Failure criteria characterization of orthotropic beams under combined loads for stiffness and strength assessment

Eduardo D. Telli[1], Gustavo P. Mezzomo[2], Joel Boaretto[1], Daniel M. De Leon[3], Ignacio Iturrioz[3]

[1]*Instituto Hercílio Randon*
R. Francisco Getúlio Vargas, 1130 - 95070-560, RS, Brazil
eduardo.telli@ihr.tec.br, joel.boaretto@ihr.tec.br
[2]*Post-Graduation Program in Design and Manufacturing Processes, University of Passo Fundo*
BR 285 Km 292,7 — Campus I, Bairro São José, 99052-900, RS, Brazil
gpmezzomo@yahoo.com.br
[3]*Applied Mechanics Group (GMAp), Federal University of Rio Grande do Sul*
Sarmento Leite Street, 425, Zip-Code 90050-170, Porto Alegre/RS, Brazil
daniel.leon@ufrgs.br, ignacio.iturrioz@ufrgs.br

Abstract. Composites are no longer promising materials in the industry and have already become a reality in several applications. Their valuable properties such as high strength/weight ratio, durability, and corrosion resistance make them suitable for a vast range of applications. Looking for composites manufacturing, pultrusion is a technique that can produce closed and open-section profiles with a broad range of fiber orientation distribution throughout different areas of the cross-section. The process involves pulling reinforcement fibers through guides, immersing them in resin material, and placing them in a heated chamber to cure, all in a continuous process. Various applications, including bridge construction, marine construction, transportation, and energy systems, have adopted these techniques. Typically, structural components can be simplified into beam-like components. These components can be exposed to different loading conditions in each application, usually in a combination of torsion, flexure, and traction, which have varying contributions to the total load. In this situation, creating new composite parts can be a difficult and time-consuming task because of the many variables involved. To address this issue, a previous investigation of this research focused on using the constant cross-section of extruded parts to analyse their free vibrational modes and frequencies. This analysis provided information on the stiffness distribution. However, this information alone does not provide all the necessary collection of data to draw robust conclusions. In this sense, this work aims to investigate structures under combined loads in attempt to extract information about their static behavior. Along with this information a static analysis is also employed to generate failure curves, which are collections of points that characterizes the resistance of each cross section studied under different load contributions of bending and torsion. Thus, stiffness and resistance of different types of cross-sections, with multiple fiber angle distributions, can give valuable insights to the designer.

Keywords: continuous fiber composites, orthotropic beams, failure criteria

1 Introduction

Composites have transcended their status as mere promising materials within the industrial landscape, evolving into indispensable realities across a multitude of applications. Their remarkable attributes, including elevated strength-to-weight ratios, durability, and corrosion resistance, position them as versatile solutions for a wide spectrum of uses [1–3]. Among the various methods of composite manufacturing, pultrusion stands out as one of the longest standing and the oldest continous process at the same time. This technique is capable of producing both closed and open-section profiles. This process facilitates a diverse distribution of fiber orientations throughout distinct regions of the cross-sectional profile, thus widening the possibilities for material's structural behavior tailoring [4].

The pultrusion process encompasses a sequential operation wherein reinforcing fibers are drawn through guides, subsequently immersed in a resin matrix, and then subjected to a curing process within a heated chamber – all carried out in an uninterrupted, continuous manner. This technique has found widespread adoption in numerous critical domains, ranging from the construction of bridges and maritime structures to transportation systems and energy infrastructure. Often, these applications involve intricate structural components that can be simplified into beam-like configurations [5].

However, the inherent complexity of real-world loading conditions, which commonly encompass a combination of torsion, flexure, and tension, necessitates a meticulous understanding of the static and dynamic behavior of these composite structures. Introducing new composite components tailored to specific applications can be a challenging and time-intensive endeavor due to the multitude of variables at play. Recognizing this challenge, the preceding study [6] investigated the free vibrational modes and frequencies of parts possessing constant cross-sections. This analysis yielded insights into the distribution of stiffness across these components, offering a foundation for comprehending their mechanical properties.

Nevertheless, the scope of the previous investigation is limited, as it exclusively addresses vibrational behavior, leaving critical gaps in our understanding of these composite structures' static characteristics. To address this gap, the present work is dedicated to a comprehensive exploration of the static behavior of composite structures, with the objective of extracting valuable insights into their static response. To do so, interaction curves for these composites sections are selected as a methodology for this exploration.

