Optimal risk-based design of a RC frame under different column loss scenarios

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Abstract. The sudden loss of a single supporting element in a reinforced concrete (RC) frame can result in disproportionate structural collapse if its design fails to confine the initial damage through resistance mechanisms. Given the significant impact of uncertainties related to material properties and geometric parameters on the behavior of these resisting mechanisms, and considering the high stakes involved in such failure events, risk optimization provides a practical approach to striking the right balance between cost-efficiency and safety. This is demonstrated herein through the optimization of a two-story, four-bay RC frame under two scenarios of column removal at the first floor: middle column and corner column. Design variables include cross-sectional depth, steel rebar areas, and concrete strength of beams and columns. Failure consequences are assessed for both the intact structure (considering beam serviceability, beam bending, shear failure of beams, and flexo-compression failure for the columns) and for both column removal scenarios (involving steel rupture of the top rebar layer at the interface between the beam and adjacent column, shear failure of beams, and flexo-compression failure of the columns). A physical and geometrical nonlinear static analysis is conducted, with sample points subjected to bay pushdown analysis. Material behavior is characterized by an elastoplastic model with isotropic hardening for the steel rebars and the Mazars μ model for the concrete (using the modified Park-Kent model for calibration reference). Failure probabilities are assessed using the Weighted Average Simulation Method, and risk optimization is performed using the Firefly Algorithm. To mitigate the computational cost arising from the nonlinearities and the high number of required sample points, Kriging is employed to generate an accurate metamodel for the limit states and reliability indexes. The optimal conventional design prioritizes resistance against bending failure at the beam ends rather than serviceability failure. Beyond a certain threshold value of local damage probability, an increase in the overall frame robustness is observed for both column loss scenarios, with the most significant improvement occurring for corner column removal. This is attributed to this scenario leading to lower resistance against steel rupture and to keep bending moments at the adjacent column close to zero.

Keywords: kriging, progressive collapse, reinforced concrete, reliability analysis, risk optimization

1 Introduction

Progressive collapse happens when an initial member failure triggers the failure of the adjacent elements, in resemblance to a cascade effect, leading to a final failure with a disproportionate higher severity in relation to the initial event. When under multiple hazards, the probability of structural collapse $P[C]$ is given as:

$$P[C] = \sum_{H} \sum_{LD} P[C|LD, H]\, P[LD|H]\, P[H] \tag{1}$$

where $P[H]$ is the probability of hazard occurrence; $P[LD|H]$ is the conditional probability of local damage for a given hazard H; and $P[C|LD, H]$ is the conditional probability of collapse for a given LD and H.

This study follows Beck et al. [1, 2], considering $P[LD|H]\, P[H]$ as the probability of local damage P_{LD} to combine column loss and intact structure scenarios in a single objective function. The cost-benefit of considering two column removal scenarios in designing a RC frame while considering the realistic nonlinear structural behavior is herein addressed.

2 Formulation and implementation

The RC frame considered is shown in Figure 1. Its beams have a span of 4.00 m, cross section width of 20 cm, concrete cover of 2.5 cm, and stirrups with a diameter of 8 mm. Each column has 3.00 m, cross section width of 20 cm, concrete cover of 2.5 cm, and stirrups with diameter of 8 mm spaced by 10 cm. Each member has longitudinal rebars with yielding strength of 510 MPa and modulus of elasticity E_S = 210 GPa. Both dead load and live load are 7.0 kN/m², and an additional 2.0 kN/m due to non-structural components over the beams is considered. Since the floors are one-directional, this leads to a nominal dead load D_n and live load L_n of 16 kN/m and 14 kN/m, respectively. The design parameters to be optimized are the mean values of: the beams cross section height, top and bottom beam rebar areas, stirrups spacing in the beams, longitudinal rebar area at the columns, and column cross section height. Hence, every design variable is a random variable. No discontinuities are considered along the elements, and the same beam and column optimal design is attributed to every beam and column, respectively.

