Reliability Analysis of Corroded Pipelines Using an Efficient Selective Monte Carlo Approach

Rodrigo S. Oliveira[1], Renato S. Motta[1], Eduardo A. de Souza Neto[2], Silvana M. B. A. da Silva[1]

[1]*Institute for Petroleum and Energy Research (i-LITPEG), Federal University of Pernambuco*
Av. da Arquitetura, Cidade Universitária, 50740-540, Recife, Pernambuco, Brazil
rodrigo.oliveira@ufpe.br, renato.motta@ufpe.br, silvana.bastos@ufpe.br
[2] *Civil Engineering Department, Engineering Central, Swansea University, Bay Campus, Fabian Way*
Crymlyn Burrows, Swansea, SA1 8EN, Wales, UK
e.desouzaneto@swansea.ac.uk

Abstract. In the present paper assessment of corroded pipelines taking into account uncertainties are considered. The possibilities for pipelines assessment, after inspection, are standards, and numerical simulations. Here, a computational tool to perform the reliability analysis of corroded pipelines is created. The tool combines together an efficient mesh generator, an in-house finite element (FE) based plasticity software and four different options for reliability computations. The corroded pipelines are modeled by axisymmetric finite elements. Deterministic validation studies are conducted considering eleven specimens from literature. Then, reliability analysis (RA) studies to assess the sensitivity of the reliability index against the geometry of corrosion-related defects are performed. This later is also accomplished with the purpose to find out the most efficient RA procedure from different methodologies such as traditional crude Monte Carlo (MC), Importance sampling (IS), selective Monte Carlo (SMC) and the first-order reliability method (FORM). To conduct this comparison, initially, semi-empirical equation from the ASME B31G standard are used. As typical, large number of function evaluations were needed for MC simulation method. Big reductions were achieved for both IS and SMC. However, SMC prove to be as accurate as CMC and simpler than IS. FORM method has demonstrated to be very computationally efficient when compared to the other approaches, slightly less accurate than MC based methods. Due to this observed behavior, FE studies were performed on SMC and FORM only. Good comparison was found. Recommendation is the use of the SMC in the assessment of corroded pipelines as it provided the best accuracy-speed compromise.

Keywords: reliability analysis, corroded pipelines, selective Monte Carlo.

1 Introduction

Corrosion can be a serious threat to the safety of oil pipelines. Severe corrosion can lead to structural failure which, in this case, may have catastrophic consequences, including loss of life, environmental disasters as well as economic and social impact. To guarantee their safety, oil pipelines must be continuously monitored and assessed in service. This requires regular inspections and the use of the collected data to assess safety either by using established standards or by means of numerical simulations. Typically, the collected in-service data contains inherent uncertainties that must be taken into account in the assessment procedure.

Standard codes have introduced alternative methods of evaluating corroded pipes taking into account sophisticated methods such as reliability analysis [1]. However, the consideration of such uncertainties may incur significant numerical simulation costs.

In this paper, we present a finite element-based computational environment developed for the estimation of reliability of corroded pipelines. Two comparative studies are carried out within the proposed environment: one to determine the most efficient approach in the treatment of uncertainties; and another one to assess the sensitivity of the reliability index against the geometry of corrosion-related defects.

The simulation tool combines an efficient mesh generator, a finite element (FE) based plasticity in-house software and four different options for reliability computations. The corroded pipelines are modeled by axisymmetric finite elements.

The FE models are created by PIPEFLAW program, developed by the PADMEC research group from UFPE, an efficient and trustworthy tool, less susceptible to human errors. The developed function receives geometric parameters as input and automates the process of geometry creation and discretization, as well as the application of loads and boundary conditions, generating a descriptive file as output, that will pass to the analysis module.

Nonlinear analysis is conducted by HYPLAS finite element code [2], a software for implicit small and large strain analysis of hyperelastic and elasto-plastic solids in plane stress, plane strain and axisymmetric states.

