Reliability-Based Design Optimization of Steel Frames Using Genetic Algorithms and Artificial Neural Networks

Laís B. Lecchi[1,2], Francisco A. Neves[1], Eduardo S. Cursi[2], Ricardo A. M. Silveira[1], Walnório G. Ferreira[3]

[1]*Dept. of Civil Engineering, Federal University of Ouro Preto*
122 St. Diogo de Vasconcelos - Ouro Preto, 35400-000, Minas Gerais, Brazil
lais.lecchi@aluno.ufop.edu.br, fassis@ufop.edu.br, ricardo@ufop.edu.br
[2]*Laboratoire de Mécanique de Normandie, National Institute of Applied Sciences of Rouen*
685 Av. de l'Université - Saint-Étienne-du-Rouvray, 76800, Normandy, France
eduardo.souza@insa-rouen.fr
[3]*Dept. of Civil Engineering, Federal University of Espírito Santo*
514 Av. Fernando Ferrari - Vitória, 29075-910, Espírito Santo, Brazil
walnorio@gmail.com

Abstract. In structural optimization, for both Deterministic Design Optimization (DDO) or Reliability-Based Design Optimization (RBDO) approaches, the nature of the objective function remains the same, as minimizing the weight, for example. However, RBDO formulation differs from DDO by the possibility of finding the optimal solution considering failure probabilities limits or target reliability indices as design constraints. Classical methods found in the literature can do reliability assessment. Nonetheless, due to convergence problems and the considerable computational effort required, it is interesting to employ other techniques like surrogate models in order to reduce the processing time. Thus, this work intents to compare a traditional double-loop RBDO analysis and a machine learning based RBDO model, by using Artificial Neural Networks (ANNs), in a single floor steel frame example. First order structural analysis is considered, and Genetic Algorithms perform the optimization. The First Order Reliability Method (FORM) calculates the reliability index. The numerical example shows how the ANN performance and accuracy are quite dependent on its architecture and on the available number of training samples.

Keywords: RBDO, Artificial Neural Networks, Genetic Algorithms, FORM, Structural Optimization

1 Introduction

The Reliability-Based Design Optimization method can balance cost and reliability assurance by accounting for uncertainties in the design process [1]. However, the difficulty in RBDO analysis is the amount of computational effort required to evaluate reliability constraints. In this way, introducing surrogate models in the process can be an efficient alternative, since they work as function approximators [2], reducing a complex system to simple operations with matrices.

This paper presents *Classification* and *Regression* ANNs models for the RBDO analysis of a single floor steel frame, in a double-loop approach, where the optimization is the outer loop and the reliability constraints evaluation the inner one. The ANN surrogates the reliability index calculation, traditionally obtained by applying the structural analysis + reliability assessment (by FORM).

1.1 Optimization with Genetic Algorithms

Optimization is finding the best result under given circumstances. Depending on the problem, the best result means maximizing or minimizing an objective function within specific design conditions previously established [3].

Genetic Algorithms are population-based metaheuristic methods for optimization, inspired by biological evolution process [4] and were first presented by Holland in 1975 [5]. The process starts by creating a population of individuals (*chromosomes*), that will be evaluated regarding the tolerances previously set. If the population does not meet these tolerances, genetic operators, such as *elitism, crossover and mutation* are applied in order to improve the quality (*fitness value*) of individuals.

1.2 Softwares

Two softwares were used:

- ***CS-ASA*** (*Computational System for Advanced Structural Analysis*): a finite element method based program, for the structural analysis [6, 7];
- **MATLAB®**: manages all the analysis stages, as calling the structural analysis program *CS-ASA*; the reliability loop (FORM algorithm), the optimization loop and ANN application [8].

2 Artificial Neural Networks

ANNs are numerical models based on the natural functioning of brain and neurons connections. It has been introduced and used as a function approximator [2], also called *surrogate model*, or *metamodel*. Figure 1 shows the typical architecture of an ANN.

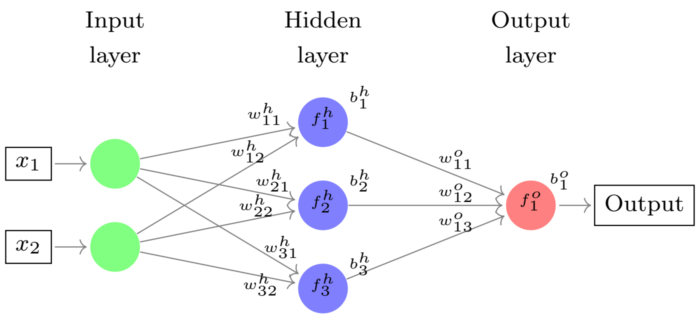

Figure 1. Hypothetical scheme of an [2]x[3]x[1] artificial neural network

The neuron is a processing unit with one or more inputs and one output [9]. Each input is weighted, summed and then added to a bias value, composing what is called activation function, according to eq. (1):

$$a^l_k = \sum_{i=1}^{n} w_{ki} x_i + b^l_k, \tag{1}$$

where a^l_k is the activation function of the k-neuron belonging to layer l; w_{ki} represents the weight applied to the input x_i of the k-neuron; b^l_k is the bias value, a constant corrective term which allows having a non-negative activation a^l_k, of the k-neuron belonging to layer l.

