Experimentally estimated bipedal model parameters to simulate human-induced vibrations on footbridges

D. V. Ruiz[1], C. Magluta[2], N. Roitman[2]

[1]*Department of Structural and Geotechnical Engineering, Polytechnic School, University of São Paulo*
Av. Professor Almeida Prado 83, 05508-070, São Paulo/SP, Brazil
dvegaruiz@usp.br
[2]*Department of Civil Engineering, COPPE, Federal University of Rio de Janeiro*
Av. Athos da Silveira Ramos 149, 21941-909, Rio de Janeiro/RJ, Brazil
magluta@coc.ufrj.br, roitman@coc.ufrj.br

Abstract. The human-structure interaction in slender structures has received increasing attention from civil engineers and researchers lately. For this reason, several pedestrian models that take into account the biodynamic parameters of the human body have been widely studied. In this paper, results from a series of experimental tests are used to obtain parameters for a bipedal walking model. The ground reaction forces (GRF) induced by different people during walking on a test footbridge were measured using force platforms. The bipedal model is then adjusted to match the measured pedestrian forces. Thus, a set of non-linear regression equations are proposed to estimate the fundamental model parameters as functions of the pedestrian's mass, height, and walking speed. The new set of empirical equations resulted in improvements in the parameter expressions with a higher R-squared compared to linear regression. Then, the numerical model is used to reproduce the experimental situations of walking people on the structure, and the predicted vertical accelerations of the structure are compared with those measured experimentally. The results show the suitability of the proposed numerical model to reproduce the vibrations induced by people on pedestrian structures.

Keywords: bipedal model, ground reaction force, human-induced vibrations

1 Introduction

The problem of excessive vibrations in pedestrian structures have led to the emergence of numerous researches with focus on modeling the human gait as realistically as possible. Traditionally, walking forces have been modeled as a moving harmonic force applied to the structure considering some of its first harmonics. However, it has been demonstrated that in slender structures this is not the most appropriated way of modeling, since the human-structure interaction phenomenon can occur [1–3]. For this reason, several pedestrian models that consider biomechanical parameters of the human body, such as mass, stiffness and damping have been proposed in the literature [4–6].

The model most commonly used to represent a pedestrian is the linear mass-spring-damper model of one-degree-of-freedom, in which the human body is modeled by a mass interconnected with a spring and a damper. A lot of research has been conducted to proposed the parameters of mass, stiffness and damping of the model based on several measurements from pedestrian tests [7–9]. On the other hand, bipedal walking models recently adopted for applications to the human-induced vibrations problem in civil engineering, have been less study and lack sufficient experimental validations. This motivates further investigations of these particular kind of models.

Bipedal walking models are those that are able to reproduce the human walking behavior, that is, the legged locomotion, in which the two phases of the gait cycle are reproduced. Several bipedal models of different levels of complexity can be found in the biomechanical literature. However, some of them have been adopted specifically to study the human-structure interaction problem. In [10] a bipedal model inspired in the inverted pendulum is applied to study the effect of structure vertical vibrations in the interaction force, concluding that pedestrians in general tend to add mass and damping to the system. In [11] the inverted pendulum model is also applied to simulate the human structure-interaction and it is found out that the larger the vibration level of the structure the larger the pedestrians step-by-step variations. In [12] a more sophisticated bipedal model that considers a spring and a damper on each leg is proposed to model the interaction between a structure an a single pedestrian, and in [13] the same model combine with a finite element model of a footbridge is proposed for the dynamic analysis of slender structures under possible crowd-induced loading. Later, in [14] a more practical model is proposed with self-determined walking speed.

Although several studies have been conducted, to consider de use of bipedal models in practice it is necessary to rely on a set of parameters that can reproduce a stable walking and GRFs as close as to the real ones as possible. In that regard, [15] provided a method for reproducing the ground reaction forces measured in rigid ground. However, the average step frequency of the population that took part of that study is 1.73 Hz, which is lower than those values reported for other populations. In [16] and [17] sets of empirical linear regression equations are proposed to reproduce the pedestrian forces measured on rigid and flexible surfaces, respectively, as function of body mass, height and walking speed. This article is an extension of those previous investigations, which aims to find more precise correlations for the bipedal model parameters.

