Probabilistic Analysis of Embankment Stability on Soft Soils using Random Fields

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Abstract. It is widely recognized that geotechnical properties vary spatially due to mineralogical composition, stress history and deposition processes, even within soil layers that appear to be homogeneous. Recently, there are studies that capture the spatial variability from input variables and random fields to create models. These models can be integrated with finite element methods (FEM) to calculate slope stability. In the present study, the objective was to conduct a probabilistic analysis using random fields applied to the stability of embankments on soft soils. The Local Average Subdivision (LAS) method was employed to generate random fields of undrained shear strength (Su) which were incorporated into the finite element analysis using Abaqus software. To validate the implementation of random fields, modeling experiments on two literature slopes were developed and compared the obtained results with the original data. The comparison revealed acceptable agreement, indicating the effectiveness of the random field implementation approach.

Keywords: inherit variability; undrained shear strength; strength reduction, Monte Carlo, Local Average Subdivision

1 Introduction

Soil variability is widely acknowledged to have a significant impact on both the local mechanical behavior of slope stability and the overall response of large structures (Griffiths et al., [1]). When using homogeneous soil models in deterministic or probabilistic analysis to represent strength and stiffness, non-conservative outcomes may arise due to the inherent randomness in soil properties (Liu et al., [2]). In such cases, Random Field Theory becomes an essential tool in slope stability analysis to incorporate more realistic soil discretization and failure modes.

Various approaches have been developed in the literature to account for spatial randomness in soils during slope stability analysis. Among these, the Random Field Finite Element Method (RFEM) proposed by Fenton and Griffiths [3] emerges as the predominant approach. It involves generating random fields of soil properties using the Local Average Subdivision (LAS) method and employs finite element analysis for stress and strain computations.

The present paper addresses the application of the Local Average Subdivision Method (LAS) to generate random fields of undrained shear strength (Su) which were introduced in finite element analysis using Abaqus [4] software. To validate the implementation of random fields, modeling experiments on two literature slopes were considered showing in general good correspondence.

2 Probabilistic Slope Stability

2.1 Random Field Theory

Soil variability can be considered to have two components (Phoon and Kulhawy, [5]): a deterministic (often depth-dependent) trend component, and a fluctuation component, following Eq.1.

$$\xi(x) = t(x) + w(x) \tag{1}$$

where, $\xi(x)$ is the geotechnical parameter to be modelled, $t(x)$ is the trend component and $w(x)$ is the fluctuation component, or noise. The fluctuation term $w(x)$ can be associated to statistical properties as mean μ, standard deviation σ, and correlation length Θ. Different methods can be found in the literature for noise generation. In the present work the model proposed by Fenton and Griffiths [3], Local Average Subdivision (LAS) was used. The LAS method divides the global averages into subdivided regions in such a way that local averages of the divisions preserve the overall global parent value. This method provides a guideline for generating the required cell-to-cell variation dictated by a correlation length structure.

2.2 Probability of Failure

When considering finite element solution to investigate probability of failure, a numerical convergence criterion can be utilized to define the failure cases coupled with the inspection of plastic zones (indicative of critical slip surface) or considerable slope deformation (Zienkiewicz et al. [6]; Matsui and San, [7]; Dyson and Tolooiyan, [8]). In fact, numerous repeated simulations of random field realizations are needed in a Monte Carlo Method (MCM) approach to establish a probability of failure P_f:

$$P_f = \frac{n_f}{n_t} \tag{2}$$

Where, n_f is the number of simulation instances reaching slope failure; n_t and is the total number of simulation instances.