The output value is calculated as a function of a^l_k and it is called *transfer function*, which is commonly chosen from a list of *S-shaped* functions [10]. A typical function used for this purpose is the *hyperbolic tangent sigmoid* type, as eq. (2) shows:

$$f(a^l_k) = \frac{2}{1 + exp(-2a^l_k)} - 1. \tag{2}$$

Determining the architecture of an ANN is not a simple task and often consists of a trial-and-error process. According to Chojaczyk et al. [2] higher the complexity of a problem, larger is the number of processing elements in hidden layers. By means of an iterative process, it is possible to find the weights w_{ki} and biases b^l_k used to train the network. The training iterative algorithm is repeated until the network outputs converge to the target values.

3 Reliability-Based Design Optimization Methodology

3.1 Reliability Assessment: First Order Reliability Method (FORM)

FORM is responsible for the reliability assessment in this work. The method is an approximation of the limit state function by a tangent hyper-surface at the design point [11]. The distance from the origin to this point is what we call the reliability index β [12], as it is possible to see in Fig. 2.

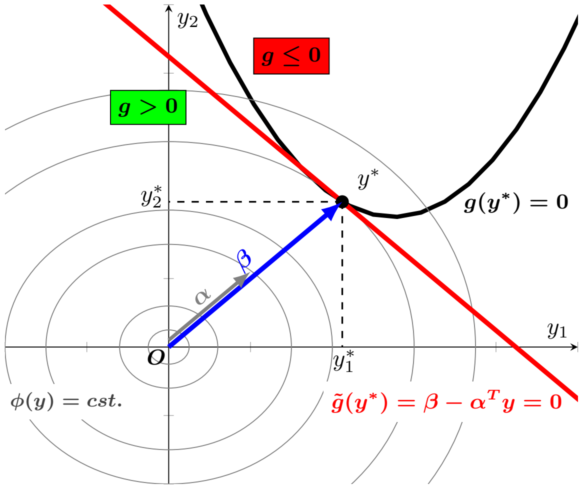

Figure 2. Scheme of the First-Order Reliability Method (FORM)

Before calculating β, we need to transform the variables from the original space $\mathbf{X}$ to an uncorrelated common standard normal space $\mathbf{Y}$ [13], as eq. (3) shows. This procedure can be done by Nataf's tranformation.

$$\mathbf{Y} = \mathbf{J}_{yx} \left\{ \mathbf{X} - \mu^{neq} \right\}, \tag{3}$$

where μ^{neq} is the normal equivalent mean of the variables. $\mathbf{J}_{yx}$ is the Jacobian matrix given by the chain rule:

$$\mathbf{J}_{yx} = \left[\frac{\partial y_i}{\partial x_k} \right] = \left[\frac{\partial y_i}{\partial z_j} \frac{\partial z_j}{\partial x_k} \right] = \mathbf{L}^{-1} (\mathbf{D}^{neq})^{-1} = \mathbf{J}_{yz} \mathbf{J}_{zx}, \tag{4}$$

in which $\mathbf{J}_{yz} = \mathbf{L}^{-1}$, $\mathbf{L}$ is the lower triangular matrix obtained from the Cholesky decomposition of the correlation matrix; $\mathbf{J}_{zx} = (\mathbf{D}^{neq})^{-1}$, $\mathbf{D}^{neq}$ is the diagonal matrix of standard deviations of equivalent normal variables.

Once we have the transformed variables, it is possible to find the most probable point of failure y^* employing the Hasofer-Lind-Rackwitz-Fiessler (HLRF) iterative algorithm. Thus, β is given by:

$$\beta = ||y^*||. \tag{5}$$

3.2 Reliability-Based Design Optimization - RBDO

In RBDO, failure probabilities limits or targets reliability indices are defined as optimization constraints:

$$P[g_i(\mathbf{X})] \leq P_f, \quad i = 1, 2, ..., n, \tag{6}$$

$$\beta_i(\mathbf{X}) \geq \beta_T, \quad i = 1, 2, ..., n, \tag{7}$$

in which $P[g_i(\mathbf{X})]$ is the failure probability of a structure for a given limit state function $g_i(\mathbf{X})$; P_f is the failure probability limit; $\beta_i(\mathbf{X})$ is the reliability index of a structure; β_T is the target reliability index.

