Preliminary studies of homogenization of NFRCM reinforcement and reinforced masonry

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Abstract. Masonry is a heterogeneous material formed by the manual and individual arrangement of units connected by joints, being one of the first construction materials, and that can have a partition or structural function. Despite apparently being a simple material, masonry is a composite material with a complex characterization. For various reasons, masonry often needs some type of reinforcement. Currently, one of the most used materials as reinforcement are the composites known as FRP (fiber reinforced polymer), which consist of fibers (usually carbon, glass, or aramid) embedded in a polymer matrix (epoxy resin, for example). This type of reinforcement is already well characterized in the literature. However, FRPs have some disadvantages, such as high cost, use of synthetic materials, and in the case of masonry, adhesion problems between the reinforcement and the substrate. Recently, fabric reinforced cementitious matrix (FRCM) with natural fibers (NFRCM) have been studied as an alternative to FRPs. There are already works that show the potential of using NFRCM as structural reinforcement, where Brazil appears in a prominent position as one of the largest producers of natural fibers in the world, as is the case of sisal and pineapple leaf fiber. For the numerical characterization of these and other composite materials, the Finite Element Method (FEM) is commonly used. In the present work, an alternative numerical technique is used: Mechanics of Structure Genome (MSG), which has been successfully employed in the homogenization of unreinforced and FRP-reinforced masonry. Thus, this work deals with a preliminary numerical study for the characterization of NFRCMs and masonry reinforced with this type of reinforcement.

Keywords: Masonry, Homogenization, Constitutive modeling, Natural fibers, NFRCM

1 Introduction

Masonry is one of the oldest building materials in the world. In fact, the vast majority of constructions conceived until the end of the 19th century and still existing today are made of masonry. World-famous examples of masonry constructions are: the Great Pyramid of Giza, in Egypt, built 2,600 years BC; the Colosseum in Rome, built between 68 and 79 AD. the Guimarães castle, Portugal, built in the 10th century; the Reims Cathedral, in the Champagne region, France, built between 1211 and 1300; and the Monadnock, in Chicago, USA, built between 1889 and 1891, a symbol of the first phase of structural masonry, which lasted until the emergence of New Architecture, with the advent of steel and concrete structures. From then on, structural masonry began to decline and would only be rediscovered after the end of the Second World War, being today one of the most used structural systems in civil construction.

Commonly formed by the manual and individual association of natural or artificial blocks – where the most commonly used materials are concrete or ceramic materials – bonded by mortar, masonry is a composite material, quasi-brittle and with lower tensile strength, where in some situations it is necessary to use reinforcement. Such situations can occur in the preservation of historic or architectural heritage buildings, in regions subject to seismic tremors, and even in buildings under greater threat of explosions. Generally speaking, masonry reinforcements can be classified into three types: internal, near to the surface and external. Internal reinforcements have the disadvantage of being more invasive. Near to the surface reinforcements – less invasive than the previous ones – also require special care, which, when observed, can be a good solution. In this work, a type of external reinforcement is considered, which consists of the application of a cementitious matrix reinforced internally by a fabric of natural wires, specifically, sisal, but it is possible to find in the literature studies with other types of natural plant fibers, for example hemp, jute and pineapple leaf fiber. Depending on the case to be dealt with, this reinforcement can be applied to just one side of the masonry wall or to both sides. The reinforcement discussed in this study, known as natural fiber reinforced cementitious matrix (NFRCM), is a new material that presents itself as an alternative to fiber reinforced polymer (FRP) reinforcements, with the advantages of being a lower cost material, lower dependence on non-renewable resources, lower pollutant emissions, lower greenhouse gas emissions, biodegradability (Wei and Meyer [1]), and in the case of masonry, with fewer adhesion problems between the reinforcement and the substrate.

However, studies on the physical properties of NFRCM are still quite limited, especially as masonry reinforcement. Such studies basically take place on two fronts: performing physical tests in the laboratory or through computational modeling. As for the numerical tests, the complexity of the problem addressed is observed, since both the masonry and the NFRCM reinforcement are composite materials. In the case of NFRCM, the complexity is even greater, since there is greater variability in the physical properties of natural fibers, the geometry involved in the problem is more complex and there is also a greater difference in scale between the cross-section of the fibers and the dimensions of the other elements, thus configuring a multiscale problem.