This study utilizes experimental data from [17] that comprises measured vertical GRFs induced by 16 different pedestrians. Results from multiple simulations performed to identify the best matches between the model's predictions and experimentally measured GRFs are used to propose a new set of empirical equations. Instead of linear functions, nonlinear equations that represent the fitted surfaces to the data are proposed. To validate the proposed equations, the experimental tests are simulated, and the predicted structure dynamic responses are compared to measured vibrations on the footbridge.

2 Pedestrian model

The bipedal model adopted is presented briefly here, as a more detailed description can be found in [17]. The model consist of a concentrated mass on top supported by two massless legs endowed with springs and dampers (see Fig.1). The leg stiffness and leg damping coefficients are given by k_l and c_l, respectively. The horizontal and vertical displacements of the center of mass (CM) are expressed in Cartesian coordinates(x_{cm}; z_{cm}).

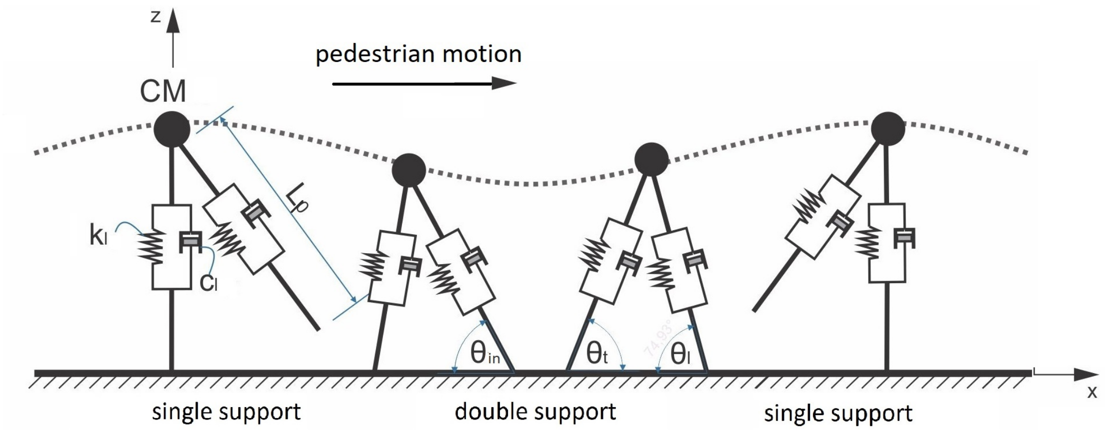

Figure 1. Biomechanical model of bipedal walking adopted to represent a pedestrian

The gait cycle is defined as the interval between successive heel impacts of the same foot and is divided into two phases, the single support phase (when only one foot is on ground) and double support phase (when the two feet are on ground). A gait cycle begins at the heel impact, which is defined by the condition in which the vertical displacement of the center of mass z_{cm} equals $L_p sin\theta_{in}$. Here, θ_{in} defines the orientation of the leg in relation to the contact surface at the heel impact and the pendulum length L_p is is the maximum length of the legs owing to the stretching of the springs. Once the double support phase begins the length of the leg in front of the CM shortens at every time interval while the length of the leg behind the CM increases until L_p is attained and it starts to oscillate. Thus, the single support phase begins until the initial condition is reached and the gait cycle is completed.

