Probabilistic Assessment of Cement Sheath Integrity in Oil and Gas Wells

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Abstract. Cementing is one of the most important stages of oil and gas well construction. It consists of displacing cement paste to the annular space between the casing and wellbore, after laying the casing string of each drilled phase. The objective is to guarantee hydraulic isolation in permeable zones and ensure the borehole structural stability. Given its importance and complexity, the cement sheath demands a robust integrity assessment, considering that if it is poorly designed or executed, it causes operational problems such as unwanted influx, so-called kick, or it can even lead to a critical event like a blowout. This work proposes a probabilistic analysis of the analytical models that quantify the interaction of the casing-cement-formation system, to evaluate the displacements and stresses acting on the interfaces between these components. The classical Mohr-Coulomb criterion is applied, which provides the limiting stress for shear failure in the cement sheath. The probability of failure is estimated using the First Order Reliability Method (FORM). Some design variables such as material and geometrical parameters of tubulars, cement and formation are randomly described, and their influence on the probabilistic response is investigated. Case studies are presented to illustrate the application of the proposed methodology in the reliability-based analysis of the cement sheath integrity, contributing to the decision-making process in well structure design.

Keywords: Cement sheath, Wellbore integrity, Probabilistic model

1 Introduction

The search for oil and gas in increasingly higher depths exposes the wells to extreme conditions, including high levels of pressure and temperature (HPHT) (Gouveia et al. [1]), which are experienced from the construction - which includes drilling, casing, cementing and completion - and throughout its lifecycle. Cementing is performed to fill the annular space between the casing and the surrounding rock formation, ensuring structural stability of the well and hydraulic isolation in permeable zones [2]. Given its significance and complexity, cementing necessitates integrity analyses and risk assessment, as a poor execution can lead to failures that require intervention operations, leading to increased costs and reduced well lifespan. As outlined by Zhang and Wang [3], during completion and production operations, the integrity of the cement sheath and its interfaces can become compromised, primarily due to stresses induced by mechanical and thermal phenomena. Many works deal with the structural analysis of cement sheath, based on numerical, analytical or experimental approaches (Xu et al. [4], De Andrade and Sangesland [5], Arjomand et al. [6], Rahman et al. [7], Valov et al. [8], Wu et al. [9]).

In the field of structural analysis, uncertainties associated with design variables, such as dimensions, loads and mechanical properties of materials, play an important role on the structural response. The incorporation of the uncertainties can be performed by using the statistical description of the variables, in the context of the structural reliability theory, which allows the estimation of the failure probability of the structure, for specified limit states. For more details, see Melchers and Beck [10]. Regarding uncertainty quantification and probabilistic analysis of cement sheath, some papers can be cited, such as Yuan et al. [11], in which a simulation-based analysis involving cyclic loads is performed for cement in injection wells. Moradi and Nikolaev [12] present an experimental characterization and statistical analysis of the compressive strength for two different cement compositions.

This paper deals with integrity analysis of cement sheath, in a reliability-based framework, taking into account the randomness of its thermo-mechanical parameters, and using the First Order Reliability Method (FORM).

2 Casing-Cement-Formation Modeling

After the cement hardening process, the casing-cement-formation system form a thick hollow cylinder subject to the internal pressure of the fluid inside the casing and the external pressure applied by the rock formation. As a result of the interaction between the materials, contact pressures arise at the casing-cement and cement-formation interfaces (Figure 1). According to Bois et al. [13], among the failure mechanisms that can occur in cement sheath, it can be cited the internal/external debonding, and cracking along radial, circumferential and longitudinal directions. Internal debonding is observed at the casing-cement interface, while external debonding occurs at the cement-formation region. Different crack patterns arise if the tensile or shear stress exceed the allowable stress levels of the material.

The integrity of the cement sheath is evaluated using the theory of elasticity or thermoelasticity combined with a failure criteria, such as the well-known Mohr–Coulomb model [14]. Contact pressures are calculated from the continuous displacement condition at the interfaces, and thus the pressure at any radial point in the cement sheath is estimated.

The thermomechanical model presented below assumes the following hypotheses: 1) casing, cement and formation are homogeneous and isotropic materials, 2) interfaces between materials are perfectly bonded, 3) temperature variation in the radial direction is constant, 4) initial stress in the cement sheath is zero and 5) composite cylinder is considered in the plane strain conditions.