The Finite Element Method (FEM) has been the most widely used numerical method for characterizing composite materials. However, in order to obtain the three-dimensional (3D) elastic constants of a heterogeneous material using the FEM, it is necessary to solve a problem that is also 3D, for each of the 6 possible boundary conditions. This means that using FEM for this purpose has a considerable computational cost, both in terms of analysis time and the hardware resources required. In order to overcome these problems and optimize the multiscale modeling of composite materials, Yu [2] developed the Mechanics of Structure Genome (MSG) technique. Almeida and Lourenço [3] applied the MSG technique in an unprecedented way to the homogenization of unreinforced masonry and Almeida and Cecchi [4] to the homogenization of FRP-reinforced masonry. Therefore, the objective of this work is to carry out a preliminary study of the application of the MSG technique to the homogenization process of NFRCM reinforcement, as well as masonry reinforced by this type of reinforcement. In addition to this introductory chapter, the structure of this article follows with Section 2 which presents more information about the MSG technique, followed by Section 3 with the presentation and discussion of numerical results, ending with Section 4 of conclusions.

2 Mechanics of Structure Genome

Mechanics of Structure Genome (MSG) is a technique recently presented by Yu [2] for multiscale constitutive modeling that can be applied to all types of composite structures including beams, plates and shells and three-dimensional (3D) structures. Since it is a semi-analytical technique, MSG considerably reduces the computational cost and maintains the same accuracy as 3D analysis, for example, by the Finite Element Method (FEM), which has its origins in the early 1960s.

The initial step of the MSG technique consists of determining a Structure Gene (SG), which can be defined as the smallest mathematical portion of the structure in the sense that it contains all the constitutive information necessary for its characterization, in the same way that, in biology, the genome is a set of information encoded in DNA that serves as the basis for the growth and development of an organism (Yu [5]). Figure 1 illustrates 3 SG possibilities for a 3D structure. Note that for a composite structure with one-dimensional (1D) heterogeneity – for example, binary composites made with two alternating layers – the SG will be a line with two segments, which can be mathematically repeated in the plane of the cross-section and then outside this plane to generate the entire structure. In the case of a composite with two-dimensional (2D) heterogeneity – for example, composites reinforced by continuous fibers – the SG will be 2D, and for composites with 3D heterogeneity – for example, composites reinforced by particles – the SG will be 3D. Although the SG for 3D bodies can present different dimensions, depending on their heterogeneity, the effective properties for the 3D structural analysis will continue to be 3D. For example, for a linear elastic analysis, the complete 6×6 constitutive matrix can be obtained from the analysis of a 1D SG of a binary composite.

For 3D bodies, SG is confused with the concept of Representative Volume Element (RVE) in micromechanics. However, it is observed that, unlike SG, the dimension of the RVE is usually determined by the necessary properties for the structural analysis, in addition to heterogeneity. For example, if 3D properties are required for a 3D analysis of a continuous fiber-reinforced composite, a 3D RVE is usually needed, whereas a 2D SG would do; in other words, MSG has the ability to obtain the complete 3D properties and local fields of heterogeneous materials by