Four different reliability methods are incorporated in the tool. They are: Monte Carlo (MC) [3], Importance sampling (IS) [4], selective Monte Carlo (SMC) [5] and the first-order reliability method (FORM) [6, 7].

Prior to the comparative studies, the FE environment is validated by means of deterministic simulations, with the results benchmarked against different sources available in the literature (experimental, 2D and 3D FE solutions with ANSYS software) [8]. The random variables considered in the reliability analysis were the operating pressure, pipeline wall yield stress, thickness and its external diameter, maximum pipeline defect depth and defect length.

In the sensitivity study, we computed the sensitivity of the reliability index with respect to defect depth and defect length. For that, MC, IS, SMC and FORM were considered. For failure function, semi-empirical equation from the ASME B31G standard were used firstly. As expected, the reliability index was found to be very sensitive to defect dimensions; a large decay was found for the critical defect case.

Predictably, a large number of function evaluations are needed for the MC simulation method. Significant reductions are achieved for both IS and SMC. However, SMC proves to be as accurate as the (far more expensive) CMC. FORM method was found to be very computationally efficient when compared to the other approaches, albeit slightly less accurate than MC based methods. Due to the observed behavior of the methods considered, FE studies were performed on SMC and FORM only. Good agreement was found between these two approaches. We conclude then that the use of the SMC is recommended in the assessment of corroded pipelines as it provided the best accuracy-speed compromise.

2 Geometry mesh generation and boundary condition

The modeling through FE requires specific knowledge and training that are not characteristic of all pipeline engineers. The process of creating good computational models for a pipe with a defect, which includes precise representation of the geometry of the defect and the generation of an appropriate mesh, with corresponding boundary and load conditions is very important. Is important to ensure a high quality mesh, which means a mesh without highly distorted elements and such that it is fine enough for a detailed and accurate result in the crucial regions (i.e. in the defect region), but not too fine, otherwise the analysis will require considerable computational time and storage [8].

Axisymmetric geometry was defined rotated around the X-axis. The starting point of the automatic process was to generate the defect region, characterized by a finer discretization. After that the region with no thickness loss is modeled with a corser mesh. A convergence study was performed to analyze the effect of mesh refinement in the results and to select the mesh that provides best compromise between solution accuracy and computational resources and can be seen in Rodrigues *et al*. [8].

Due to the problem longitudinal symmetry only half of the axisymmetric pipe model is created. In order to establish this condition, the UX degree of freedom was constrained for the nodes along the thickness in the left extremity of the defect box, as shown in Figure K. Following the same procedure, the UX degree of freedom was also constrained for the nodes in the right extremity of the pipe.

Internal pressure was the only type of loading considered in this work. However, when closed-end pipes are analysed, the inside pressure generates an axial load, which can be computed by establishing force equilibrium. Finite element models incorporate this effect by the application of a longitudinally traction pressure on the end of the pipe model. Internal and axial pressures are also shown in Fig. 1.

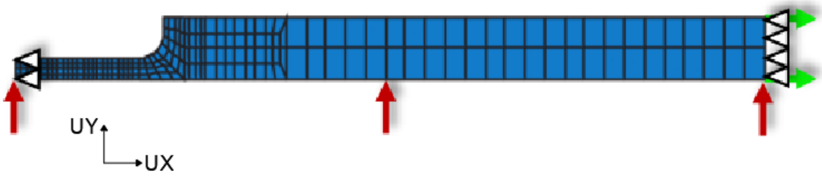

Figure 1. Finite element model [11].

3 HYPLAS

The HYPLAS software [2] is a finite element code for implicit small and large strain analysis of hyperelastic and elasto-plastic solids in plane stress, plane strain and axisymmetric states.

The exact Newton-Raphson algorithm, possibly coupled with a cylindrical arc-length procedure, is used in HYPLAS to solve the nonlinear system of global finite element equilibrium equations for nodal displacements. This method is chosen due to its quadratic rate of asymptotic convergence. At the Gauss point level, an operator split-based elastic predictor/plastic corrector algorithm is used as the state-update procedure for elasto-plastic materials. During the plastic corrector stage of this algorithm, the full Newton-Raphson method is used to solve the corresponding (generally nonlinear) return mapping equations.