The MCM stabilization criteria can be directly defined through the median P_f verification with its variance at a desired confidence level, following Eq. 3 (Melchers and Beck [9]).

$$\bar{P}_f - k * \sigma_{\bar{P}_f} \leq P_f \leq \bar{P}_f + k * \sigma_{\bar{P}_f} \quad \text{with} \quad \sigma_{\bar{P}_f} = \sqrt{\frac{P_f(1-P_f)}{n}} \tag{3}$$

where, $\bar{P}_f$ is the approximated probability of failure; k is the normal standard deviate dependent on the desired confidence level ($k = 1.96$ for 95% confidence level) and n is the number of realizations.

3 Implementation

Starting from a theoretical distribution, either normal or log-normal, with mean and standard deviation of the parameter under consideration, as well as the correlation length, the software RFEM by Fenton and Griffiths [3] was used to generate random fields of undrained strength (Su). These fields are defined according to a mesh specification (mesh size), resulting in a spatial distribution of properties by coordinates. This spatial distribution was then imported as tabular data to compose a distribution law to be considered as the constitutive parameters in the Abaqus [4] software. CAX4P elements were considered in the presented analysis. The CAX4P element is used in axisymmetric cases, with four nodes and bilinear deformation, also considering pore pressure. The number of elements and the details of geometric sections can be viewed in subsequent items.

The subdivision of the geometries in the software should generally follow the same premises as the established random field. That is, when the finite element geometry in the software Abaqus is divided into multiple distinct element partitions, material strength parameters are assigned for each element based on the coordinates of an associated random field cell.

When mapping a random field to the finite element domain, the effects of the local averaging process must be considered. It is crucial to verify whether the criterion of local means is preserved, particularly when a log-normal distribution is used. Random fields generated by LAS, requiring a lognormal distribution must be transformed from the standard normal Gaussian with as follows (Dyson and Tolooiyan, [8]):

$$X_i = \exp\left[\mu_{\ln X} + s_{\ln X} G(x_i)\right] \tag{4}$$

where Xi is the transformed soil property of the ith element; xi is coordinates of the center point of the ith element; G(xi) is the standard normal random field; and μ_{lnX} and σ_{lnX} are the mean and standard deviation of the underlying normal distribution lnX, determined by Eqs. (10) and (11) as follows:

$$\mu_{\ln X} = \ln \mu - \frac{1}{2} s_{\ln X}{}^2 \tag{5}$$

$$s_{\ln X}{}^2 = \ln\left(1 + \frac{s^2}{\mu^2}\right) \tag{6}$$

After verifying the generated random fields, a Python routine was implemented to enable the automatic generation of the required realizations for the Monte Carlo Method (MCM). This routine facilitates the repetitive process of generating multiple realizations with different parameter values, allowing for efficient probabilistic analyses and uncertainty quantification in the subsequent simulations.

3.1 Palomino-Tamayo, Awruch e Rodríguez-Calderón [10] section

Palomino-Tamayo, Awruch, and Rodríguez-Calderón [10] conducted a probabilistic analysis of slope stability using the Finite Element Method (FEM), the Monte Carlo Method (MMC), and the LAS method for generating stochastic fields. The study was implemented using the RFEM routines developed by Fenton and Griffiths (2008), considering a typical section with a slope inclination of 2:1. Additionally, boundary conditions are identified, including a fixed base and lateral sliding supports in the vertical direction. The mesh is a square with a side length of 0.05 m. The horizontal direction is discretized into 60 elements (3 m), and the more extensive vertical direction consists of 15 elements (0.75 m). In total, there are 610 eight-node plane elements with the assumption of plane deformation.