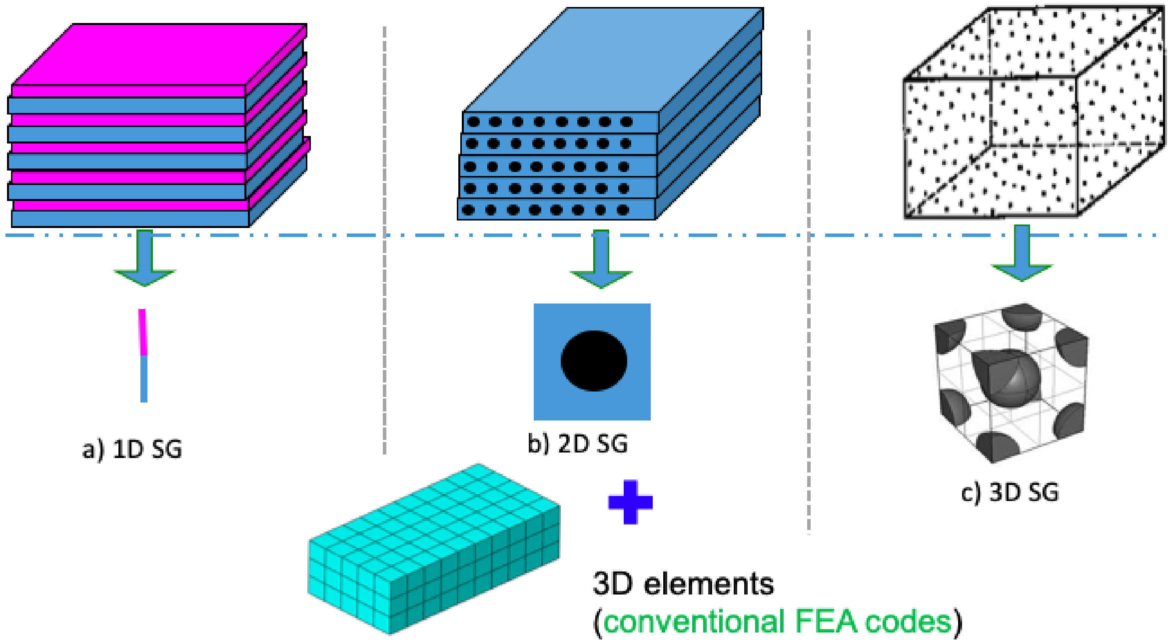

Figure 1. SG for 3D structure, Yu [2]

treating 1D and 2D heterogeneities. It should be noted that for a heterogeneous material with a 3D RVE with periodic boundary conditions, both the RVE and MSG analysis will give exactly the same results for effective properties and local fields.

Once the SG is defined, the second step is carried out, which consists of the analysis of the homogenization of the SG, having as a product of this second phase the constitutive relations of the elements of the macroscopic analysis, which can be beam elements, plates, shells and solids. The constitutive relation found serves as input for the macroscopic structural analysis, which results in the global behavior of the original structure. Finally, the macroscopic behavior serves as input for the dehomogenization process, where the displacement, stress and strain fields are obtained on the microscopic scale of the original structure.

MSG was developed based on the principle of minimum information loss, which states that the homogenized model can be built by minimizing the information loss between the original model and the homogenized model. For linear elastic materials, the information can be the strain energy density; in other words, the aim is to minimize the difference between the strain energy of the material stored in the SG and that stored in the structural model of the structural analysis. Macroscopic coordinates x_i and microscopic coordinates y_i relate to each other as $y_i = x_i/\delta$, where δ is a small parameter to describe the SG. This relation between macroscopic and microscopic coordinates is classic in asymptotic methods. However, since the value of δ does not change the results, $\delta = 1$ can be assumed.

The kinematics of the original model as a function of the model to be built can be written as:

$$u_i(x, y) = \overline{u}_i(x) + \delta\chi_i(x, y), \tag{1}$$

where u_i is the displacement field of the heterogeneous medium, $\overline{u}_i$ the displacement field of the homogenized medium and χ_i the difference between the two fields, commonly known in micromechanics as fluctuating functions.

The deformation field of the original model can be written as:

$$\varepsilon_{ij}(x, y) = \overline{\varepsilon}_{ij}(x) + \chi_{(i,j)}, \tag{2}$$

where the subscript has the operation as in $A_{(i,j)} = \frac{1}{2}(\frac{\partial A_i}{\partial y_j} + \frac{\partial A_j}{\partial y_i})$ and $\overline{\varepsilon}_{ij} = \frac{1}{2}(\frac{\partial \overline{u}_i}{\partial y_j} + \frac{\partial \overline{u}_j}{\partial y_i})$.

The kinematic variables of the homogenized model as a function of the terms of the original model can be written as:

$$\overline{u}_i = \langle u_i \rangle \text{ and } \overline{\varepsilon}_{ij} = \langle \varepsilon_{ij} \rangle, \tag{3}$$

where $\langle \cdot \rangle$ represents the average in the EG domain, and imply the following prescriptions in the fluctuating function:

$$\langle \chi_i \rangle = 0 \text{ and } \langle \chi_{(i,j)} \rangle = 0. \tag{4}$$

The principle of minimum information loss seeks to minimize the difference between the deformation energy of the original model and the homogenized model:

$$\Pi = \langle \frac{1}{2} C_{ijkl} \varepsilon_{ij} \varepsilon_{kl} \rangle - \frac{1}{2} C^*_{ijkl} \overline{\varepsilon}_{ij} \overline{\varepsilon}_{kl}. \tag{5}$$

