

Dynamic Analysis of a Prestressed Concrete Railway Bridge Considering Cyclic Fatigue Load due to Railway Traffic

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Abstract. The dynamic interaction between railway train composition, track, and bridge considering increased deflections under fatigue loading is realized through computational analyses. The twenty five equations of motion of three dimensional railway vehicle are obtained through dynamic equilibrium. The railway train composition model is obtained by the eight vehicle's association. The railway track irregularities are represented by harmonic functions. The mechanic contact between wheels and rails model is based on Hertz, Kalker, Coulomb, Vermeulen and Johnson's theories. The rails are modeled with Euler-Bernoulli beam's. The sleepers and the ballast are modeled with spring-damper systems. The railway bridge structure in prestressed concrete is mainly composed by two simply supported I-beams, which are represented with three-dimensional frame elements based on the Euler-Bernoulli assumption. The Rayleigh's method is used in structural damping. The equations of motion of structural systems are implemented in Matlab software. Static analyses are presented considering vertical deflections due to permanent load and prestress effects. Dynamic analyses are presented considering the displacements of the sleepers, the ballast and the bridge. Results of vertical deflections considering cyclic load fatigue influences at the reinforced concrete beams are presented and analyzed.

Keywords: vehicle-track-bridge-interaction, mechanic contact, track irregularities, finite element method, concrete fatigue.

1 Introduction

The phenomenon of fatigue occurs when a material loses its original strength due to successive cyclical loads. In reinforced concrete structures, the fatigue depends on the load amplitude and the number of cycles as well as on the stress level, Olsson and Pettersson [1].

The railway bridges structures are submitted to $(10^5 \text{ to } 10^6 \text{ cycles})$ in their live in service, being very important to investigate the dynamic behavior of the materials used in design. In prestressed concrete structures, the fatigue of cables and concrete is investigated. The most used methodologies to studied the fatigue in these materials are Paris law, Mechanics Fracture, Palmgren-Miner and Continuous Damage Mechanics.

This work are presented a fatigue study based on cyclic load effects according to Brazilian standard NBR-6118. Firstly, are simulated the static response of the railway bridge with respect to permanent load and prestress effects. The bridge are modeled with finite elements, and the train composition are modeled through dynamic of multi-body systems. These numerical models are coupled through mechanic contact with wheels and rails.

Secondly, are calculate the vertical deflection of the bridge considering permanent load, prestress effects, and traffic load simulations. Finally, the cyclic load effects are estimated according to Brazilian standard.

2 Railway vehicle and train models

The railway vehicle model is composed of a car body, front and rear bogies, and four wheelsets, interconnected by springs and dampers that represent the suspension systems. This model is capable of representing the bouncing, rolling, and pitching motions of the car body, and bogies, and also the bouncing, lateral, rolling and yawing motions of the four wheelsets, totalizing 25 degrees-of-freedom.

The train composition model is composed of an association of 8 railway vehicles (totalizing 200 degreesof-freedom). It represents a urban Brazilian composition employed for passenger transportation composed of two Electrical Train Units. The Fig. 1 illustrates a urban Brazilian composition and railway vehicle model.



Figure 1. Urban Brazilian composition and vehicle model.

The equations of motion of the train composition can be written in matricial form below:

$$[M_{T}][\ddot{U}_{T}(t)] + [C_{T}][\dot{U}_{T}(t)] + [K_{T}][U_{T}(t)] = [P_{T}(t)].$$
(1)

Where $[M_T]$, $[C_T]$, and $[K_T]$ are the mass, damping, and stiffness matrices, respectively, and $\{\dot{U}_T(t)\}$, $\{\dot{U}_T(t)\}$, $\{U_T(t)\}$, and $\{P_T(t)\}$ are the acceleration, velocity, displacement and external forces vectors, respectively. The formulation and its respective developments are presented in Beghetto and Abdalla Filho [3].

As the bridge considered in this work is a short single-span structure, the assumption is made that the train composition moves with constant speed along the bridge. Further, it is assumed that track irregularities are the only external sources of load acting on the train, and therefore back onto the bridge. Vertical and lateral irregularities are both represented by sinusoidal functions, while rotational track irregularities are represented by cosinusoidal functions. The latter produce yawing motions and hunting phenomenon.

Track irregularities oscillations excite the degrees-of-freedom of the train, which in turn are used to calculate the acting forces. Such forces are responsible for the dynamic behavior of the train and, therefore, of the whole composition while passing along the bridge.

These vehicle forces are transmitted to the rails at the wheel-rail contact points, considering wheels conicity, and wheelsets rolling motion. Applying a wheel-rail contact model which combines Hertz theory, Timoshenko and Goodier [4], and Kalker's linear theory, Zaazaa and Schwab [5], vertical forces, lateral forces, longitudinal forces and moments are applied to the rails. These forces are transferred to the sleepers, to the ballasts, and finally to the bridge's beams.