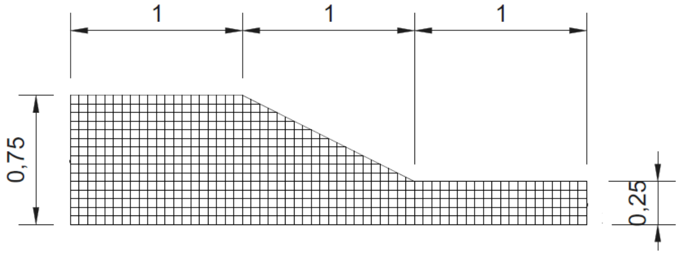


Figure 1. Palomino-Tamayo, Awruch, and Rodríguez-Calderón [10] section

An elastoplastic model with the Mohr-Coulomb criterion was considered by the authors varying undrained shear strength (Su), Young's modulus (E), and unit weight (γ). For sake of simplicity in the present paper only the variability of the undrained shear strength (Su) was considered, following a normal distribution, with 2 kPa as mean value (μ) and 0.5kPa as standard deviation (σ), vertical and horizontal correlation distances (Θh and Θv) of 1m. The following fixed parameters were adopted: friction angle (ϕ) = 0; dilation angle (ψ) = 0; Poisson's ratio (ν)=0, Young's modulus (E) =100.000kPa; unit weight (γ) of 20kN/m^3. These set of parameters were used in the RFEM software to generate random fields. Also, in order to validate the implementation made in the Abaqus software, the results of the probabilistic analysis carried out in the RFEM itself were also evaluated.

Figure 2 shows the evolution of the failure probability according to the number of realizations (n). A reference of 4000 iterations was established for the comparative analysis. The RFEM software resulted in a failure probability of 4.7%. On the other hand, the implementation in Abaqus characterized a failure probability of 2.2%. For the Abaqus results, it was possible to establish a deviation for the probabilities obtained using a 95% confidence interval (Eq. 3).

Initially, it should be noted that there are differences in the failure criteria between the numerical modeling software RFEM and Abaqus. For the RFEM program, there is an option to set the convergence tolerance and the maximum number of simulations (with a default maximum of 500 iterations). In the case of Abaqus, there are tolerance criteria for deformations on the order of 10^{-8}, among other predefined criteria. Despite considering these criteria and seeking more accurate results, perfect congruence was not achieved. Therefore, inherent differences exist between the software, encompassing the type of elements considered, the interpolation function used, and the internal resolution structure.

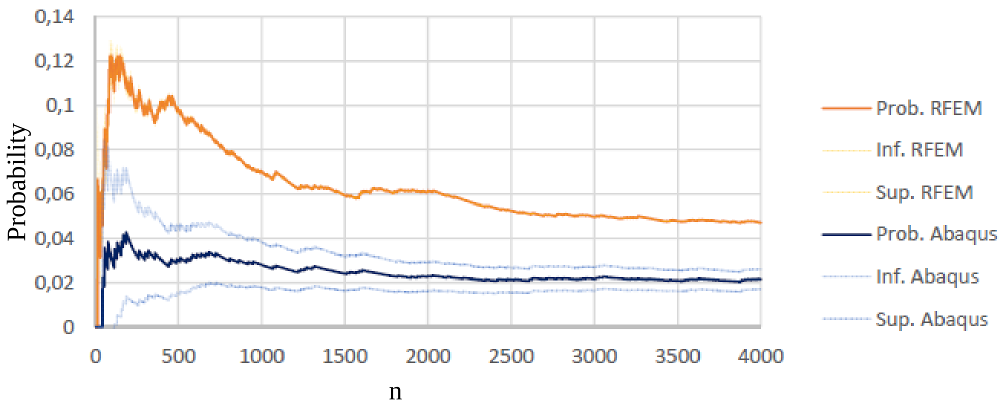
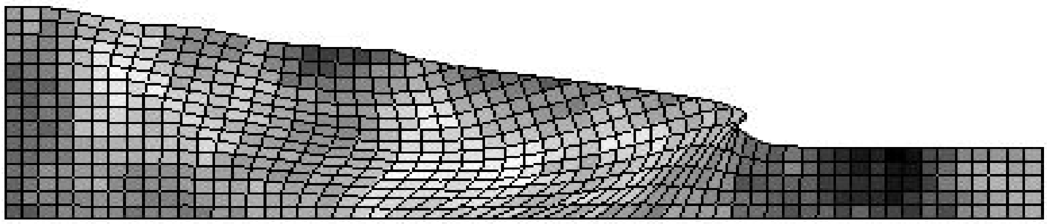
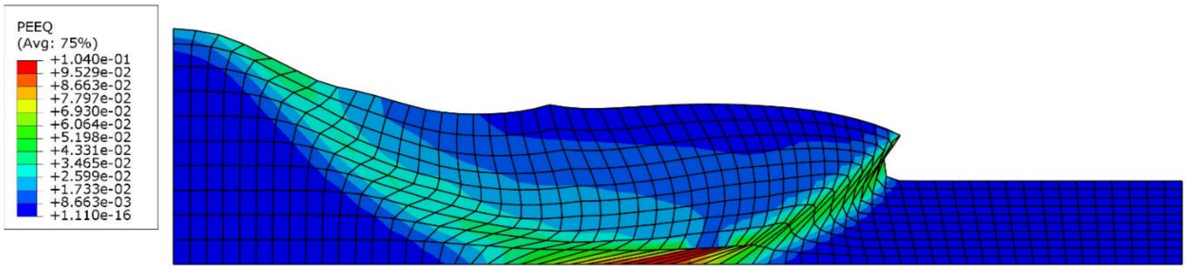