Interaction curves, also known as strength envelopes, are widely used in structural engineering and mechanics to provide a graphical representation of the combined effect of multiple loadings on a structural component or system. Particularly diffused over the metallic and concrete structures domain, these interaction curves are frequently used to build technical information and assess buckling withstanding performance of beams as it can be checked in [7]. These curves play a crucial role in the design and analysis of structures, particularly those subjected to complex and concurrent loading conditions [8].

This study aims to unravel a more nuanced comprehension of these structures' performance under varying loading conditions. In doing so, it aspires to furnish a holistic framework that advances the design, analysis, and optimization of composite structures manufactured through the pultrusion technique.

2 Methodology

The methodology adopted in this study is based on numerical models using the Finite Element Method (FEM) for each geometry investigated. Using this tool, the load configuration was sistematically varied in every simulation run so that a series of specific combinations of values for torsion and flexure loads were obtained. These load combinations have one thing in common: the maximum Tsai-Wu failure criteria value found in the model is 1. In other words, the first step was to obtain the admissible load of each geometry by applying one kind of load separately. Once the flexure and torsion ultimate loads, F_{Adm} and T_{Adm}, were defined, a fraction of one of these loads was applied to the model and the other load was systematically increased up to failure criteria reaches the load combination that violates the safe envelope. These data were gathered and the interaction curves were obtained. Below, a more detailed description about the FEM model and the geometries studied are shown.

2.1 Numerical Model

Using the Finite Element Method (FEM) to analyze composite structures presents distinct characteristics compared to its application in the analysis of isotropic materials like steel and aluminum, as highlighted by [9]. In this work, models were built with quadratic 20-node hexaedrical elements to perform a linear static analysis. Boundary conditions are equivalent to a cantilever beam, where the nodes of one end have all their degrees of freedom (DOF) restricted to zero, while at the other, rigid beam elements are used to connect the end's face nodes to a master node which is used to apply Torsion (T) and Flexural force (F). Figure 1 illustrates the model for the bulk cantilever beam case.

All models of this work were built using ANSYS Mechanical APDL [10]. The study maintains consistent material properties throughout its procedures, which were determined using a micromechanical method employing the online tool "Mech G-Comp" as outlined in the work by [11]. The material composition consists of 78% weight of "Fiberglass S2" and 22% epoxy resin. For the determination of failure criteria, the Tsai-Wu failure criteria was chosen for its capability of differing traction and compression forces. The equation can be written as in equation 1 [12].

$$F_1\sigma_1 + F_2\sigma_2 + F_3\sigma_3 + F_4\sigma_4 + F_5\sigma_5 + F_6\sigma_6 +$$

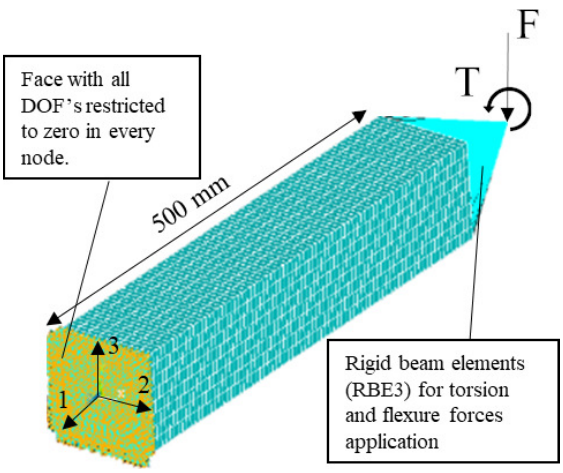

Figure 1. Example of FEM model: bulk cantilever beam example.

$$F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{33}\sigma_3^2 + F_{44}\sigma_4^2 + F_{55}\sigma_5^2 + F_{66}\sigma_6^2 \leq 1. \tag{1}$$

Now let the failure strength in tension and compression in the directions of anisotropy be R_{1t}, R_{1c}, R_{2t}, R_{2c}, R_{3t}, R_{3c} and the shear strengths in the planes of symmetry be R_{23}, R_{12} and R_{31} the coefficients of the criterion are as in equation 2:

$$F_1 = \frac{1}{R_{1t}} - \frac{1}{R_{1c}}; \quad F_2 = \frac{1}{R_{2t}} - \frac{1}{R_{2c}}; \quad F_3 = \frac{1}{R_{3t}} - \frac{1}{R_{3c}}; \quad F_4 = F_5 = F_6 = 0$$
$$F_{11} = \frac{1}{R_{1t}R_{1c}}; \quad F_{22} = \frac{1}{R_{2t}R_{2c}}; \quad F_{33} = \frac{1}{R_{3t}R_{3c}}; \quad F_{44} = \frac{1}{R_{23}^2}; \quad F_{55} = \frac{1}{R_{31}^2}; \quad F_{66} = \frac{1}{R_{12}^2}. \tag{2}$$

Table 1 shows the material properties applied in this work. The failure strength in directions of anisotropy are depicted in Table 2.