3 Track and bridge models

The track are composed of two steel rails TR68, modeled with Euler-Bernoulli finite elements. The sustainable wooden sleepers (*eucalyptus citriodora*), are modeled with discrete springs (ks=291,727MN/m), and discrete dampers (cs=0,5kN.s/m), Correa [6]. And the ballast is modeled with discrete springs (kb=48MN/m), and discrete dampers (cb=144kN.s/m), Calçada [7].

These systems are interconnected to parallel simply supported prestressed concrete I-beams thirty meters long. Each beam is modeled by thirty 3D Euler-Bernoulli frame elements of equal length, Bathe [8]. The equations of motion of the bridge's systems can be written in matricial form below:

$$[\boldsymbol{M}_{B}][\boldsymbol{\ddot{U}}_{B}(t)] + [\boldsymbol{C}_{B}][\boldsymbol{\dot{U}}_{B}(t)] + [\boldsymbol{K}_{B}][\boldsymbol{U}_{B}(t)] = [\boldsymbol{P}_{B}(t)].$$
⁽²⁾

Where $[M_B]$, $[C_B]$, and $[K_B]$ are the mass, damping, and stiffness matrices, respectively, and $\{\dot{U}_B(t)\}$, $\{\dot{U}_B(t)\}$, $\{U_B(t)\}$, and $\{P_B(t)\}$ are the acceleration, velocity, displacement and external forces vectors, respectively. A consistent mass matrix is adopted. The damping matrix $[C_B]$ is calculated using to Rayleigh's proportional damping, Bathe [8]. The theoretical definitions and procedures for constructing the frame element matrices can be found in Craig [9]. Properties of the bridge correspond to those of a railway bridge which is actually in



operation. The material and dynamical properties of the railway bridge model are presents in Beghetto and Abdalla Filho [3]. The Fig. 2 shows the railway bridge cross section and finite element model.

Figure 2. Railway bridge cross section and finite element model.

4 Static analyses

The calculation of deflections in reinforced concrete beams is quite complicated due to the reduction of the moment of inertia due to cracking and also due to the effects of concrete creep. Most of the final deformation is reached after 3 or 4 years, although the total stabilization of creep occurs in about 10 to 15 years, Pfeil [10].

The permanent load of the railway track and bridge can be computed with dead load of the railway track and self weight of the bridge. The dead load of the railway track per meter can be computed by: steel rails model TR-68, (2x67,56kg/m); wooden sleepers, (114kg/sleeper); and graded crushed granite ballast (2016kg/m).

The self weight of the bridge per meter can be computed by: slab and parapets in reinforced concrete, (3920kg/m); the prestressed concrete beams (5325kg/m) in section A, near to supports, and (2959,4kg/m) in section B, like illustrated to Fig. 2. The permanent load totalizing (111,693kN/m) near to supports, and (88,486kN/m) in others sections.

With the purpose to compensate the vertical deflection due to permanent load, the prestressed of the beams occurs in two steps: in 7 days concrete age in cables C1, C2 and C3 with effective load 1500kN in each cable, and 28 days concrete age in cables C4, C5, C6, C7 and C8 with effective load 1500kN in each cable. The cables used are CP175-RB. The total prestress effect produces upward deflection (0,0365m). The Fig. 3, shows the half lateral view of bridge beam, the positioning of parabolic cables, and cross section at mid span.



Figure 3. Half lateral view of bridge beam and cross section at mid span.

Considering the differential equation of elastic line, in Timoshenko and Gere [11], subjected to permanent load, the elastic modulus of concrete (E=29,43GPa), and moment of inertia (I=1,49m⁴) near to supports, and (I=0,93m⁴) in others sections, the maximum vertical deflection calculated is near to (-0,034m).

The effective static vertical deflection (without train traffic load), can be computed algebraically by total prestress effect and permanent load, which correspond to upward deflection (0,0025m).

5 Dynamic analyses

This section is concerned to the dynamic study of a railway track and bridge under a passing train composition considering cyclic load fatigue influences. Firstly, are presented in Fig. 4, the dynamic response of vertical displacements of the sleepers and ballast systems subjected to train composition.



Figure 4. Vertical displacements of the systems on the mid span.

In the Fig. 4 it is possible to observe the transient response of the systems, which are produced by track irregularities. The permanent response correspond to traffic of train's wheelset. In general, the response of the systems are different, because each dynamic component have different stiffness and dampers properties.

The Fig. 5 shows the vertical deflection time history of the bridge in its mid span at the resonance condition considering new structure (firsts cycles), without prestress effect.