Figure 2. Probability analysis stabilization - Palomino-Tamayo, Awruch, and Rodríguez-Calderón [10] section

Furthermore, aiming to validate the implementation in Figure 3, using the same seed described for each model (the same random field), an inspection of the failure surface is presented, where a good agreement between models can be observed. This inspection was performed for different seeds throughout the modeling, demonstrating similar behavior.



(a)



(b)

Figure 3. Failure surface Palomino-Tamayo, Awruch, and Rodríguez-Calderón [10] (a) RFEM; (b) Abaqus

3.2 Liu et al. [2] section

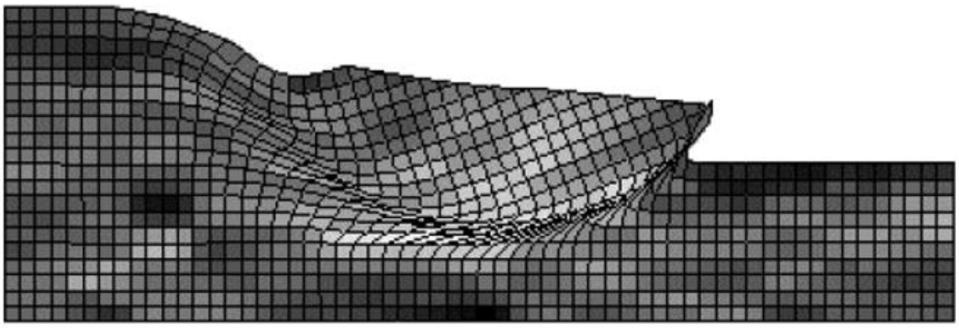
The article by Liu et al. [2] compared the stability results of slope analyses between two-dimensional (2D) and three-dimensional (3D) analyses. The authors pointed out that the 3D finite element analysis, using random fields, adequately reflects the spatial variability of soil properties. However, it often requires more time than the 2D analysis. This time is significantly increased when performing probabilistic analysis and implementing methodologies to identify safety factors such as the Strength Reduction Factor (SRF). For this reason, in their research, the authors selected a 2D cross-section analysis to be the less advantageous (or more pessimistic) inside a 3D model. The term "more pessimistic" refers to the cross-section with the lowest average value of undrained shear strength. The selection of the most pessimistic cross-section was obtained by generating a three-dimensional random field of undrained shear strength and calculating the mean value of Su. The study was executed using the RFEM routines developed by Fenton and Griffiths [3] only to generate the 3D random fields. Then, they were imported to analysis in Abaqus software.

