structure Genome (MSG), and Finite-Volume Direct Averaging Micromechanics (FVDAM) are presented and compared. The analyzed masonry model consists of blocks, mortar, and FRP structural reinforcement. The masonry pattern is a running bond, and the mortar thickness is 10 mm. Regarding the mechanical properties, both the block and the mortar have the same Poisson's ratio of 0.2 and the following Young's moduli: 5 GPa for the block and 1 GPa for the mortar. The reinforcement has a thickness of 1.2 mm and is centered within the mortar along the horizontal length of the masonry (Fig. 2). In this example, three different reinforcements were considered, labeled as S, M, and H, all with the same Poisson's ratio of 0.4, and different Young's moduli of 145 GPa, 210 GPa, and 300 GPa, respectively. The results of the reinforced masonry obtained with MSG (Almeida and Cecchi [7]), 2D FEM (Barcieri and Cecchi [30]), and FVDAM (Escarpini Filho and Almeida [17]) were compared among themselves and with the unreinforced masonry.

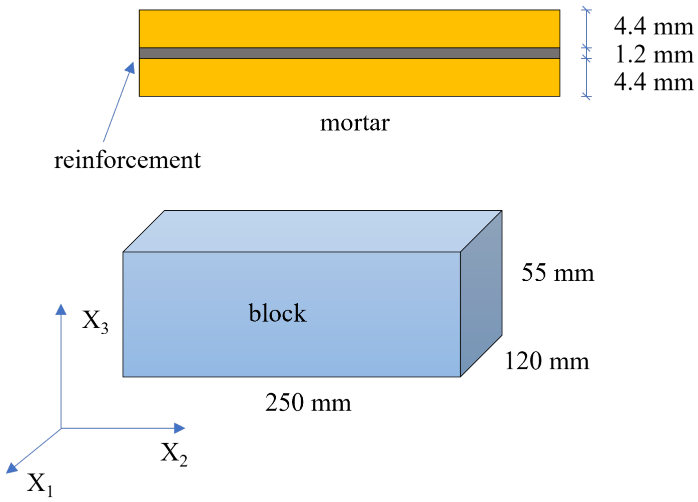

Figure 2. Geometry of the masonry components.

In this example, in addition to studying masonry with and without reinforcement, the behavior of effective properties was also examined due to variations in the E_b/E_m ratio from 5 to 90. Figures 3, 4, 5 and 6 respectively show the effective components of the composite C_{22}, C_{33}, C_{23}, and C_{44}. It's worth noting that in Escarpini Filho and Almeida [17], the results of FVDAM for unreinforced masonry were not included, being presented for the first time here. The relative differences found for the coefficient C_{22}, compared to FVDAM, were less than 0.2% for MSG (S model with E_b/E_m = 90) and less than 1.9% for FEM (H model with E_b/E_m = 40) (Fig. 3). In Figure 4 for the coefficient C_{33}, the highest percentage difference encountered was 0.06% between FVDAM and MSG for the H model with E_b/E_m = 80, and 0.42% between FVDAM and FEM for the S reinforcement model with E_b/E_m = 90, demonstrating good agreement between the responses. Regarding the effective coefficient C_{23}, the largest differences compared to FVDAM were 0.73% for MSG and 3.84% for FEM, both for the H reinforcement model with E_b/E_m = 90 (Fig. 5). Finally, for the C_{44} property (Fig. 6), the percentage differences between FVDAM/MSG were 0.06% (S reinforcement and E_b/E_m = 90), and FVDAM/FEM was 0.33% (H reinforcement model with E_b/E_m = 5).

5 Conclusions

In this study, three numerical techniques were employed to evaluate the effective properties of a reinforced masonry Representative Volume Element (RVE): FVDAM, MSG, and FEM. The first method served as the baseline for comparisons, demonstrating substantial agreement with the others. In the case examined here, the effective properties of reinforced masonry were derived for three distinct types of reinforcement (S, M, and H), enabling an exploration of the effective properties' behavior in relation to the ratio between the elasticity modulus of the block and mortar. As depicted in the graphs, it is apparent that FVDAM displayed a strong correlation with both MSG and FEM.