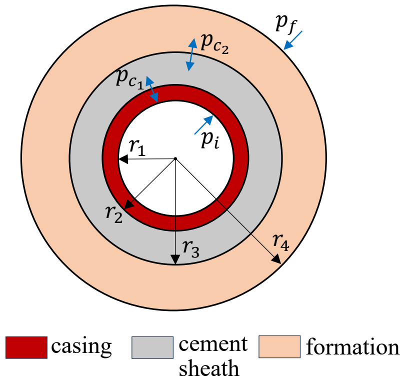

Figure 1. Conceptual model for interaction between casing-cement-formation system

Based on Hooke's law and considering temperature and thermal expansion, the constitutive equations for elastic isotropic material are defined as follows:

$$\begin{cases} \varepsilon_r = \dfrac{1}{E}\left[\sigma_r - \nu\left(\sigma_\theta + \sigma_z\right)\right] + \alpha\Delta T \\ \varepsilon_\theta = \dfrac{1}{E}\left[\sigma_\theta - \nu\left(\sigma_r + \sigma_z\right)\right] + \alpha\Delta T \\ \varepsilon_z = \dfrac{1}{E}\left[\sigma_z - \nu\left(\sigma_r + \sigma_\theta\right)\right] + \alpha\Delta T. \end{cases} \tag{1}$$

where E defines the Young's modulus, ν is the Poisson's ratio, α refers to thermal expansion coefficient and ΔT represents temperature variation. Assuming plane strain condition for the composite cylinder (i.e. $\varepsilon_z \approx 0$), the axial stress σ_z becomes dependent only on the tangential (hoop) and radial components, whose associated strains can be defined as follows:

$$\begin{cases} \varepsilon_\theta = \dfrac{1}{E}\left[\left(1-\nu^2\right)\sigma_\theta - \left(\nu+\nu^2\right)\sigma_r + \left(1+\nu\right)\alpha E\Delta T\right] \\ \varepsilon_r = \dfrac{1}{E}\left[\left(1-\nu^2\right)\sigma_r - \left(\nu+\nu^2\right)\sigma_\theta + \left(1+\nu\right)\alpha E\Delta T\right] \end{cases} \tag{2}$$

Once these strain fields are defined, Lamé equations for thick-walled cylinders are applied, allowing the definition of radial and tangential stresses in casing (σ_{rs}, $\sigma_{\theta s}$), cement (σ_{rc}, $\sigma_{\theta c}$) and formation (σ_{rf}, $\sigma_{\theta f}$):

$$\begin{cases} \sigma_{rs} = \dfrac{p_i r_1^2}{r_2^2 - r_1^2}\left(1 - \dfrac{r_2^2}{r^2}\right) - \dfrac{p_{c_1} r_2^2}{r_2^2 - r_1^2}\left(1 - \dfrac{r_1^2}{r^2}\right) \\ \sigma_{\theta s} = \dfrac{p_i r_1^2}{r_2^2 - r_1^2}\left(1 + \dfrac{r_2^2}{r^2}\right) - \dfrac{p_{c_1} r_2^2}{r_2^2 - r_1^2}\left(1 + \dfrac{r_1^2}{r^2}\right) \\ \sigma_{rc} = \dfrac{p_{c_1} r_2^2}{r_3^2 - r_2^2}\left(1 - \dfrac{r_3^2}{r^2}\right) - \dfrac{p_{c_2} r_3^2}{r_3^2 - r_2^2}\left(1 - \dfrac{r_2^2}{r^2}\right) \\ \sigma_{\theta c} = \dfrac{p_{c_1} r_2^2}{r_3^2 - r_2^2}\left(1 + \dfrac{r_3^2}{r^2}\right) - \dfrac{p_{c_2} r_3^2}{r_3^2 - r_2^2}\left(1 + \dfrac{r_2^2}{r^2}\right) \\ \sigma_{rf} = \dfrac{p_{c_2} r_3^2}{r_4^2 - r_3^2}\left(1 - \dfrac{r_4^2}{r^2}\right) - \dfrac{p_f r_4^2}{r_4^2 - r_3^2}\left(1 - \dfrac{r_3^2}{r^2}\right) \\ \sigma_{\theta f} = \dfrac{p_{c_2} r_3^2}{r_4^2 - r_3^2}\left(1 + \dfrac{r_4^2}{r^2}\right) - \dfrac{p_f r_4^2}{r_4^2 - r_3^2}\left(1 + \dfrac{r_3^2}{r^2}\right) \end{cases} \tag{3}$$