To minimize Π, the homogenized model is considered as follows (considering that C^*_{ijkl} and $\overline{\varepsilon}_{ij}$ do not vary). χ_i can be solved from the following variational expression:

$$\min_{\chi_i \in Eq.(4)} \langle \frac{1}{2} C_{ijkl} \varepsilon_{ij} \varepsilon_{kl} \rangle = \min_{\chi_i \in Eq.(4)} \langle \frac{1}{2} C_{ijkl} (\overline{\varepsilon}_{ij} + \chi_{(i,j)})(\overline{\varepsilon}_{kl} + \chi_{(k,l)}) \rangle. \tag{6}$$

From the variational calculus, it can be concluded that χ_i must satisfy the following Euler-Lagrange equation:

$$(C_{ijkl}(\overline{\varepsilon}_{kl} + \chi_{(k,l)}))_{,j} = 0, \tag{7}$$

together with the prescriptions of Eq. (4). χ_i is then solved as a function of $\overline{\varepsilon}_{kl}$:

$$\chi_k = H_k^{mn} \overline{\varepsilon}_{mn}. \tag{8}$$

The volumetric average strain energy can be written as:

$$U = \langle \frac{1}{2} C_{ijkl} (\overline{\varepsilon}_{ij} + H^{mn}_{(i,j)} \overline{\varepsilon}_{mn})(\overline{\varepsilon}_{kl} + H^{st}_{(k,l)} \overline{\varepsilon}_{st}) \rangle. \tag{9}$$

The effective constitutive matrix can be obtained as:

$$C^*_{ijkl} = \frac{\partial \overline{\sigma}_{ij}}{\partial \overline{\varepsilon}_{kl}} = \frac{\partial^2 U}{\partial \overline{\varepsilon}_{ij} \partial \overline{\varepsilon}_{kl}}; \tag{10}$$

$$C^*_{ijmn} = \langle C_{ijmn} + C_{ijkl} H^{mn}_{(k,l)} \rangle. \tag{11}$$

More details regarding the presented formulation can be found in Yu [2], Liu et al. [6] and Almeida and Cecchi [4].

3 Numerical results

The first step was to obtain the elastic constants of the natural fiber reinforced cementitious matrix (NFRCM) by means of the homogenization process using the Mechanics of Structure Genome (MSG) technique. For this purpose, we considered a two-dimensional (2D) Structure Gene (SG) formed by the cementitious matrix with a (continuous) inclusion of a sisal yarn, similar to the illustration in Figure 1b. Both the cementitious matrix and the sisal yarn were considered homogeneous and isotropic materials, where the Young's modulus and Poisson's ratio of the cementitious matrix were considered to be $E_{cm} = 14,429$ MPa and $\nu_{cm} = 0.2$, and for the sisal yarn $E_s = 7,142$ MPa and $\nu_s = 0.2$, respectively. These values were obtained from De Carvalho Bello et al. [7, 8].

Table 1 shows the elastic constants of the NFRCM for different values of sisal yarn volume fraction (VF). It can be seen that direction 1 corresponds to the longitudinal direction of the sisal yarns and directions 2 and 3 define the SG, with direction 2 being horizontal and 3 vertical. Since the Young's modulus of the sisal yarn is lower than that of the cementitious matrix – less than half – the respective elastic constant of the composite decreases as the volume fraction of sisal increases, but the greatest reduction is 1.45% in the constants E_2 and E_3 for a volume fraction of sisal of 2%. As expected, Young's modulus in the longitudinal direction of the yarns (E_1) is higher than in the perpendicular directions (E_2 and E_3), reaching 0.44% higher for a volume fraction of 2%. It can also be seen that the homogenization process considered periodic boundary conditions in all 3 directions. However, considering periodicity only in directions 1 and 2, the maximum difference is less than 0.03%.