In Fig. 5 it is possible to observe the oscillatory vibrations are produced by traffic of train composition, and by track irregularities. Also the beat motion occurs at the resonance condition. The damping of the system is observed by the logarithm decay of motion, mainly after the train composition has crossed the bridge at the final times. The constant values of vertical deflections near to (-0,034m) represent the influence of permanent load in structure, without prestress effect.

The maximum deflection obtained in this analysis is near to (-0,0358m), without prestress effect. The resonance of the bridge occurs when the frequency of the traffic train composition coincides with a first natural frequency of vertical vibration of the bridge. Considering 2,5% of damping ratio, this damped frequency is near to 4,5Hz.

The purpose of prestressed structures is to compress members, reducing tensile stresses and compensating deflections produced by permanent loads and the creep effects.

5.1 Considerations of cyclic loading in reinforced concrete structures

The low-cycle fatigue (10^{0} to 10^{2} cycles) corresponds to structures subjected to earthquakes. The high-cycle fatigue (10^{3} to 10^{4} cycles) corresponds to airport pavements and bridges, and (10^{5} to 10^{6} cycles) to highway and railway bridges, and highway pavements. And the super-high-cycle fatigue (10^{7} to 10^{9}) corresponds to sea structures.

According to Brazilian standard NBR-6118, in section of dynamic actions and fatigue, subsection ultimate limit state: The modifications introduced by repeated loads can significantly affect the structures from the point of view of their behavior in service, particularly with regard to the appearance of cracks that do not exist under static actions, to the aggravation of cracking already existing and the increased of deformations.

The increase in deformations is progressive under cyclic dynamic actions and add to the increase of deformations resulting from creep. In the absence of conclusive experimental data, the cycle effect can be estimated by the expression:

$$a_n = a_1 \cdot \left[1, 5 - 0, 5 \cdot e^{(-0, 05, n^{0.25})} \right]$$
(3)

where (a_1) is the deflection in the first cycle due to the maximum load including effects of shear strains, and (n) is the number of cycles. It is possible to observe that the increased deflections under fatigue loading are exponential functions.

The Fig. 6 shows the vertical deflection time history of the bridge in its mid span at the resonance condition considering firsts cycles (new structure), high-cycle fatigue (10⁵ cycles) and (10⁶ cycles), corresponding to railway bridges, without prestress effect.



Figure 6. Vertical deflections on the mid span considering firsts cycles and high-cycle fatigue.

In Fig. 6 it is possible to observe the total response of the system in different cycles fatigue conditions. These are composed by steady state response and transient response. The oscillatory vibrations are produced by traffic of train composition, and by track irregularities. Also the beat motion occurs at the resonance condition. The effect of permanent load contribute to increase deflection due to creep.

The maximum deflection obtained at the firsts cycles, correspond to new structure is near to (-0,0358m). When considering high-cycle fatigue (10^5 cycles), the maximum deflection is near to (-0,0463m), and (-0,050m) at the (10^6 cycles), according to Brazilian standard NBR-6118, when neglected the prestress effect.

6 Conclusions

The oscillatory response presented in the railway track and also in bridge are produced by traffic of train composition, and by track irregularities. The presence of damping of the system is observed by the decay of motion. The prestress effect produce upward deflection (0,0365m), which to purpose to compensate the vertical deflection due to permanent load (-0,034m).

The maximum deflection considering the traffic of train composition obtained at the firsts cycles, correspond to new structure is near to (-0,0358m). When considering high-cycle fatigue (10⁵ and 10⁶ cycles), the maximum deflections obtained are (-0,0463m) and (-0,050m), respectively, according to Brazilian standard NBR-6118. The maximum deflection allowed by Brazilian standard NBR-6118, is calculate as (L/350), which results in (-0,0857m), accordance with analyses presented in this work.

The increased deflections under fatigue loading are exponential functions. In the beams structures members, the increased deflections are proportional to reduction of flexure rigidity. This can be explained to the reduction of the moment of inertia due to cracking and also creep effects.

The resistance at 10 million cycles, for compression, tension or bending, is approximately 55% to 60% of the resistance under static load. In reinforced concrete beams, cracking is produced by the repeated application of approximately half the static cracking load, but complete failure does not occur unless the cyclic load is between 60% and 70% of the static load. Failure due to cyclic stress in prestressed beams almost always occurs due to cable fatigue and only rarely due to concrete crushing. Cyclic stress cracking of concrete can be avoided by limiting the maximum stress to half the static stress required to produce cracking. The failure of the structure, in relation to fatigue, is the result of the accumulation of damage caused by a multiplicity of load cycles, with different frequencies and amplitudes, Driemeier [12].

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