From the force balance, the pedestrian movement in vertical direction is obtained by

$$
\begin{aligned}
m_h\ddot{z}_{cm} + \left(\alpha c_l \sin^2\theta_l + (1-\alpha)c_l\sin^2\theta_t\right)\dot{z}_{cm} + \left(-\alpha c_l\cos\theta_l\sin\theta_l\ + (1-\alpha)c_l\cos\theta_l\sin\theta_t\right)\dot{x}_{cm} \\
+\left(\frac{k_l\left(\sqrt{\left(n_s d - x_{cm}\right)^2 + z_{cm}^2} - L_p\right)}{\sqrt{\left(n_s d - x_{cm}\right)^2 + z_{cm}^2}} + \frac{k_l\left(\sqrt{\left(x_{cm} - \left(n_s - 1\right)d\right)^2 + z_{cm}^2} - L_p\right)}{\sqrt{\left(x_{cm} - \left(n_s - 1\right)d\right)^2 + z_{cm}^2}}\right) \times z_{cm} \\
= -m_h g
\end{aligned} \tag{1}
$$

m_h is the mass of the human body, θ_l and θ_t are the orientations of the legs at a specified time, n_s is the step number, α is a time dependent damping coefficient, d is the step length, $\ddot{z}_{cm}$, $\dot{z}_{cm}$ and z_{cm} are the acceleration, velocity and displacement of the CM in vertical direction, $\dot{x}_{cm}$ and x_{cm} the velocity and displacement in horizontal direction and mhg is the gravitational force acting at the body's CM.

3 Experimental tests

A series of experimental tests were conducted at the Structures Laboratory of Federal University of Rio de Janeiro (LabEst-COPPE/UFRJ). The experimental tests were designed to measure the forces produced by pedestrians during walking on a prototype footbridge in a scenario as close as to real life as possible. The pedestrians parameters and vertical accelerations on the structure were measured simultaneously. Before the pedestrian tests, free vibration tests were carried out to obtain the modal parameters of the structure. More detail information about modal testing, pedestrian tests and structure description can be found in [17].

The pedestrians tests consisted of two experimental programs. The first program was performed with single pedestrians walking on the structure, while the second one was performed with pedestrians' groups walking on the structure, as can be seen in Fig.2. Sixteen volunteers participated in the tests.

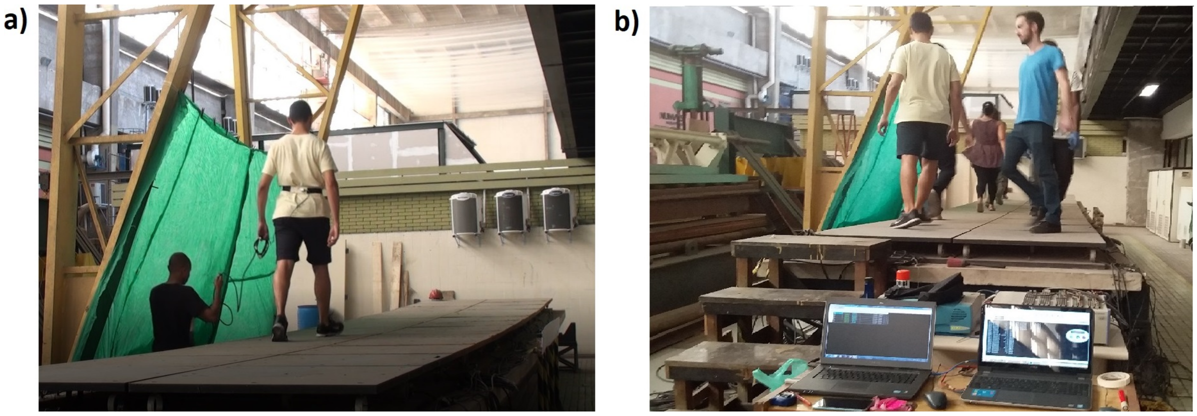

Figure 2. A typical pedestrian walking tests during a)single pedestrian testing b)pedestrian group testing