Equations 4 and 5 define the radial displacements at casing outer radius and cement inner radius, respectively:

$$\delta_{rso} = \frac{r_2}{E_s}\left[\left(1 - \nu_s^2\right)\left(\frac{2r_1^2}{r_2^2 - r_1^2}p_i - \frac{r_2^2 + r_1^2}{r_2^2 - r_1^2}p_{c_1}\right) + \left(\nu_s + \nu_s^2\right)p_{c_1} + \left(1 + \nu_s\right)\alpha_s E_s \Delta T\right] \tag{4}$$

$$\delta_{rci} = \frac{r_2}{E_c}\left[\left(1 - \nu_c^2\right)\left(\frac{r_3^2 + r_2^2}{r_3^2 - r_2^2}p_{c_1} - \frac{2r_3^2}{r_3^2 - r_2^2}p_{c_2}\right) + \left(\nu_c + \nu_c^2\right)p_{c_1} + \left(1 + \nu_c\right)\alpha_c E_c \Delta T\right] \tag{5}$$

Regarding the cement-formation interface, radial displacements at cement outer radius and at formation inner radius are defined by eq. 6 and 7, as follows:

$$\delta_{rco} = \frac{r_3}{E_c}\left\{\left(1 - \nu_c^2\right)\left[p_{c_1}\left(\frac{2r_2^2}{r_3^2 - r_2^2}\right) - p_{c_2}\left(\frac{r_3^2 + r_2^2}{r_3^2 - r_2^2}\right)\right] + p_{c_2}\left(\nu_c + \nu_c^2\right)\right\} + \left(1 + \nu_c\right) r_3 \alpha_c \Delta T \tag{6}$$

$$\delta_{rfi} = \frac{r_3}{E_f}\left\{\left(1 - \nu_f^2\right)\left[p_{c_2}\left(\frac{r_3^2 + r_4^2}{r_4^2 - r_,^2}\right) - p_f\left(\frac{2r_4^2}{r_4^2 - r_3^2}\right)\right] + p_{c_2}\left(\nu_f + \nu_f^2\right)\right\} + \left(1 + \nu_f\right) r_3 \alpha_f \Delta T \tag{7}$$

Imposing displacement compatibility conditions at both interfaces, by means of eq. 4 and 5, and eq. 6 and 7, respectively, the contact pressures can be obtained.

2.1 Probabilistic approach proposed

The probabilistic model for cement sheath failure is developed based on the Mohr–Coulomb criterion, widely adopted for brittle materials [15]:

$$\tau = c + \sigma_n \tan\phi \tag{8}$$

where τ is the shear stress, σ_n the normal stress, c is the cohesion of material and ϕ is the internal friction angle. Al-Ajmi and Zimmerman [16] present the Mohr–Coulomb criterion in the form $\sigma_1 = \sigma_c + q\sigma_3$, in which σ_1 and σ_3 are the maximum and minimum principal stresses respectively, σ_c is the compressive strength of the material and q is a parameter related to internal friction angle. These parameters can be calculated by the following equations [17]:

$$\begin{cases} q = \tan^2\left(45 + \dfrac{\phi}{2}\right) = \dfrac{1+\sin\phi}{1-\sin\phi} \\ \sigma_1 = \max\left[\dfrac{1}{2}\left(\sigma_\theta + \sigma_z \pm \sqrt{(\sigma_z - \sigma_\theta)^2 + 4\sigma_{\theta z}^2}\right), \sigma_r\right] \\ \sigma_3 = \min\left[\dfrac{1}{2}\left(\sigma_\theta + \sigma_z \pm \sqrt{(\sigma_z - \sigma_\theta)^2 + 4\sigma_{\theta z}^2}\right), \sigma_r\right]. \end{cases} \tag{9}$$

The limit state equation is defined in subsection 3.2. The random variables adopted refers to mechanical properties of the cement sheath. The First Order Reliability Method (FORM) is employed to the estimation of reliability index and, consequently, the probability of failure. According to Silva et al. [18], the method is based on the transformation of the original random variables (r.v.) into equivalent normal ones. It also involves the linearization of the limit state equation. The main advantage of the method lies in its ability to use all the statistical information of the r.v., dealing with any statistical distributions, including correlation between pairs of variables. The reliability problem is formulated as a constrained nonlinear optimization problem, which is iteratively solved by the HLRF algorithm. Further details about the method are presented in Melchers and Beck [10].