In order to simplify the problem, the present study did not perform an analysis of the more pessimistic 2D cross-sections described by Liu et al. [2]. Only the basic cross-section described by the authors was used, considering the Mohr-Coulomb elastoplastic model with the following parameters: undrained shear strength (Su), following a log-normal distribution, with 22 kPa as mean value (μ) and 6.75kPa as standard deviation (σ), vertical correlation distance (Θv) of 2m and horizontal correlation distance (Θh) of 10m. The following fixed parameters were considered: friction angle (ϕ) = 0; dilation angle (ψ) = 0; Poisson's ratio (ν)=0.49, Young's modulus (E) =100.000kPa; unit weight (γ) of 18kN/m3. These set of parameters were used in the RFEM software to generate random fields and then they were imported to Abaqus software. The mesh is a linear quadrilateral element. In total, there are 1034 elements with the assumption of plane deformation. The horizontal direction has 30 m, and the more extensive vertical direction consists of 10 m. Also, in order to validate the implementation made in the Abaqus software, the results of the probabilistic analysis executed in the RFEM itself were also evaluated.

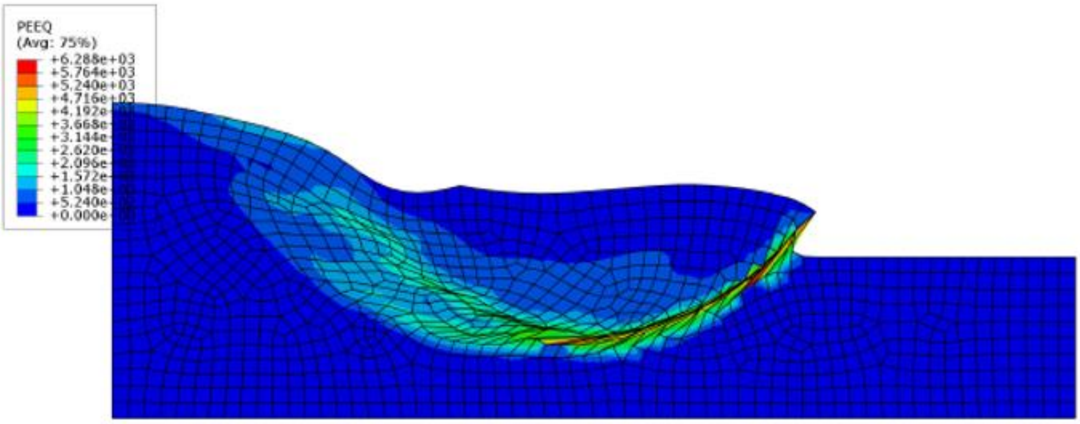
Using RFEM, a failure probability of 0.58% was found, while Abaqus yielded a probability of 0.02% from

5000 iterations. As observed in the results obtained from the model by Palomino-Tamayo, Awruch, and Rodríguez-Calderón [10], the probability obtained using Abaqus showed a less conservative value than that presented by the RFEM program.

Finally, in Figure 4, an example of the displacement surface is also provided for both previously mentioned software. Similar surfaces formed by the same random field are compared, with the same configuration of the failure modes.



(a)



(b)

Figure 4. Failure surface – Liu et al. [2] (a) RFEM; (b) Abaqus

4 Conclusions

The present paper presented the results of implementing random fields in the Abaqus software by verifying two cases from the literature that used the RFEM software as reference. Initially, it is pointed out that there are differences in the failure criteria between the numerical modeling software RFEM and Abaqus. Distinct tolerance criteria, different finite elements, interpolation functions, and internal resolution structures which lead to different results for the calculated probabilities. In general, the probability values obtained by the RFEM software are more conservative. However, when observing the deformation surfaces with the same seed (same random field), there was good correspondence between models. It is worth highlighting that the advantage of the implementation using the Abaqus software lies in its broader range of finite elements and constitutive models, allowing more sophisticated and complex analysis.

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