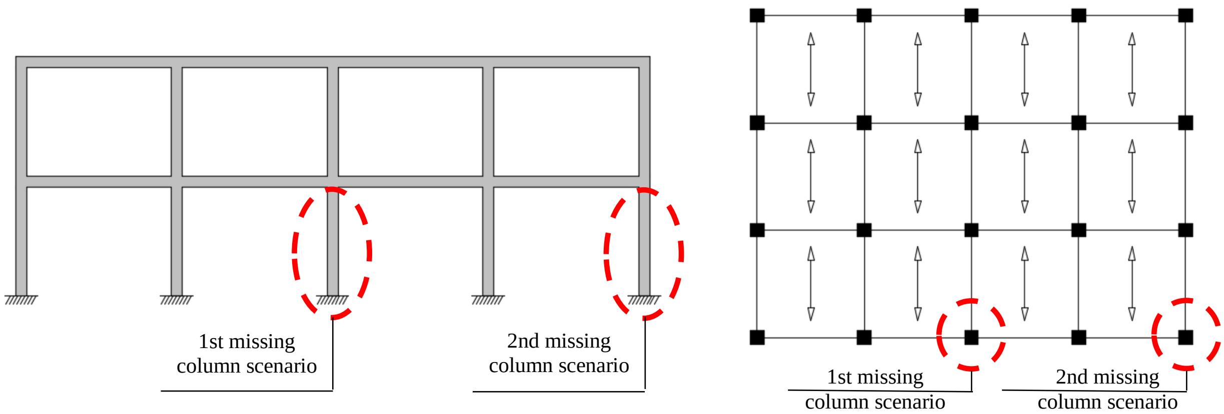

Figure 1. Studied frame

2.1 Risk optimization

The risk optimization problem follows the formulation proposed by Beck et al. [1, 2] with a total expected cost C_{TE} adapted to the RC frame studied herein:

$$C_{TE} = C_M + \sum_{i=1}^{NI} k_i \, C_M \, P_{fi} + \sum_{j=1}^{NCL} k_j \, C_M \, P_{fj} P_{LD} \tag{2}$$

where C_M is the manufacture cost; the superscripts I and CL stands for intact and column loss scenarios, respectively; P_{fi} is the failure probability of the i-th failure mode. Failure consequences are assessed for the intact structure (considering beam serviceability, beam bending, shear failure of beams, and flexo-compression failure for the columns) and for both column removal scenarios (involving steel rupture of the top rebar layer at the interface between the beam and adjacent column, shear failure of beams, and flexo-compression failure of the columns).

The SINAPI database is adopted to estimate C_M in R$, where unencumbered prices for São Paulo regarding the period of July 2023 are considered, and later is converted do €. Hence, C_M is composed by cost of formwork, obtainment of concrete, pouring of concrete, obtainment of steel rebars, and placing of steel rebars. For a given failure mode, the expected cost of failure C_{ef} is given by the product of a cost multiplier k times C_M times the probability P_f that the considered failure mode occurs. Thus, for CL the probability of local damage P_{LD} also multiplies $k \times C_M \times P_f$. The multipliers k are chosen according to the order of severity of each failure mode and regarding the real life ratio between the cost of the building and the cost of reconstruction after failure [1, 2]. Therefore, k is assumed equal to 10 for serviceability failure, 30 for beam bending failure, 40 for steel rupture at catenary action, and 80 for each fragile and severe failure mode.