4 Deterministic validation

For the studied cases the elements were defined as 2D axisymmetric solids constituted of a nonlinear material with elastic modulus 200GPa and Poisson’s ratio 0.3. In this work, 11 experimental burst tests obtained from literature [9, 10] and simulated by ANSYS [11] in Rodrigues *et al.* [8] were used for the framework validation. These specimens and their respective materials with its nominal Yield and Ultimate Tensile stresses are listed in Tab. 1.

Table 1. Specimens materials and yield and ultimate tensile stresses.

Specimen ID	Material (API 5L)	Yield Stress (MPa)	Ultimate Tensile Strength (MPa)
ET 1.1	X60	452	542
ET 1.2	X60	452	542
ET 2.1	X60	452	542
ET 2.2	X60	452	542
ET 3.1	X60	452	542
ET 3.2	X60	452	542
ET 4.1	X60	452	542
ET 4.2	X60	452	542
ET 5.1	X60	452	542
IDTS2	X80	601	684
IDTS8	X80	589	731

For each case, the true stress×strain curve was built according to Ramberg-Osgood model [12]. However, an accurate representation of these curves requires information regarding material hardening. Tab. 2 shows the geometrical parameters for the 11 cases analyzed.

Table 2. Geometrical parameters for the analyzed cases.

Specimen ID	Defect type	PL (m)	OD (m)	DL (mm)	d (mm)	t (mm)	r (mm)
ET 1.1	Long external	2	323.9	467	6.67	9.94	3
ET 1.2	Long external	2	323.9	306	6.67	9.66	3
ET 2.1	Long external	2	323.9	395	6.67	9.71	3
ET 2.2	Long external	2	323.9	350	6.67	9.71	3
ET 3.1	Long external	2	323.9	433	6.67	9.91	3
ET 3.2	Long external	2	323.9	484	6.67	9.79	3
ET 4.1	Long external	2	323.9	500	6.67	9.79	3

ET 4.2	Long external	2	323.9	528	6.67	9.74	3
ET 5.1	Long external	2	323.9	256	6.67	9.8	3
IDTS2	Short external	1.8	458.8	39.6	5.39	8.1	3.5
IDTS8	Short external	1.8	458.8	40	3.75	8	3.2

The results obtained and the error in relation to the experimental results are shown in the Tab. 3. Apart from our framework (considering HYPLAS) and experimental [9, 10] results, 3D and 2D FE results carried by ANSYS taken from Rodrigues *et al*. [8] are also provided.

Table 3. Failure pressure results obtained by ANSYS and HYPLAS.

Case	Failure pressure (MPa)				Error (%)	
	Experimental [9, 10]	Ansys (2D) [8]	Ansys (3D) [8]	HYPLAS	Ansys (2D) [8]	HYPLAS
ET 1.1	11.92	12.6	12.72	12.97	5.70	8.81
ET 1.2	14.07	11.76	12.07	12.36	-16.42	-12.19
ET 2.1	12.84	11.75	11.92	12.20	-8.49	-5.02
ET 2.2	13.58	11.9	11.99	12.34	-12.37	-9.13
ET 3.1	12.13	12.54	12.66	12.91	3.38	6.43
ET 3.2	11.91	11.99	12.11	12.35	0.67	3.66
ET 4.1	11.99	11.99	12.11	12.33	0.00	2.80
ET 4.2	11.3	11.77	11.78	12.09	4.16	7.03
ET 5.1	14.4	12.57	13.01	14.75	-12.71	2.42
IDTS2	22.68	21.96	23.06	23.29	-3.17	2.68
IDTS8	24.2	24.38	25.04	24.01	0.74	-0.78

As it can be seen, the results were similar between Ansys and HYPLAS for the axisymmetric (2D) model. The mean error for Ansys was 6.16%, in comparison with the experimental results, while the mean error for HYPLAS was 5.54%. This result shows that HYPLAS is a good tool for assessment of corroded pipelines.