In single pedestrian tests the volunteers were asked to walk over the structure at three speeds: slow, normal, and fast. The speed is established according to the pedestrian's criterion. Each volunteer made three rounds in a closed-loop path at each of the walking speeds. Pedestrian parameters such as the mass, height and leg length of each volunteer were measured before testing. The GRFs were recorded by force plates that covered the whole structure's surface (see Fig.2) with a sampling frequency of 50 Hz and filtered with a 12.5 Hz low-pass filter. In addition, the pedestrians CM's acceleration was measured approximately by an accelerometer fastened firmly to a belt around each pedestrian's waist. The pedestrians' walking speed, step frequencies and step lengths were estimated from the tests. The average walking speed for each pedestrian test was estimated from the initial and final times recorded by the force plates and the distance traveled by the pedestrian. The pacing rate was obtained from the spectrum of accelerations measured at the pedestrian's CM. The step length was obtained as the ratio of the walking speed to the pacing rate.The pedestrian parameters obtained experimentally are summarized in Table 1.

In the experimental program involving pedestrian groups, the volunteers were asked to walk in line at their preferred walking speed, following the same path of the previous program. The walking frequency of each

Table 1. Summary of pedestrian parameters measured experimentally

Statistic	m_h (kg)	h (m)	L_l (m)	f_s (Hz)	w_s (m/s)	d (m)
Mean	68.1	1.68	0.91	1.88	1.23	0.63
Min.	89.9	1.84	1.05	1.57	0.88	0.49
Max.	53.8	1.57	0.82	2.27	1.66	0.73
StD	11.0	0.08	0.06	0.18	0.22	0.06

m_h: body mass, h: height, L_l: leg length, f_s: step frequency, w_s: walking speed, d: step length.

pedestrian's group was estimated through the frequency spectrum of accelerations measured on the structure. The walking speed was estimated as the ratio of the total distance traveled by the group to the time consumed.

4 Regression model to obtain the fundamental parameters

The required parameters for the bipedal model are the mass of the pedestrian (m_h), the pendulum length (L_p), the step frequency (f_p), the walking speed ($\dot{x}_{cm}$), the leg damping ratio (ξ_l) and leg stiffness (k_l). There is also required to establish proper initial conditions, these are the vertical velocity of the CM at the beginning of double support phase ($\dot{z}_{cm(0)}$) and the initial orientation of the leg (θ_0). The identification of the model parameters was performed through numerical-experimental correlation of GRFs. For this purpose multiple simulations were performed to find the best possible matches between predicted and measured pedestrian forces. A detailed description of the methodology is presented in [17] and for this reason it is not presented here.

In [17], linear regression functions were proposed to obtained L_p, d, ξ_l, k_l, θ_0 and $\dot{z}_{cm(0)}$ as functions of m_h, $\dot{x}_{cm}$ and the pedestrian's height (h). It should be noted that the step length (d) can be expressed as $\dot{x}_{cm}/f_p$, so the walking speed can be modified to obtain a desired step frequency. In this study, with the aim of finding expressions leading to a better fit, a curve fitting process was carried out on the empirical data with the help of MATLAB [18]. Second-order and third-order regression expressions are found to lead to a better fit. Figure 3 shows the regression curves proposed for the bipedal model parameters and the empirical data. The quality of the correlations are evaluated by means of the determination coefficient (R^2), where a value of R^2=1 indicates that the model perfectly predicts the outcome and a value of R^2=0 indicates that the model does not predict the outcome. The best correlations found are shown in Table 3, which are proposed as a new numerical model to express the bipedal model parameters. The regression model proposed by [17] is also shown for the sake of comparison.

As can be seen in Table 3, the values of R^2 corresponding to expressions proposed here are higher than those proposed by [17], which means an improvement in the predictive model. Although there is still a weak correlation for the initial orientation of the leg and leg damping, these parameters have been reported to have little influence on the GRF profile in certain ranges. In [12] it is stated that damping has little influence in the range of 5–8%. It is also reported little effect on the dynamic response of footbridge when initial orientation of the leg is in the range of 68–70°.

5 Simulation of pedestrians walking on the test footbridge

To validate the proposed new set of regression equations for the model's parameters, the experimental tests are simulated and numerical vibration responses are compared to the measured ones. The numerical model to simulate walking pedestrians on a structure was used as a previous part of this research in [17]. Thus, more details can be found there. It is presented briefly here for the sake of clarity.