2.2 Reliability analysis

The Weighted Average Simulation Method (WASM), proposed by Rashki et al. [4], is used herein to estimate the failure probabilities P_f required to compute the C_{TE}. This technique is appropriate for optimization problems involving random design variables since the estimation of P_f depends only on the index function $I(\boldsymbol{x})$ and the weight index $W(\boldsymbol{x})$ of the n_{sp} sample points, with $\boldsymbol{x}$ being the random variable vector. Therefore, changing the mean value of the candidate for optimal design only requires the re-evaluation of the weight index $W(\boldsymbol{x})$.

$$P_f = \frac{\sum_{k=1}^{n_{sp}} I(\boldsymbol{x}_k)\, W(\boldsymbol{x}_k)}{\sum_{k=1}^{n_{sp}} W(\boldsymbol{x}_k)} \quad (3)$$

The uncertainties adopted in this work are addressed in Table 1. A total of 7 million sample points are used to estimate every P_f for 2000 optimal candidates, which are generated via Latin Hypercube Sampling over the design domain. These optimal candidates are used to elaborate a metamodel for every $\hat{\beta} = -\Phi^{-1}(\hat{P}_f)$, reducing even further the computational cost to compose C_{TE}.

Table 1. Uncertainties considered

Variable	Distribution	Mean (μ)	Standard deviation (σ)	Coefficient of variation (δ)	Reference
Beams cross section height (h_B)	Normal	To be optimized*	1 mm	-	[5]
Bottom rebar area (A_B)	Normal	To be optimized*	-	0.05	[5, 6]
Top rebar area (A_T)	Normal	To be optimized*	-	0.05	[5, 6]
Spacing between the beam’s stirrups (s_t)	Normal	To be optimized*	-	0.05	Assumed
Rebar area of the columns (A_C)	Normal	To be optimized*	-	0.05	[5, 6]
Columns cross section height (h_C)	Normal	To be optimized*	1 mm	-	[5]
Beam concrete strength (${f'_c}_B$)	Lognormal	To be optimized*	-	0.12	[7, 8]
Column concrete strength (${f'_c}_C$)	Lognormal	To be optimized*	-	0.12	[7, 8]
Yielding strength (f_y)	Normal	510 MPa	-	0.05	[6, 8]
Concrete’s self-weight (γ_c)	Normal	25 kN/m³	-	0.05	Assumed
Ultimate steel strain (ε_{su})	Normal	0.20	-	0.14	[9]
Dead load (D)	Normal	1.05 D_n	-	0.10	[10]
50-year live load (L_{50})	Gumbel	1.00 L_n	-	0.25	[10]
Arbitrary point in time live load (L_{apt})	Gamma	0.25 L_n	-	0.55	[10]
Model error (E_M)	Lognormal	1.107	-	0.229	Obtained

2.3 Structural analysis

In order to estimate the probabilities of failure, a metamodel via kriging is employed [11], which requires the evaluation of a sufficient number of support points by an accurate model of structural analysis. The finite element method based on positions proposed by Coda [12] is used herein, where layered 2D beam elements are adopted.

Each beam is discretized into 3 finite elements with a eight-degree of approximation, and each column into 1 finite elements with the same degree. A total of 15 layers with 1 integration point each is used to discretize the cross-sections, being 13 layers for the concrete core and one for each steel reinforcement. An uniaxial model with isotropic hardening is used to represent the elastoplastic behavior of the longitudinal rebars, while μ-Model [13] is used to represent the damage evolution and the unilateral behavior of the concrete. Stirrups cannot be explicitly considered, but its influence on the ductility of the confined concrete is regarded by considering the resulting uniaxial curve from the Modified Park-Kent Model [14] to calibrate the parameters of the μ-Model.
Two structural analysis are carried out for every sample point: one for the intact structure (I), where an increasing uniform load is applied over each beam, and one for the column loss scenario (CL), where the uniform load is increased only over the beams directly affect by the column removal. Due to the symmetry in the structural geometry and loading conditions, only half of the structure is modelled for both scenarios.

3 Results

The following results were obtained considering 2500 support points for metamodeling the structural response (thus the limit states), which allowed the obtainment of 15 million sample points to guarantee the estimative of additional 2500 support points for metamodeling the reliability indexes of each failure mode considered. Firefly algorithm [16] was used for the risk optimization (40 fireflies, 100 iterations + auxiliary extensive search). Table 2 shows the optimal design for every value of P_{LD} ranging from P_{LD}^{min} = 5E-6 [1, 2] until P_{LD} = 1.0.