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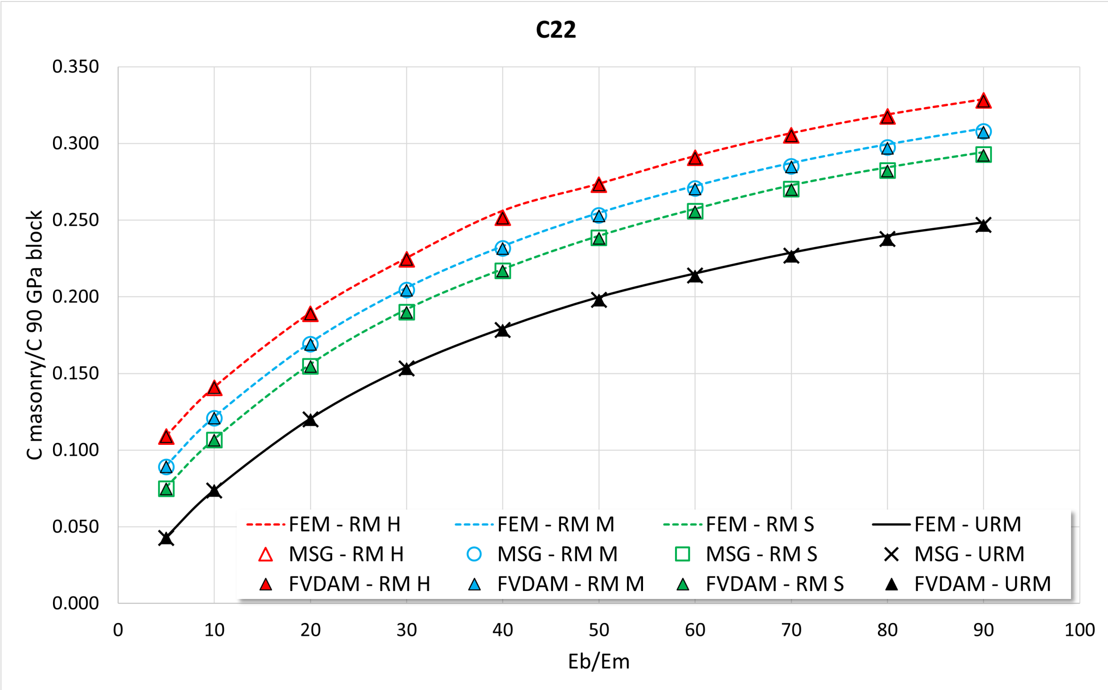

Figure 3. Relative coefficient C_{22} for reinforced and unreinforced masonry.