Table 1. Elastic properties of the composite material considered.

Property	Nomenclature	Value
Young Modulus	E_1 [GPa]	55.84
Young Modulus	$E_2 = E_3$ [GPa]	17.92
Poisson Ratio	$\nu_{12} = \nu_{31}$	0.0.27
Poisson Ratio	ν_{23}	0.35
Shear Modulus	$G_{12} = G_{31}$ [GPa]	6.205
Shear Modulus	G_{23} [GPa]	3.89

Table 2. Stress limits of the composite material considered.

Property	Nomenclature	Value
Ultimate Traction	R_{1t} [MPa]	2000
Ultimate Traction	$R_{2t} = R_{3t}$ [MPa]	62
Ultimate Compression	R_{1c} [MPa]	965
Ultimate Compression	$R_{2c} = R_{3c}$ [MPa]	155
Ultimate Shear	$R_{12} = R_{31}$ [MPa]	93.08
Ultimate Shear	R_{23} [MPa]	46.15

2.2 Geometries

All geometries considered for this study are presented in Figure 2, from 1 to 5, on the same dimension scale. The 5 cases have same cross sectional area of 10000 mm^2 and the fibers are oriented at 0°, which corresponds to the length direction of the beam (in green), except for geometry 1, which has a layer of material (in orange) oriented peripherally at 45°.