Table 2. Optimal values for each design variable according to P_{LD}

Variable	Middle column loss scenario			Corner column loss scenario		
P_{LD}	5×10^{-6}	10^{-2}	1	5×10^{-6}	10^{-2}	1
h_B (mm)	424	466	492	424	449	498
A_B (cm²)	3.30 (~3ϕ12)	3.30 (~3ϕ12)	8.01 (~4ϕ16)	3.30 (~3ϕ12)	3.30 (~3ϕ12)	11.84 (~3ϕ22)
A_T (cm²)	6.66 (~4ϕ14)	10.02 (~4ϕ18)	8.47 (~4ϕ17)	6.67 (~4ϕ14)	10.54 (~4ϕ14)	11.55 (~3ϕ22)
s_t (mm)	200	110	106	200	106	105
h_C (mm)	300	300	300	300	300	367
A_C (cm²)	4.60 (~4ϕ12)	4.60 (~4ϕ12)	6.91 (~4ϕ15)	4.60 (~4ϕ12)	9.97 (~4ϕ18)	9.00 (~4ϕ17)
${f'_c}_B$ (MPa)	45.0	32.8	45.0	45.0	42.5	43.2
${f'_c}_C$ (MPa)	30.0	36.7	44.4	30.0	37.6	40.2
C_M (€)	3724.36	4307.07	4912.88	3724.63	4817.17	5966.90
C_{TE} (€)	3760.10	4396.07	5079.74	3760.37	5131.24	6048.47

Optimal conventional scenario is guided by bending failure at the beam ends (negative moments), with optimal reliability index of ~3.4 and an increased top reinforcement (Figure 2a and 2b). Since the beam has a small length (4 m), this optimal detailing leads to a much greater reliability index for the beam serviceability failure (maximum vertical displacement).
Table 2 shows, for the studied frame, that the adopted formulation results in greater overall robustness for corner column loss scenarios (increased cross-section depth, greater reinforcement area, greater concrete strength, and reduced stirrup spacing at the beams). According to the nonlinear structural analysis, the ultimate load for steel rupture after sudden loss of a corner column is found to be 75% of this corresponding value for middle column loss. This is possibly due to the lack of an efficient Vierendeel action (small number of storeys), which reduces the amount of alternative load paths for this case. Hence, more material has to be allocated in order to provide a similar optimal reliability index of ~3.4 against this failure mode (Figure 2a and 2b). Figure 2d shows that the expected cost of steel rupture grows faster for the corner column loss scenario.