The test footbridge is modeled as a grid composed of frame elements using the finite element (FE) method. The structure is assumed to behave linear, therefore, the modal superposition technique is applied. The structure vibration response is determined by integrating the uncoupled equation of motion for each vibration mode j, expressed by

$$\ddot{Y}_j(t) + 2\xi_i\omega_j\dot{Y}_j(t) + \omega_j^2 Y_j(t) = \Phi_j^T p(t) \tag{2}$$

where ξ_j, ω_j are the damping rate and frequency of the $j-th$ vibration mode, $\ddot{\mathbf{Y}}$, $\dot{\mathbf{Y}}$, and $\mathbf{Y}$ the generalized coordinates of the acceleration, velocity and displacement vectors in the direction of the applied force, and $\mathbf{p(t)}$ is the forced induced by the pedestrian. Only the two first vibration modes of the structure are considered in the

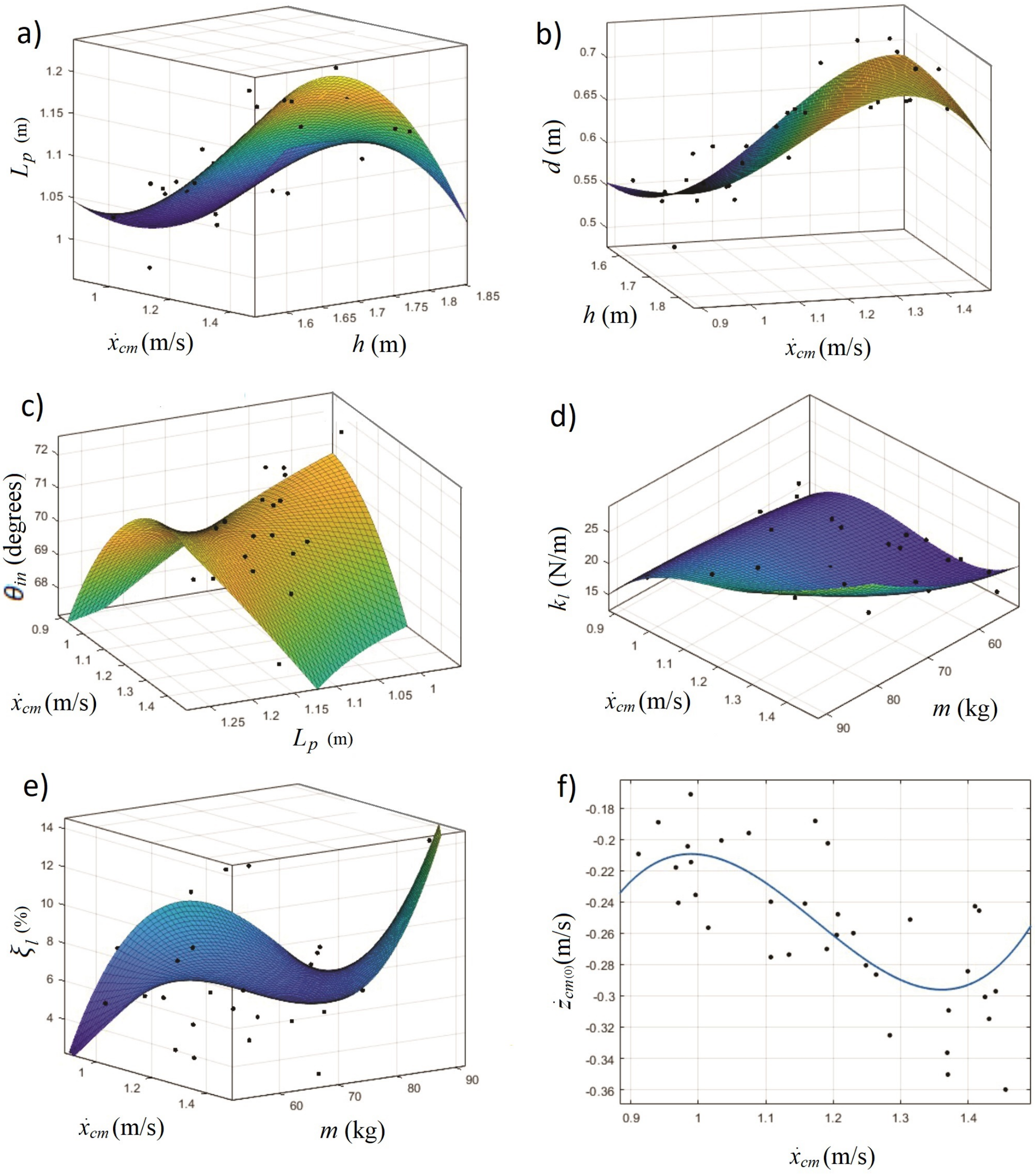

Figure 3. Empirical parameters and regression model for a) pendulum length (L_p), b) step length (d), c) initial orientation of the leg (θ_{in}), d) leg stiffness (k_l), e) leg damping rate (ξ_l) and f) vertical velocity at initial state of the gait cycle ($\dot{z}_{cm(0)}$.)

analysis. The GRFs are obtained first through the integration of Eq.1, and then the forces are applied at the contact points according to the pedestrians motion along the footbridge.

To simulate a more realistic walking pattern, intra-subject variability is included in the model by simulating each step with a different speed. For this purpose, the gait speed is generated randomly within a range from a uniform distribution, which results in a different set of parameters for the simulation of each step, producing the desired effect.