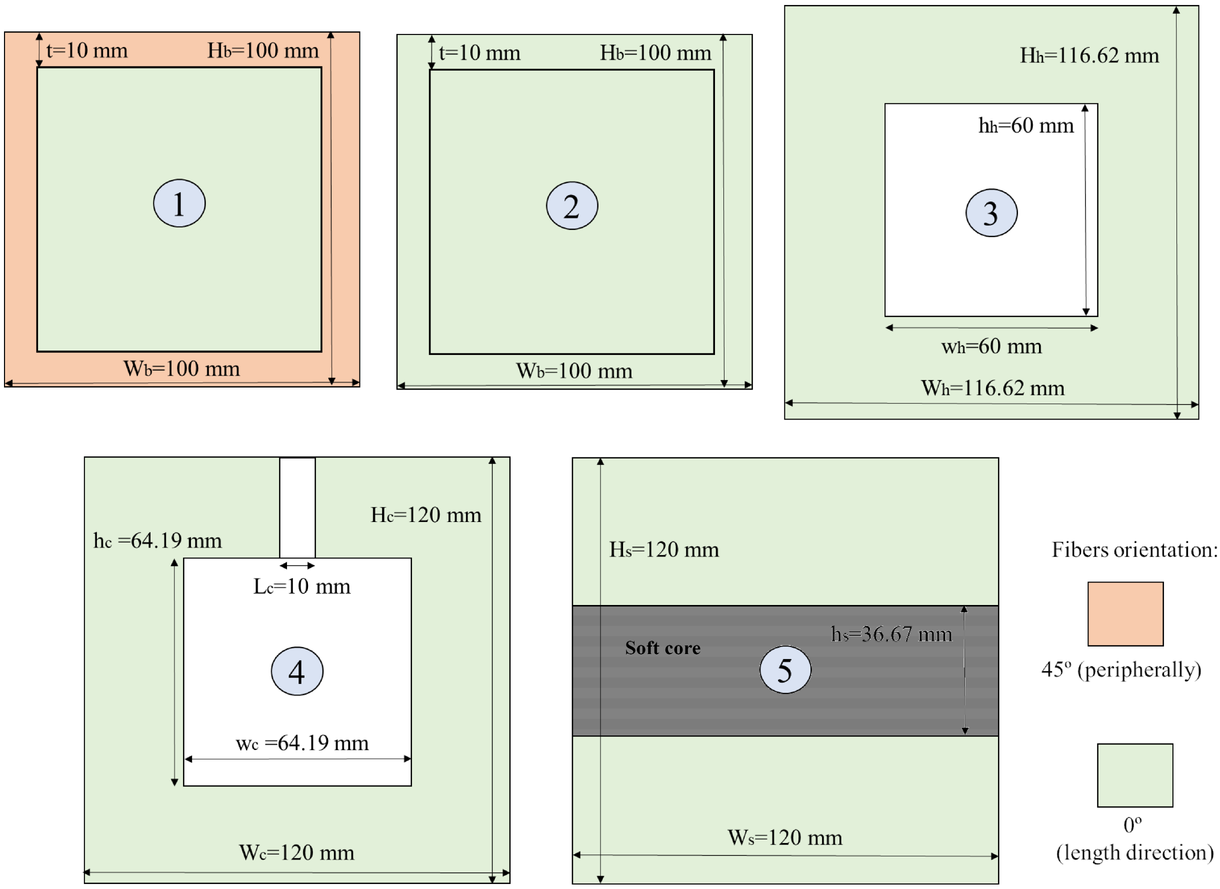

Figure 2. Geometries considered for this study with their respective dimensions (on same scale) and fiber orientation.

3 Results

All five considered geometries shown in the previous subsection were analysed applying a varying combination of loads. It was explained in the methodology section that for each geometry an interaction curve that represents a kind of safe envelope for loads combination has been built. As it can be seen in figure 3, a collection of curves that represents the combination of flexure and torsion that returns a Tsai-Wu failure criteria equal to unity is presented. Each point of each curve represents a condition which the component fails under a specific loading condition. In other words, each curve represents a strenght envelope and the points below each curve can be considered a feasible solution (a solution that the component would theoretically not fail). Since for all geometries the common constant parameter was the cross sectional area (and the volume), they still have different geometrical properties as moment of inertia and torsion constant. Consequently, the different geometries presents F_{Adm} and T_{Adm} distincts.

These results show that, for instance, geometry number 1 the worst choice for flexure load condition, while the geometry number 5 is the example that performed better.

For a better reading of the results, Figure 4 shows the same information with a normalized collection of

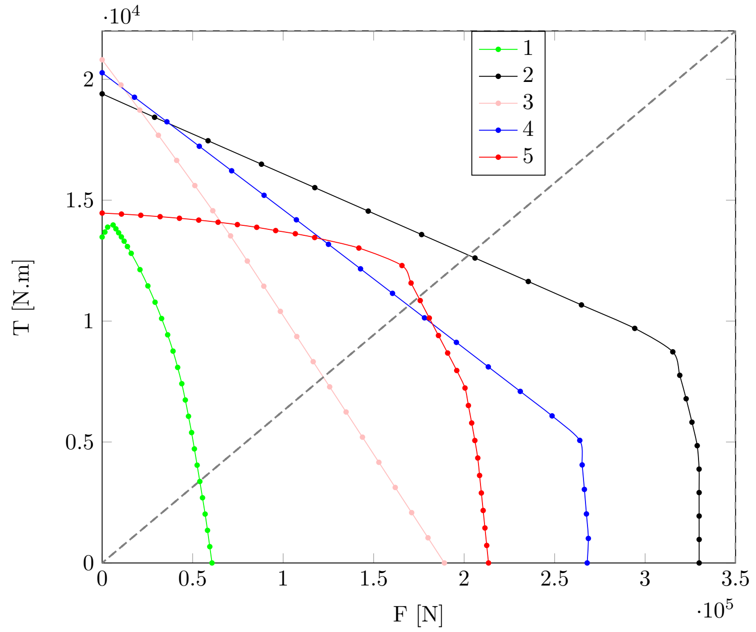

Figure 3. Fail curves for each geometry considered in this work.

combined loads. Then the x and y axis are ranging between 0 and 1 and corresponds to F/F_{Adm} and T/T_{Adm}, respectively.

These results are interesting because they depict each geometry's ability to withstand combined loads. For instance, we can see that geometry number 5 can deal with an increasing flexure load without needing to decrease torsion loads near the ultimate value. Although geometry 5 is not the best choice for flexure or torsion loads, these geometry presented the smoother transition between the applied loads.