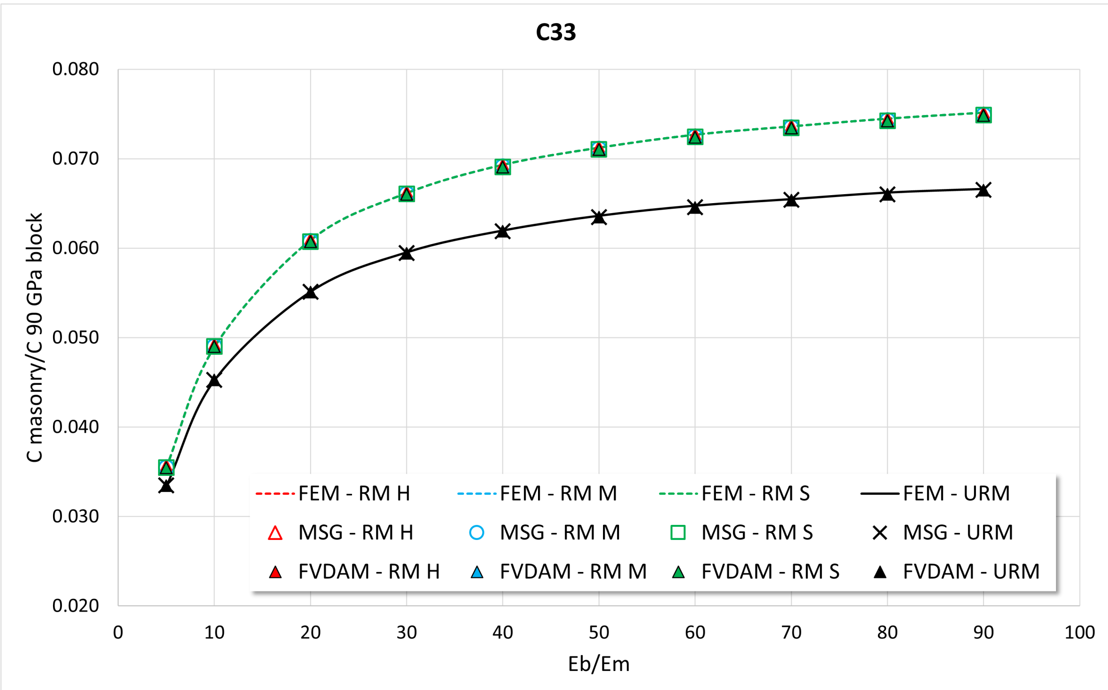

Figure 4. Relative coefficient C_{33} for reinforced and unreinforced masonry.

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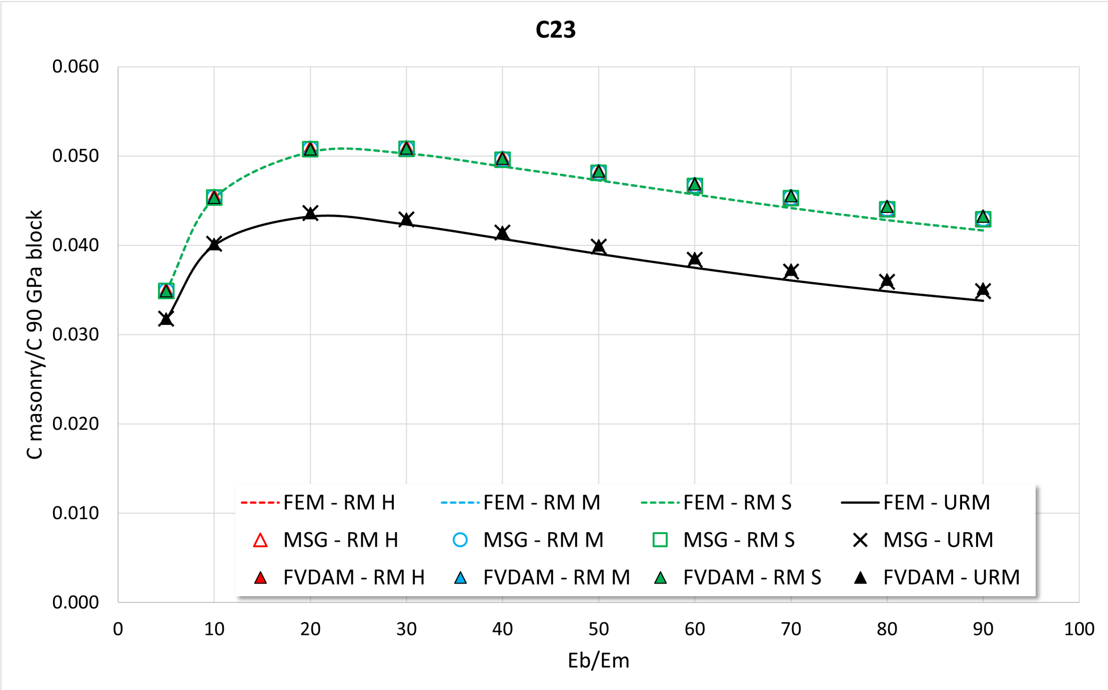

Figure 5. Relative coefficient C_{23} for reinforced and unreinforced masonry.