5 Semi-empirical assessment methods

The structural integrity assessment of pipelines can also be computed through semi-empirical methods. There are semi-empirical methods available in standards. Some of these have been widely used due to their simplicity, such as the models proposed by ASME B31G [13], which is considered here for comparison purposes in the reliability analysis studies.

The failure pressure equation according to this is given by

$$P_b^{B31G} = P_{bi}\left[\frac{1-(2/3)(d/t)}{1-(2/3)(d/t)M^{-1}}\right]$$

where

$$M = \sqrt{1 + 0.8\left(\frac{L}{D}\right)^2\left(\frac{D}{t}\right)} \tag{2}$$

$$P_{bi} = \frac{1.1\sigma_y 2t}{D} \tag{3}$$

In equations above, d is the maximum depth of the corrosion area, t is the thickness of the pipe, L is the length of the defect, D is the external diameter of the pipe and σ_y is the yield stress of the steel.

6 Reliability analysis

Through reliability analysis the reliability index and the probability of failure (Pf) of an undesirable event can be obtained. To perform it, a failure (limit state) function and the random variables have to be defined. The failure function considered here is the pipe burst, and its limit state function (G) is shown in eq. (4).

$$g(X) = P_b - P_o \tag{4}$$

Where P_b is the failure pressure and P_o is the operating pressure. Apart from P_o, other random variables considered here are: wall thickness t, maximum defect depth d, defect length L, yield stress σ_y, and external diameter D. A parametric study on reliability index involving d and t will be conducted on the next section. Overall statistics considered for each random variable to perform reliability analysis are shown in Tab. 4, where SD is the standard deviation. These data were obtained from [14].

Table 4. Distributions and moments for each random variable.

Variable	Distribution	Case	Mean	SD	COV (%)
σ_y (MPa)	Lognormal	1-5	410.7	32.86	8
D (mm)	Normal	1-5	406.4	0.41	0.1
t (mm)	Normal	1-5	12.7	0.13	1
P_o (MPa)	Gumbel	1-5	17.28	1.2	7
d (mm)	Weibull	1	2.17	1.09	50
		2	2.67	1.33	50
		3	3.02	1.51	50
		4	3.32	1.66	50
		5	3.59	1.79	50
L (mm)	Weibull	1	173.9	87	50
		2	213.5	106.8	50
		3	242	121.1	50
		4	265.3	132.7	50
		5	287	143.6	50

6.1 FORM

The FORM (First Order Reliability Method) [6, 7] is one of the main methods for structural reliability analysis. It approximates the reliability problem around the most probable failure point (MPP) to an equivalent standard normal problem. The MPP $|V\,*|$ is the shortest distance point from the limit state to the origin of the standard space. The minimum distance $\beta = |V^*|$, is called the reliability index. The standard space is a space of equivalent standard normal distribution of uncorrelated random variables. In the standard space the limit state is linearly approximated around the MPP [15]. The probability of failure is approximated by $Pf = \Phi(-\beta)$, where Φ is the cumulative distribution function of the standard normal distribution.

6.2 MC

The Monte Carlo (MC) method is robust and simple, but may require a large number of simulations, which may make it unfeasible in some problems due to the high computational cost for calculating the failure probability. In the MC method, the Pf is numerically approximated as

$$Pf_{MC}(x) = \frac{1}{ns}\sum_{i=1}^{ns} \quad [G(x) < 0] \tag{5}$$

6.3 Importance Sampling

Monte Carlo with Importance Sampling (IS) [4] is to use a sampling function $h_X(x)$, instead of the original probability density function (PDF) $f_X(x)$, to generate more points of the sample inside the failure domain. To keep the Pf estimate unbiased, the weights $f_X(x)/h_X(x)$ appear in the equation and it becomes

$$Pf_{IS}(x) = \frac{1}{ns}\sum_{i=1}^{ns} \frac{f_X(x)}{h_X(x)}[G(x) < 0] \tag{6}$$