Table 1. Elastic constants of the NFRCM

Engineering Constants	VF = 0.5%	VF = 1.0%	VF = 1.5%	VF = 2.0%
E_1	14393	14356	14320	14283
E_2	14378	14325	14273	14220
E_3	14378	14325	14272	14219
G_{12}	5.99	5.97	5.95	5.93
G_{13}	5.99	5.97	5.95	5.93
G_{23}	5.99	5.97	5.94	5.92
ν_{12}	0.20	0.20	0.20	0.20
ν_{13}	0.20	0.20	0.20	0.20
ν_{23}	0.20	0.20	0.20	0.20

The masonry considered in this study was the same as that of Almeida and Lourenço [3], that is, a running bond texture with: Standard Italian clay blocks (UNI 5628/65), 250 mm $\times$ 120 mm $\times$ 55 mm, $E_b = 11,000$ MPa and $\nu_b = 0.20$; and mortar with a thickness of 10 mm, $E_m = 2,200$ MPa and $\nu_m = 0.25$. It is observed that the wall thickness is 120 mm. This type of masonry has already been used by several authors, such as Anthoine [9], Milani et al. [10], Milani [11] and Bertolesi et al. [12]. For masonry homogenization, the 2D SG presented in Figure 2 was considered, discretized with 34,823 elements (35,140 nodes), where the processing time was 6 seconds, using an Intel Xeon CPU E5-2697 v3 workstation of 2.60 GHz (2 64-bit processors) and 256 GB RAM. The elastic constants obtained by the homogenization process are shown in the second column of Table 2, where directions 1, 2 and 3 correspond, respectively, to the vertical, horizontal in-plane and perpendicular to the plane of the masonry wall.

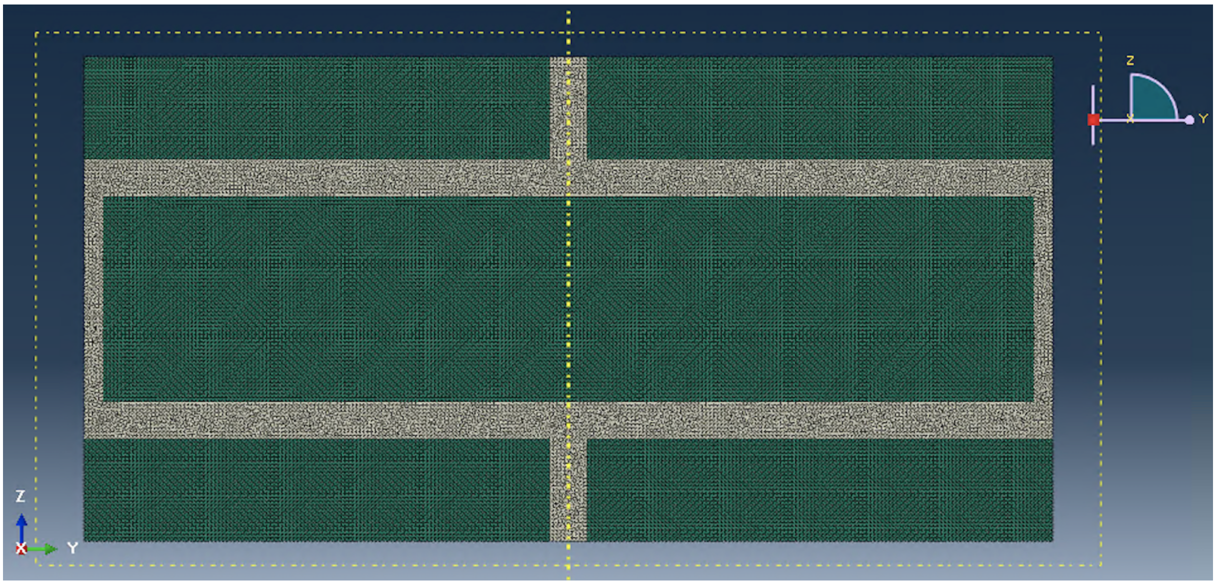

Figure 2. Discretization of the 2D structure gene of masonry

With the elastic constants of the reinforcement of NFRCM and the masonry in hand, the third stage of homogenization is carried out, which consists of the numerical characterization of the reinforced masonry, where a reinforcement with a thickness of 10 mm was considered, according to Carozzi and Poggi [13] and De Carvalho Bello et al. [8], with a volumetric fraction of 2% of fibers, a value closer to that used by Olivito et al. [14]. It should be noted that this reinforcement can be considered on only one side of the masonry, i.e. asymmetrically (asymmetric RM), or on both sides, i.e. symmetrically (symmetric RM). For this third homogenization process, it was decided to adopt a one-dimensional (1D) SG, thus considering the reinforced masonry as a laminate of orthotropic layers, where the NFRCM reinforcement consists of one layer (with a thickness of 10 mm) and the masonry another layer (with a thickness of 120 mm), this in the case of asymmetric RM; for symmetric RM, there is one NFRCM layer, one masonry layer and another NFRCM layer, forming a sandwich laminate, see Figure 1a. It is also observed that the reinforcement of NFRCM was considered with its yarns in the vertical, horizontal or double layer direction, where this consists of a layer with the yarns vertically and another with the yarns horizontally, each one with a thickness of 5 mm. In this preliminary study of homogenization of reinforced masonry, the double layer configuration is the closest to a fabric reinforced cementitious matrix reinforcement.