3 Numerical Applications

Two examples are presented in order to verify the analytical approach adopted (case 01) and illustrate the evaluation of the probability of failure in cement sheath (case 02).

3.1 Case 01

This example is based on Xu et al. [4] and refers to a gas production well drilled in four phases, as depicted in Figure 2. The interest lies on the last section, with drilled diameter of 8.5 inches, cased with a 7-inch liner, 35 lb/ft, P-110 grade tubing. The geometric and mechanical parameters adopted are presented in Table 1, in SI units.

The integrity of the cement sheath is evaluated for different scenarios, in which the internal pressure acting on the casing, referred as WHCP (Wellhead Casing Pressure), ranges from 10 to 70 MPa and 60 °C increase in wellbore temperature. Figures 3 and 4 illustrate the radial and tangential stress distributions, respectively, acting on the cement sheath, where can be seen the agreement to the reference values. A quasi-linear evolution of the stresses along the distance to the wellbore axis is observed. It is noticed that the maximum stress values occur at the casing-cement interface, indicating that the occurrence of failure is likely to initiate in this region. As expected, it is observed that the higher the WHCP, the greater the radial and tangential stresses acting on the cement sheath.

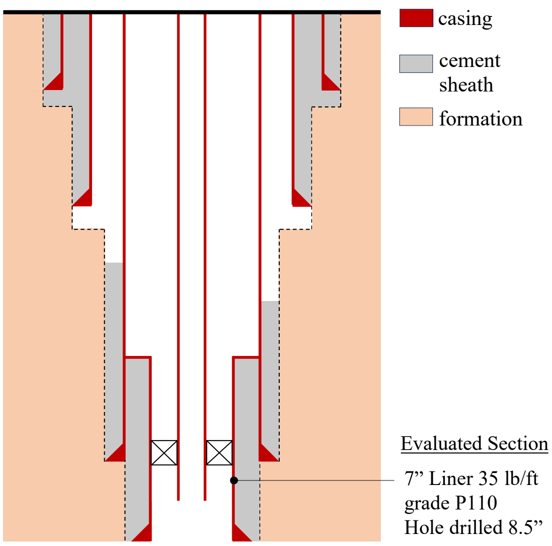

Figure 2. Well schematics

Table 1. Geometrical and material properties for case 1

Parameter	Casing	Cement	Formation
Inner Radius (mm)	76.25	88.90	107.95
Outer Radius (mm)	88.90	107.95	1079.50
Young's Modulus, E (GPa)	200	5.57	20
Poisson's Ratio, ν (-)	0.27	0.15	0.23
Thermal Expansion Coefficient, α (°C^{-1})	1.30E-05	1.00E-05	1.20E-05

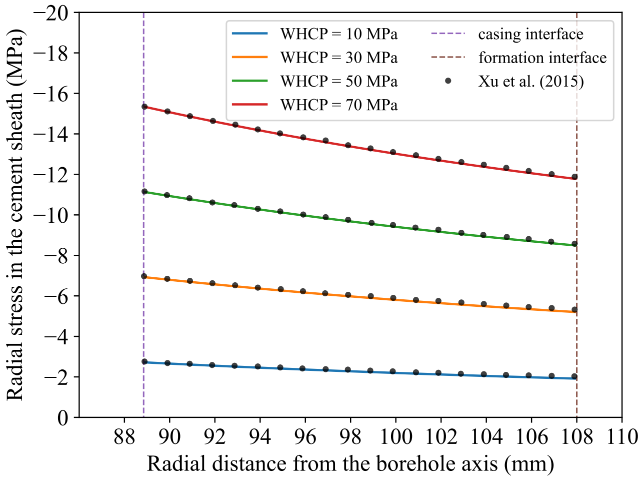

Figure 3. Radial stress distribution in the cement sheath along the borehole considering different WHCP