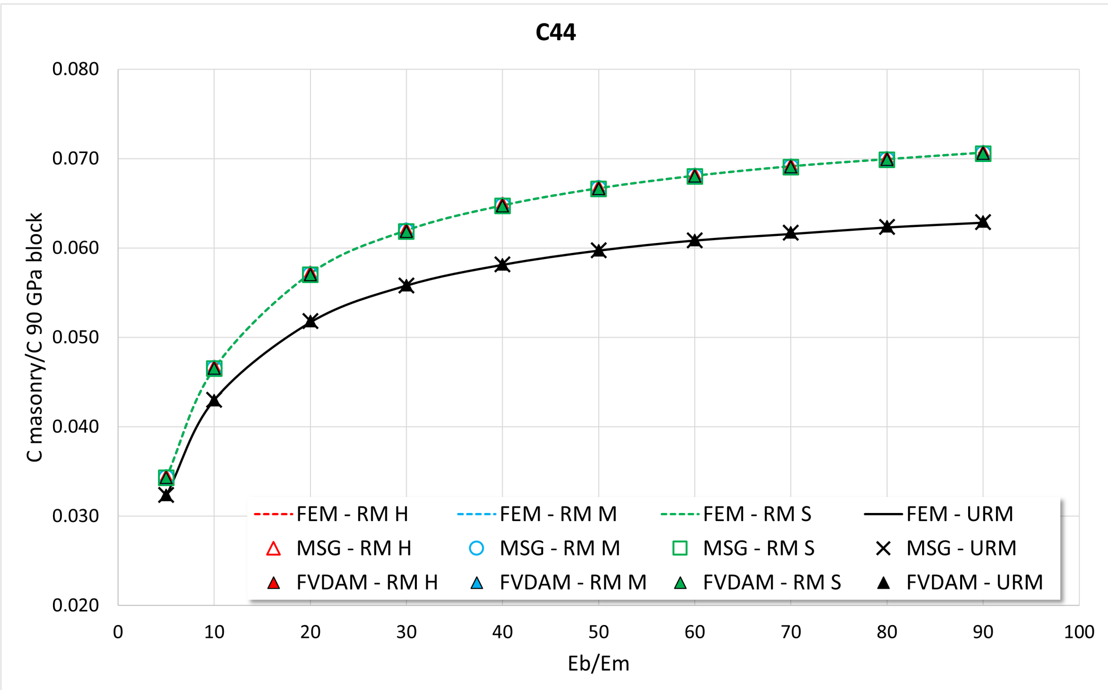

Figure 6. Relative coefficient C_{44} for reinforced and unreinforced masonry.

References

[1] A. Rekik and F. Lebon. Homogenization methods for interface modeling in damaged masonry. *Advances in Engineering Software*, vol. 46, n. 1, pp. 35–42. CIVIL-COMP, 2012.

[2] A. Cecchi and K. Sab. A multi-parameter homogenization study for modeling elastic masonry. *European Journal of Mechanics - A/Solids*, vol. 21, n. 2, pp. 249–268, 2002a.

[3] A. Cecchi and K. Sab. Out of plane model for heterogeneous periodic materials: the case of masonry. *European*