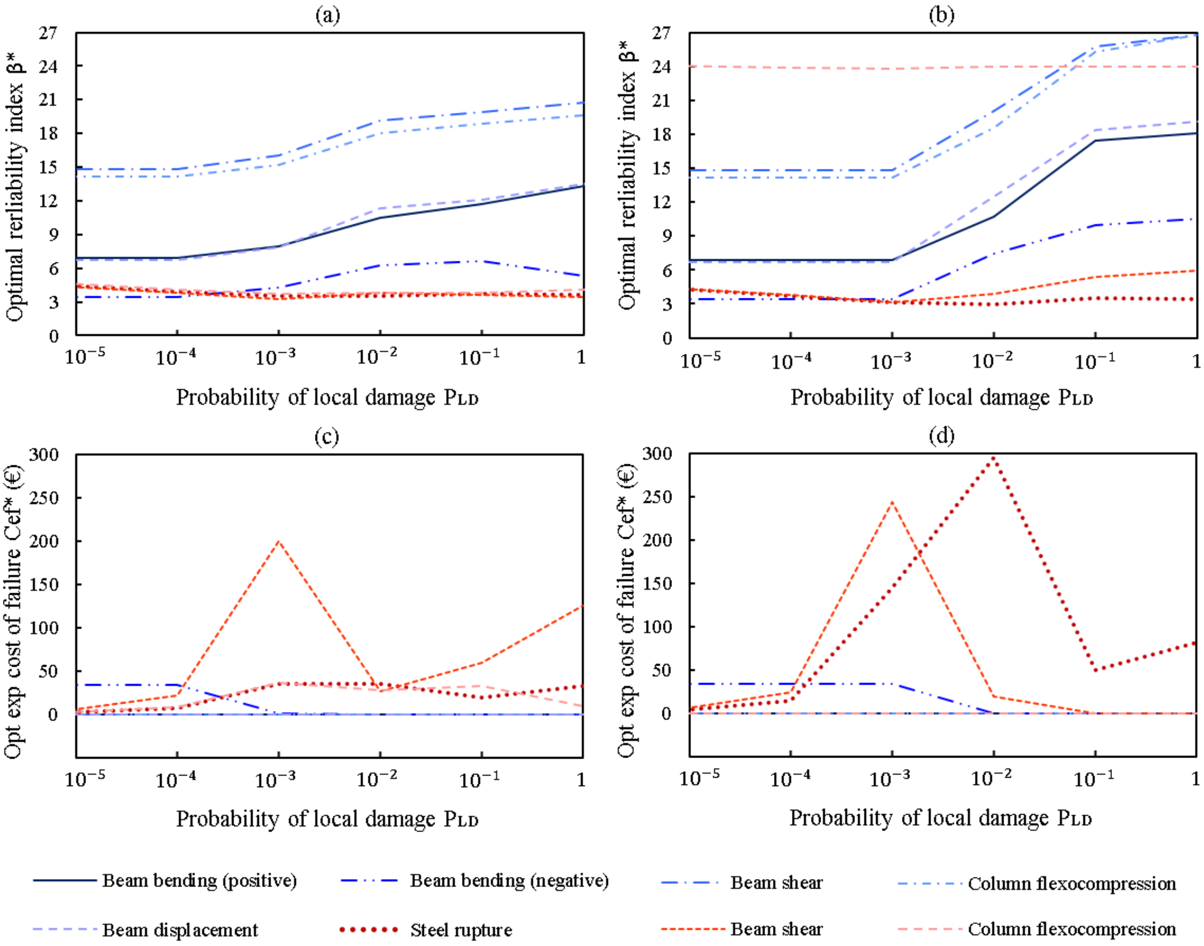

Figure 2. Optimal reliability indexes x P_{LD}

Optimal beam detailing against steel rupture for the middle column scenario provides a similar reliability index for shear failure. However, the additional strength against steel rupture for loss of a corner column leads to a shear resistance considerably increased in terms of the actual shear demand (optimal reliability index of ~5.9, Figure 2b). Besides, Figures 2c and 2d shows that shear failure is the first column loss scenario failure mode to be addressed. The greater increase in expected cost of shear failure happens due to the optimal conventional design not being able to withstand the significantly increased shear demand after a sudden column removal.

Middle column removal leads to both axial forces and bending moments at the adjacent columns significantly increased, so a greater loading demand is observed for this scenario (optimal reliability index of ~4.1). Bending moments at the adjacent column for corner column removal are significantly smaller, so loading demand is smaller and mostly due to axial forces. Nonetheless, optimal column design for this scenario has greater robustness in order to decrease the column horizontal drift, simultaneously assisting to mitigate premature steel rupture, reducing the bending moment demand and increasing column strength. In this case, the optimization algorithm shows that ensuring a near zero probability of adjacent column failure is more cost-effective.

4 Conclusions

This manuscript shows how the behavior of the optimal design for the considered RC frame suddenly changes after the consideration of two column loss scenarios starts to have a positive cost-benefit. The optimal conventional design prioritizes resistance against bending failure at the beam ends rather than serviceability failure. Beyond a certain threshold value of local damage probability, an increase in the overall frame robustness is observed for both column loss scenarios, with the most significant improvement occurring for corner column removal. This is attributed to this scenario leading to lower resistance against steel rupture and to keep bending moments at the adjacent column close to zero.

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