In a double-loop approach to RBDO analysis, the optimization loop is the outer loop and the evaluation of reliability constraints is the inner one. It is noteworthy that, in this paper, the main idea of using an ANN is to replace the traditional inner loop, which employs reliability assessment and structural analysis. It is important to mention that, despite the efficiency that a surrogate model can present, obtaining samples to train a neural network is not an effortless task.

4 Numerical Example

A RBDO analysis is made for the single floor steel frame shown in Fig. 3. The problem has 8 random variables, whose statistical characteristics are in Table 1, including the applied loads *D*, *L* and *W*; the section properties: area A, inertia I_x and plastic section modulus Z_x; material properties: Young's modulus E and yield strength F_y. A notebook with a *Intel(R) Core(TM) i7-7500U CPU 2.70* GHz processor and 16 GB of RAM executes this numerical example.

The optimizer searches among 18 W-shapes (specified in section 4.2) from the AISC database (2017), to satisfy a target reliability index and the objective function, which is minimizing the total mass. Haldar and Mahadevan [14] studied this frame, but originally as a reliability problem.

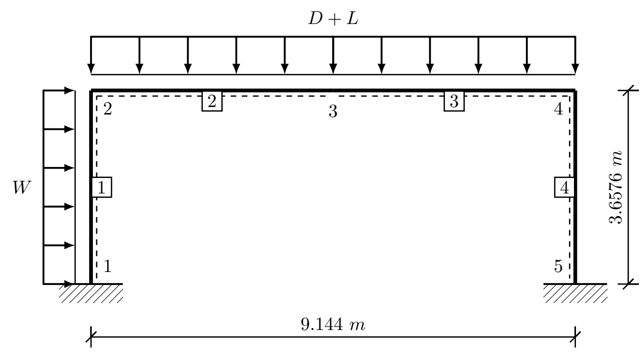

Figure 3. Single Floor Steel Frame

Table 1. Statistical properties of random variables [14]

Variable	Unit	Mean	Coefficient of variation	Distribution function
D	kN/m	6.42	0.10	Normal
L	kN/m	0.73	0.25	Ext. Value - Type 1 (largest)
W	kN/m	5.98	0.37	Ext. Value - Type 1 (largest)
A	cm^2	W-Shapes list	0.05	Normal
I	cm^4	W-Shapes list	0.05	Normal
Z_x	cm^3	W-Shapes list	0.05	Normal
E	MPa	199947.96	0.06	Normal
F_y	MPa	273.03	0.11	Normal

4.1 Limit State Function

FORM verifies one ultimate limit state, which is flexure and axial force acting on element 4, node 4. Eqs. 8 or 9 limit the interaction of efforts, as cited by AISC specification [15]:

- If $\frac{P_r}{P_c} \geq 0.2$

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \tag{8}$$

- If $\frac{P_r}{P_c} < 0.2$

$$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \tag{9}$$

where P_r: required axial strength; P_c: available axial strength; M_r: required flexural strength; M_c: available flexural strength; x: major axis bending; y: minor axis bending. Lecchi *et al.* [16] present the formulas to calculate P_c and M_c.

4.2 Design Variables

The variables are W-shapes, taken as discrete by GA optimizer, varying from 1 to 18 and then mapped to the shapes table, whose characteristics like linear mass, area (A), inertia (I_x) and plastic section modulus (Z_x) are used in the process. The 18 W-shapes considered were: W250x17.9, W200x19.3, W310x21.0, W250x22.3, W200x22.5, W150x22.5, W310x23.8, W150x24.0, W250x25.3, W200x26.6, W130x28.1, W310x28.3, W250x28.4, W150x29.8, W200x31.3, W250x32.7, W200x35.9, W150x37.1.

Three possibilities were studied: A- considering all elements with the same W-shape *(1 optimization variable)*; B- considering beam elements and columns elements with different W-shapes *(2 optimization variables)*; C- considering beam elements with the same W-shape, but allowing columns with different W-shapes *(3 optimization variables)*.

4.3 Design Constraints

Besides the lateral constraints of the previous item, there is also the reliability constraint given by a target value. Three scenarios were proposed: $\beta_{T,1} = 2.0$, $\beta_{T,2} = 2.5$ and $\beta_{T,3} = 3.0$.