Journal of Mechanics - A/Solids, vol. 21, n. 5, pp. 715–746, 2002b.
[4] A. Cecchi, G. Milani, and A. Tralli. Validation of analytical multiparameter homogenization models for out-of-plane loaded masonry walls by means of the finite element method. *Journal of Engineering Mechanics*, vol. 131, n. 2, pp. 185–198, 2005a.
[5] A. Cecchi, G. Milani, and A. Tralli. Out-of-plane loaded CFRP reinforced masonry walls: Mechanical characteristics by homogenization procedures. *Composites Science and Technology*, vol. 65, n. 10, pp. 1480–1500, 2005b.
[6] P. Asteris, D. Cotsovos, C. Chrysostomou, A. Mohebkhah, and G. Al-Chaar. Mathematical micromodeling of infilled frames: State of the art. *Engineering Structures*, vol. 56, pp. 1905–1921, 2013.
[7] F. P. Almeida and A. Cecchi. Mechanics of structure genome applied in the homogenization of masonry reinforced by frp repointing technique. *Computers Structures*, vol. 253, pp. 106576, 2021.
[8] G. Milani, E. Milani, and A. Tralli. Upper bound limit analysis model for FRP-reinforced masonry curved structures. part I: Unreinforced masonry failure surfaces. *Computers Structures*, vol. 87, n. 23, pp. 1516–1533, 2009.
[9] G. Milani. Fe homogenized limit analysis model for masonry strengthened by near surface bed joint FRP bars. *Composite Structures*, vol. 92, n. 2, pp. 330–338, 2010.
[10] G. Milani and P. B. Lourenço. Simple homogenized model for the nonlinear analysis of FRP-strengthened masonry structures. II: Structural applications. *Journal of Engineering Mechanics*, vol. 139, n. 1, pp. 77–93, 2013.
[11] S. S. Ali and A. W. Page. Finite element model for masonry subjected to concentrated loads. *Journal of Structural Engineering*, vol. 114, n. 8, pp. 1761–1784, 1988.
[12] A. Anthoine. Derivation of the in-plane elastic characteristics of masonry through homogenization theory. *International Journal of Solids and Structures*, vol. 32, n. 2, pp. 137–163, 1995.
[13] W. Yu. A unified theory for constitutive modeling of composites, 2016.
[14] F. P. Almeida and P. B. Lourenço. Three-dimensional elastic properties of masonry by mechanics of structure gene. *International Journal of Solids and Structures*, vol. 191-192, pp. 202–211, 2020.
[15] M. A. A. Cavalcante, S. P. C. Marques, and M.-J. Pindera. Parametric Formulation of the Finite-Volume Theory for Functionally Graded Materials—Part I: Analysis. *Journal of Applied Mechanics*, vol. 74, n. 5, pp. 935–945, 2006.
[16] M. Gattu, H. Khatam, A. S. Drago, and M.-J. Pindera. Parametric Finite-Volume Micromechanics of Uniaxial Continuously-Reinforced Periodic Materials With Elastic Phases. *Journal of Engineering Materials and Technology*, vol. 130, n. 3, pp. 031015, 2008.
[17] R. S. Escarpini Filho and F. P. Almeida. Reinforced Masonry Homogenization by the Finite-Volume Direct Averaging Micromechanics—FVDAM. *Composite Structures*, vol. 320, pp. 117185, 2023.
[18] C. Sun and R. Vaidya. Prediction of composite properties from a representative volume element. *Composites Science and Technology*, vol. 56, n. 2, pp. 171–179, 1996.
[19] J. Fish. Practical multiscaling, 2014.
[20] J. Aboudi, S. M. Arnold, and B. A. Bednarcyk. Dedication. In J. Aboudi, S. M. Arnold, and B. A. Bednarcyk, eds, *Micromechanics of Composite Materials*, pp. v. Butterworth-Heinemann, Oxford, 2013.
[21] Y. Bansal and M.-J. Pindera. A Second Look at the Higher-Order Theory for Periodic Multiphase Materials . *Journal of Applied Mechanics*, vol. 72, n. 2, pp. 177–195, 2005.
[22] Y. Bansal and M.-J. Pindera. Finite-volume direct averaging micromechanics of heterogeneous materials with elastic–plastic phases. *International Journal of Plasticity*, vol. 22, n. 5, pp. 775–825, 2006.
[23] J. Aboudi, M.-J. Pindera, and S. Arnold. Higher-order theory for functionally graded materials. *Composites Part B: Engineering*, vol. 30, n. 8, pp. 777–832, 1999.
[24] J. Aboudi, M.-J. Pindera, and S. M. Arnold. Linear Thermoelastic Higher-Order Theory for Periodic Multiphase Materials . *Journal of Applied Mechanics*, vol. 68, n. 5, pp. 697–707, 2001.
[25] J. Aboudi, M.-J. Pindera, and S. M. Arnold. Higher-order theory for periodic multiphase materials with inelastic phases. *International Journal of Plasticity*, vol. 19, n. 6, pp. 805–847, 2003.
[26] H. Bufler. Theory of elasticity of a multilayered medium. *Journal of Elasticity*, vol. 1, pp. 125–143, 1971.
[27] M.-J. Pindera. Local/global stiffness matrix formulation for composite materials and structures. *Composites Engineering*, vol. 1, n. 2, pp. 69–83, 1991.
[28] M. A. A. Cavalcante and M.-J. Pindera. Generalized Finite-Volume Theory for Elastic Stress Analysis in Solid Mechanics—Part I: Framework. *Journal of Applied Mechanics*, vol. 79, n. 5, pp. 051006, 2012.
[29] M. V. O. Araujo, E. N. Lages, and M. A. A. Cavalcante. Energy analysis of continuum elastic structures by the generalized finite-volume theory. *Acta Mechanica*, vol. 232, pp. 4625–4643, 2021.
[30] A. Barbieri and A. Cecchi. 2D homogenisation procedure in masonry walls strengthened by FRP repointing technique. In *3rd WSEAS International Conference on APPLIED and THEORETICAL MECHANICS*, volume 1, pp. 228–236. Spain, 2007.