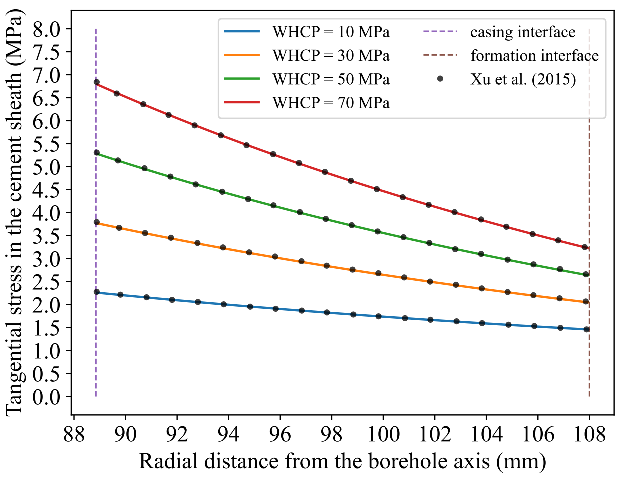

Figure 4. Tangential stress distribution in the cement sheath along the borehole considering different WHCP

3.2 Case 02

Moradi and Nikolaev [12] provide an example of probabilistic analysis of cement based on the allowable material shear stress. The mechanical properties – Young's modulus, Poisson's ratio and compressive strength – are described as r.v. Tables 2 and 3 define the geometry of the problem, and present the deterministic and statistical parameters considered.

The limit state equation, defined below, based on the Mohr–Coulomb failure criterion (Eq. 8) is used to calculate the failure probability of the cement sheath, where null or negative values of G indicate failure

$$G\left(\sigma_c, E_c, \nu_c\right) = \left(\sigma_c + q\sigma_3\right) - \sigma_1 \tag{10}$$

Table 2. Geometrical and material properties for case 2

Parameter	Casing	Cement	Formation
Inner Radius (mm)	108.40	122.25	176.21
Outer Radius (mm)	122.25	176.21	255
Compressive strength, σ_c (MPa)	-	r.v	-
Young's Modulus, E (GPa)	200	r.v	30
Poisson's Ratio, ν (-)	0.27	r.v	0.21
Thermal Expansion Coefficient, α (°C^{-1})	1.30E-05	1.00E-05	1.00E-05

Table 3. Random mechanical properties of the cement

Parameter	Distribution type	Distribution parameters
Young's Modulus, E (GPa)	lognormal	Mean = 15 GPa Standard deviation = 0.15
Poisson Ratio, ν (-)	lognormal	Mean = 0.2 Standard deviation = 0.05
Compressive strength, σ_c (MPa)	Weibull	Shape parameter = 9.506 Scale parameter = 13.59

The probability of failure is estimated for different internal pressures within the casing, considering the calculated stresses at the interface region between the casing and cement, which represents the most critical area as observed in Case Study 01. In Figure 5, it can be observed that for pressure values up to 10 MPa, the probability of failure exhibits a linear increase, with a value close to $10^{-2.5}$ (0.32%). However, for higher values, it is noticeable an exponential growth, reaching $10^{-0.9}$ (12.59%) for an internal casing pressure of 12 MPa.

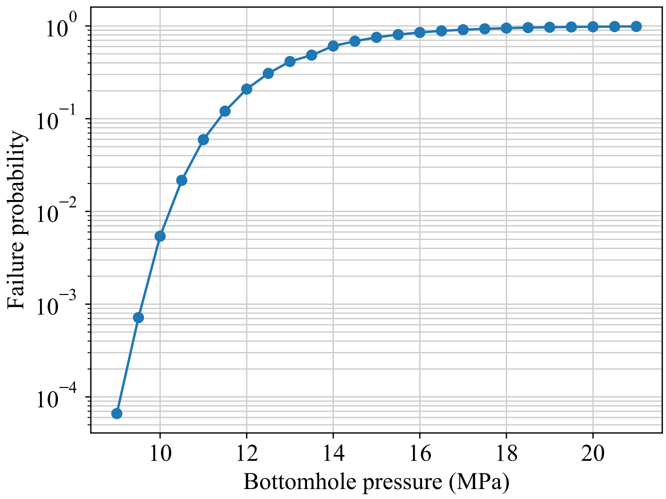

Figure 5. Probability of cement sheath failure for different internal pressures in the casing

4 Conclusions

In this study, the authors presented an analytical approach to the casing-cement-formation interaction based on a probabilistic approach. Work in this direction will contribute to disseminating the philosophy of probabilistic design applied to the integrity of oil wells. It is worth to mention that this study is comprised into the scope of the first author's thesis, and further developments are already in course, such as: poro-thermo-mechanical description of the cement, analysis of different failure modes related to stresses and displacements at the cement phase, and more complex configurations involving multiple casing and cement elements.