4.4 Objective Function

The objective function is finding the minimum mass of the structure $M(\mathbf{X})$ (Eq. (10)), where n is the number of variables; m_i is the linear mass for a given W-shape; l_i is the length of the bar:

$$M(\mathbf{X}) = \sum_{i=1}^{n} m_i l_i, \tag{10}$$

4.5 Optimization Algorithm Setting

Besides the default options already set in MATLAB®, a population size (‘PopulationSize’) of 12 individuals was set for GA.

4.6 Results

As we proposed to perform RBDO of the structure considering 3 different reliability targets, six ANNs were trained, i.e., three for *classification* and three for *regression*, considering the minimum number of samples needed to arrive at same mass found in the literature, by the traditional approach (structural analysis + FORM loop) [16]. Bayesian optimization performs the search for the best ANN architecture. The dataset was divided in 70% for training, 15% testing and 15% for validation. Moreover, cross-validation with kfold = 5 was set.

The input of the ANNs was combinations of W-shapes, randomly chosen among the 5832 combinations obtained by permuting the 18 W-shapes list tested in this problem. The *classification* ANN predicts whether a sample violates or not the reliability constraint directly. The *regression* ANN predicts the reliability index value for an input. Table 2 shows the summary of results for the ANNs. The Root Mean Square Error (RMSE) assesses the performance of the Regression ANN and the accuracy percentage is shown for the Classification ANN. The column ‘Architecture’ contains the neurons present in each hidden layer.

In terms of objective function, the same mass was found for both ANNs types and the traditional approach

Table 2. ANN - Summary of results

	Regression ANN			*Classification* ANN		
β_T	**Samples**	**RMSE**	**Architecture**	**Samples**	**Accuracy**	**Architecture**
2.0	400	0.85787	[4]x[3] n	400	81.8 %	[14] n
2.5	2100	0.46173	[5]x[298]x[15] n	2200	89.7 %	[44] n
3.0	1800	0. 6073	[299] n	2500	91.4 %	[201] n

(structural analysis + FORM loop), specified in Lecchi *et al.* [16], as we can see in Tables 3, 4 and 5, for the three cases studied in this paper. The tables also show the reliability indices calculated using FORM, and those predicted by the regression ANN (*r-ANN*).

Table 3. Case A: one design variable

β_T	Mass (kg)	W-shape	β_i (FORM)	β_i (r-ANN)
2.0	345.6	W310x21.0	2.933	2.481
2.5	345.6	W310x21.0	2.933	2.583
3.0	391.7	W310x23.8	3.713	3.375

Table 4. Case B: two design variables

β_T	Mass (kg)	W-Columns	W-Beam	β_i (FORM)	β_i (r-ANN)
2.0	317.3	W310x21.0	W250x17.9	2.729	2.426
2.5	317.3	W310x21.0	W250x17.9	2.729	2.549
3.0	337.8	W310x23.8	W250x17.9	3.468	3.302

Table 5. Case C: three design variables

β_T	Mass (kg)	W-Column 1	W-Beam	W-Column 4	β_i (FORM)	β_i (r-ANN)
2.0	306.0	W250x17.9	W250x17.9	W310x21.0	2.626	2.423
2.5	306.0	W250x17.9	W250x17.9	W310x21.0	2.626	2.535
3.0	316.2	W250x17.9	W250x17.9	W310x23.8	3.348	3.438

Table 6 shows the time spent in RBDO using structural analysis + FORM inner loop, for each case A, B and C, being 217.8 hours in total. As GA is the optimizer, the analysis was performed 3 times to confirm that the minimum mass found was the global minimum.

For the ANNs, getting samples for the training task is the most time-consuming part, being 95.4 hours the time spent to calculate the 2500 samples, which was the maximum number requested to train the classification ANN, for $\beta_{T,3} = 3.0$. Once we have the samples, finding the best architecture and training the network requires less than 20 minutes. Finally, RBDO analysis using GA and the ANN model takes less than 2 minutes to arrive at the minimum mass. Thus, the RBDO-ANN model takes almost half of the time spent on the traditional approach, being more efficient in solving the problem.

Table 6. Traditional approach analysis time

Time $(hours)$	Case A	Case B	Case C	$\sum$
Mean $(1x)$	13.5	23.9	35.2	72.6
Total $(3x)$	40.5	71.7	105.6	217.8

5 Conclusions

As we can see in Results subsection, using an Artificial Neural Network in this RBDO analysis is effective to find the minimum mass, for both regression or classification ANN types. The regression ANN model needed less samples than the classification ANN to arrive at the minimum mass, for the cases of $\beta_{T,2} = 2.5$ and $\beta_{T,3} = 3.0$, but the reliabilities indices calculated are not so close to the response obtained by FORM.

Moreover, in terms of time spent, the RBDO-ANN model seems to be more efficient when compared to the double-loop – GA + structural analysis + FORM assessment approach.

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