Table 2 presents the elastic constants obtained, where it is observed that the reinforcement with the vertical yarns is what most increases the Young's modulus of the vertical direction E_1, with values of 8% for masonry reinforced on only one face (asymmetric RM) and almost 15% in masonry reinforced on both faces (symmetric RM), this compared to unreinforced masonry. Considering the yarns in the horizontal direction, Young's modulus in this direction E_2 increases by 5% and 9.2%, for asymmetric and symmetric RM, respectively. Considering the reinforcement in the double layer configuration, it can be seen that the Young's modulus values practically coincide with the average between the values of the vertical and horizontal configurations. It can also be seen that the elastic constant G_{12} shows the greatest increase, 9.7% for asymmetric RM and 18.1% for symmetric RM.

Table 2. Elastic constants for reinforced and unreinforced masonry

Engineering Constants	**Unreinforced masonry**	**Asymmetric reinforced masonry**			**Symmetric reinforced masonry**		
		Vertical	Horizontal	Double layer	Vertical	Horizontal	Double layer
E_1	7001	7563	7558	7560	8044	8034	8039
E_2	8681	9107	9112	9110	9472	9481	9477
E_3	9361	9628	9628	9628	9866	9866	9866
G_{12}	2617	2872	2872	2872	3091	3091	3091
G_{13}	2721	2839	2839	2839	2949	2949	2949
G_{23}	3597	3709	3710	3709	3811	3812	3811
ν_{12}	0.17	0.17	0.17	0.17	0.17	0.17	0.17
ν_{13}	0.16	0.16	0.16	0.16	0.16	0.16	0.16
ν_{23}	0.19	0.19	0.19	0.19	0.19	0.19	0.19

4 Conclusions

The subject of this work is to carry out a preliminary study regarding the homogenization of a new type of structural reinforcement formed by a cementitious matrix reinforced internally by a fabric of natural fibers, known in the literature as NFRCM, as well as the homogenization of reinforced masonry for this type of reinforcement. This study used a new multiscale modeling technique developed by Yu [2] called Mechanics of Structure Genome (MSG). In the case study presented in Section 3 of Numerical Results, the three-dimensional (3D) elastic constants of the NFRCM reinforcement were obtained by solving a two-dimensional (2D) problem; it is observed that obtaining these constants by the Finite Element Method (FEM) implies approaching a 3D problem, with a much higher computational cost and with the imposition of periodic boundary conditions that could represent some difficulty for the user; in the case of the MSG technique, the imposition (or not) of these boundary conditions is already implicit in its formulation. The masonry homogenization by MSG also consists of the solution of a 2D problem; in addition to the results presented in this work, this application has already been made by Almeida and

Lourenço [3] and Almeida and Cecchi [4]. Finally, the homogenization of reinforced masonry (RM) by NFRCM is done by solving a one-dimensional (1D) problem, where the reinforcement applied to only one face (asymmetric) or both faces (symmetric) is considered. The NFRCM reinforcement was considered with the sisal yarns vertically, horizontally and in a double layer, one vertically and one horizontally. It is observed that the increase in Young's modulus in the vertical and horizontal direction are maximum when the yarns are considered in these directions, with values of 8% and 5% for asymmetric RM and 15% and 9.2% for symmetric RM. The Young's modulus values obtained with the double layer reinforcement coincide with the average value of the vertical and horizontal configurations.

The continuation of the present study follows on two fronts: one, the development of a better strategy for considering the natural fiber fabric that reinforces the cementitious matrix of the NFRCM, similar to what was done by Liu et al. [6]; another in the application of another recent homogenization technique, finite-volume direct averaging micromechanics (FVDAM), used by Escarpini Filho and Almeida [15] in the homogenization of unreinforced and FRP (fiber reinforced polymer) reinforced masonry.

Acknowledgements. Authors would like to thank the financial support of CNPq – National Council for Scientific and Technological Development, Brazil (#409028/2021-5).

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