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References

[1] L. P. Gouveia, E. T. Lima Junior, J. P. L. Santos, W. W. M. Lira, J. L. R. Anjos, and F. L. Oliveira. Probabilistic assessment of api casing strength in serviceability limit state. *Journal of Petroleum Exploration and Production Technology*, vol. 10, pp. 2089–2104, 2020.
[2] S. T. Moradi and N. I. Nikolaev. Considerations of well cementing materials in high-pressure, high-temperature conditions. *International Journal of Engineering, Transactions C: Aspects*, vol. 29, n. 9, pp. 1214–1218, 2016.
[3] Z. Zhang and H. Wang. Effect of thermal expansion annulus pressure on cement sheath mechanical integrity in hpht gas wells. *Applied Thermal Engineering*, vol. 118, pp. 600–611, 2017.
[4] H. Xu, Z. Zhang, T. Shi, and J. Xiong. Influence of the whcp on cement sheath stress and integrity in hthp gas well. *Journal of Petroleum Science and Engineering*, vol. 126, pp. 174–180, 2015.
[5] J. De Andrade and S. Sangesland. Cement sheath failure mechanisms: Numerical estimates to design for long-term well integrity. *Journal of Petroleum Science and Engineering*, vol. 147, pp. 682–698, 2016.
[6] E. Arjomand, T. Bennett, and G. D. Nguyen. Evaluation of cement sheath integrity subject to enhanced pressure. *Journal of Petroleum Science and Engineering*, vol. 170, pp. 1–13, 2018.
[7] S. H. A. Rahman, N. N. Zulkarnain, and N. Shafiq. Experimental study and design of experiment using statistical analysis for the development of geopolymer matrix for oil-well cementing for enhancing the integrity. *Crystals*, vol. 11, n. 2, 2021.
[8] A. Valov, S. Golovin, V. Shcherbakov, and D. Kuznetsov. Thermoporoelastic model for the cement sheath failure in a cased and cemented wellbore. *Journal of Petroleum Science and Engineering*, vol. 210, pp. 109916, 2022.
[9] Y. Wu, J. Zhou, J. Yang, W. Qin, T. Zhang, and Z. Wu. A study on the integrity evaluation of cement sheaths for deep wells in deep water. *Energies*, vol. 15, n. 16, 2022.
[10] R. E. Melchers and A. T. Beck. *Structural reliability analysis and prediction*. Wiley, 3rd edition, 2018.
[11] Z. Yuan, P. Gardoni, J. Schubert, and C. Teodoriu. Cement failure probability analysis in water injection well. *Journal of Petroleum Science and Engineering*, vol. 107, pp. 45–49, 2013.
[12] S. T. Moradi and N. I. Nikolaev. Assessment of the cement failure probability using statistical characterization of the strength data. *Journal of Petroleum Science and Engineering*, vol. 164, pp. 182–188, 2018.
[13] A.-P. Bois, A. Garnier, G. Galdiolo, and J.-B. Laudet. Use of a Mechanistic Model To Forecast Cement-Sheath Integrity. *SPE Drilling & Completion*, vol. 27, n. 02, pp. 303–314, 2012.
[14] Y. Fei and G. Deli. Prediction of sustained production casing pressure and casing design for shale gas horizontal wells. *Journal of Natural Gas Science and Engineering*, vol. 25, pp. 159–165, 2015.
[15] J. F. Labuz and A. Zang. Mohr–coulomb failure criterion. *Rock mechanics and rock engineering*, vol. 45, pp. 975–979, 2012.
[16] A. M. Al-Ajmi and R. W. Zimmerman. Relation between the mogi and the coulomb failure criteria. *International Journal of Rock Mechanics and Mining Sciences*, vol. 42, n. 3, pp. 431–439, 2005.
[17] L. Zhang, P. C. Cao, and K. C. Radha. Evaluation of rock strength criteria for wellbore stability analysis. *International Journal of Rock Mechanics and Mining Sciences*, vol. 47, n. 8, pp. 1304–1316, 2010.
[18] T. B. Silva, E. T. Lima Junior. Junior, and L. P. Gouveia. Structural reliability applied to analytical modeling of the tensile strength of standard api casing connections. *Brazilian Journal of Petroleum and Gas*, vol. 13, n. 4, 2019.