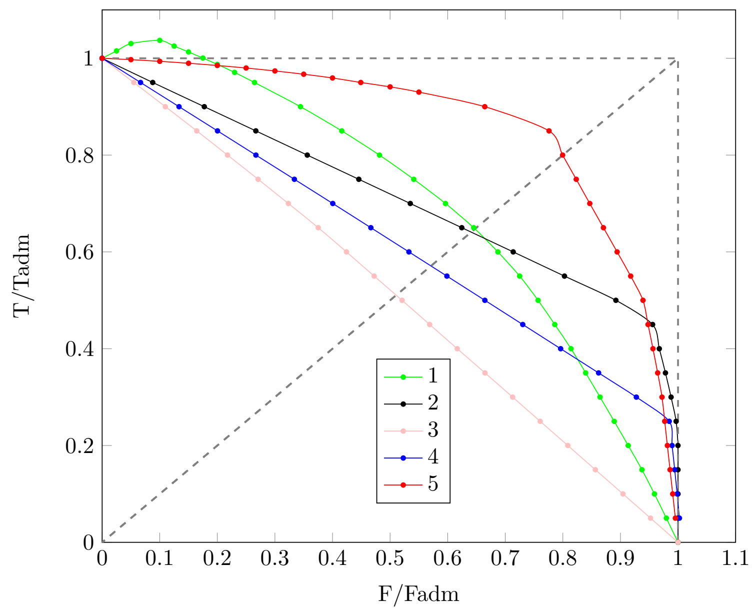

Figure 4. Normalized fail curves of each geometry considered in this work.

4 Conclusions

The features analysed here are useful for the designer when composite structures stand combined loads. The insights observed applying the proposed methodology are important in the sense that the replacement of metallic components by similar ones in composite material is increasingly evident in industry, especially in automotive and aeronautics. The simple notion that different geometries for cross sections can completely change the behavior of the structure against a selection of combined loads is of great value for the definition of components for the industry.

It is important to mention that the results presented here are in a preliminar stage and some limitations can be estimated, for example, the accounting for local stress effects that can significantly dictate the interaction curve behavior. A thorough investigation on the behavior of such geometries must be done in order to draw a definitive conclusion on the drawbacks and advantages of each cross section. However, as many components applied in industry are susceptible to more than one kind of load, the results presented in this work show that simple geometries with very few modifications can incurr in huge changes in the static behaviour.

Acknowledgements. Authors are grateful to the financial support of the National Fund for Scientific and Technological Development (FNDCT) of National Council for Scientific and Technological Development (CNPq) under the grant number 407382/2022-4.

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] R. M. Jones. *Mechanics of Composite Materials*. Taylor & Francis Inc, 1975.
[2] A. K. Kaw. *Mechanics of Composite Materials*. CRC-Press, 1997.
[3] T. W. Clyne and D. Hull. *An introduction to composite materials*. Cambridge university press, 2019.
[4] H. Lengsfeld, J. Lacalle, T. Neumeyer, and V. Altstädt. *Composite technology: prepregs and monolithic part fabrication technologies*. Carl Hanser Verlag GmbH Co KG, 2021.
[5] A. Vedernikov, A. Safonov, F. Tucci, P. Carlone, and I. Akhatov. Pultruded materials and structures: A review. *Journal of Composite Materials*, vol. 54, n. 26, pp. 4081–4117, 2020.
[6] E. D. Telli, P. M. Montanari, da G. S. Silva, de L. Abreu Correa, and I. Iturrioz. Wave propagation analysis as a global stiffness measurement in the investigation of fiber orientation effect of orthotropic beam. *Mecanica Computacional*, vol. 39, n. 6, pp. 147–156, 2022.
[7] I. Arrayago and K. J. Rasmussen. Buckling curves for cold-formed stainless-steel columns and beams. *Journal of Structural Engineering*, vol. 147, n. 10, pp. 04021149, 2021.
[8] T. V. Galambos. *Guide to stability design criteria for metal structures*. John Wiley & Sons, 1998.
[9] E. J. Barbero. *Finite Element Analysis of Composite Materials using Abaqus®*. CRC press, 2023.
[10] M. K. Thompson and J. M. Thompson. *ANSYS mechanical APDL for finite element analysis*. Butterworth-Heinemann, 2017.
[11] C. C. Angrizani, de E. A. W. Menezes, F. F. Luz, A. A. Muller, and S. C. Amico. Uso de software acadêmico mech-gcomp para o ensino de compósitos particulados. *Revista de Ensino de Engenharia*, vol. 36, n. 1, 2017.
[12] P. Kohnke. Ansys mechanical apdl theory reference. *Canonsburg, PA, USA: ANSYS Inc*, vol. 40, 2013.