6.4 Selective Monte Carlo

An efficient way to reduce the number of MC simulations is to evaluate only the sample points in the failure domain. The Selective Monte Carlo (SMC) [5] consists of searching for these points through the concept of Pareto optimality [16] applied to the random variables, which work as objective functions. If the failure direction of a certain random variable (RV) is known, *i.e.*, if it is known that the failure domain is in the negative or positive direction of that RV, then it can be known which values of it are closer to the failure domain. The Pareto optimal points, in this context, are those most extreme in the failure direction of all RVs. Thus, the result is identical to that of the MC if the failure function is monotonic, i.e., if $f(x_1, x_2, \dots, x_n) \leq f(x_1^*, x_2^*, \dots, x_n^*)$ for any $x_i \leq x_i^*, i = 1, 2, \dots, n$ or $f(x_1, x_2, \dots, x_n) \geq f(x_1^*, x_2^*, \dots, x_n^*)$ for any $x_i \leq x_i^*, i = 1, 2, \dots, n$.

7 Parametric studies

After the validation of the numerical scheme, a parametric study involving different defect sizes are here carried out in order to investigate their influence on the pipeline reliability index. Solutions are given for FORM, MC, SMC and IS. Results are provided in terms of β values and the number of functions evaluations required to compute it. Five different combinations for d and L are considered. They are given in Tab. 4.

Firstly, solutions are provided in Tab. 5 considering reliability index β according to ASME B31G standard.

Table 5. Results of β, FORM iterations and function evaluations for each case, using B31G equation.

Case	β				FORM iterations	Function evaluations			
	FORM	MC	IS	SMC		FORM	MC	IS	SMC
1	3.2569	3.2597	3.2235	3.2597	9	111	618400	1731	7460
2	2.9058	2.8479	2.8953	2.8479	9	111	181800	1681	5421
3	2.6129	2.5524	2.5841	2.5524	8	98	74400	1618	3672
4	2.3610	2.3320	2.3493	2.3320	6	72	40400	1582	3298
5	2.1440	2.1462	2.1440	2.1462	5	59	24800	1469	2774

The main observations are that as typical, large number of function evaluations were needed for MC. Big reductions were achieved for both IS and SMC. However, SMC prove to be as accurate as MC. FORM method was the most computationally efficient when compared to the other approaches, slightly less accurate than MC based methods. Due to the performance found in the investigated methods, FE studies based on the developed framework are performed on SMC and FORM only. The results are those indicated in Tab. 6. The HYPLAS results were less conservative, presenting higher β values, than those obtained by B31G except for case 5, as shown in Tab. 5 and Tab. 6.

Table 6. Results of β, FORM iterations and function evaluations for each case using HYPLAS.

Case	β		FORM iterations	Function evaluations	
	FORM	SMC		FORM	SMC
1	4.3927	4.4239	14	99	5164
2	3.4177	3.3827	5	36	4322
3	2.8384	2.8275	5	36	3564
4	2.4310	2.3882	5	36	2588
5	2.1115	2.1226	8	52	2199

8 Conclusions

The assessment of corrosion pipelines using FE and reliability analysis can be more accurate than semi-empirical and deterministic methodologies. In this study, the software HYPLAS was benchmarked against different sources available in the literature (experimental, 2D and 3D FE solutions with ANSYS software) for corroded pipelines problems using axisymmetric models. Similar results were found for HYPLAS and ANSYS, where the mean error of HYPLAS was 5.54%, against 6.16% obtained by ANSYS, in comparison with the experimental results. In general, in the reliability analysis, ASME B31G equation results were more conservative than HYPLAS simulations. This was expected, due to the simplifications considered in B31G equation. When using FE, both methods applied to calculate the reliability index, FORM and SMC, presented good agreement. However, SMC is recommended in the assessment of corroded pipelines as it provided the best accuracy-speed compromise.

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