For validation purpose, regression model for bipedal model parameters, some of the experimental tests involving pedestrian groups were simulated. Experimental tests with 3, 5, 7 and 10 people walking on the footbridge were selected and simulated using the nonlinear regression model proposed in this study. The dynamic responses at mid-span of the structure obtained using this model are presented in Table 3 in a comparison with experimental measurements in terms of RMS and peak accelerations. It is also shown the comparison with simulation results using the linear regression model proposed in [17].

Table 2. Regression equations for bipedal model parameters.

Proposed in [17]		Proposed in this study	
Equations	**R^2**	**Equations**	**R^2**
$L_p[\mathrm{~m}]=0.52+0.11 h+0.34 \dot{x}_{cm}$	0.73	$L_p[\mathrm{~m}]=48.69-51.8 h-51.79 \dot{x}_{cm}+ 14.16 h^2+51.39 h \dot{x}_{cm}+7.85 \dot{x}_{cm}^2-12.99 h^2 \dot{x}_{cm} -3.40 h \dot{x}_{cm}^2-0.59 \dot{x}_{cm}^3$	0.80
$d[\mathrm{~m}]=-0.16+0.28 h+0.27 \dot{x}_{cm}$	0.77	$d[\mathrm{~m}]=0.64+0.03 \bar{h}+0.07 \bar{\dot{x}}_{cm}-0.001 \bar{h}^2 -0.007 \bar{h} \bar{\dot{x}}_{cm}-0.005 \bar{\dot{x}}_{cm}^2-0.001 \bar{h}^2 \bar{\dot{x}}_{cm}- 0.008 \bar{h} \bar{\dot{x}}_{cm}^2-0.014 \bar{\dot{x}}_{cm}^3$	0.82
$\theta_{\text{in}}\left[^{\circ}\right]=63.1+9.67 L_p-3.19 \dot{x}_{cm}$	0.16	$\theta_{\text{in}}\left[^{\circ}\right]=114.7-49.53 L_p-34.86 \dot{x}_{cm}- 14.94 L_p^2+78.03 L_p \dot{x}_{cm}-23.27 \dot{x}_{cm}^2$	0.27
$k_l\left[\mathrm{Nm}^{-1}\right]=233 m_h+2989$	0.50	$k_l\left[\mathrm{Nm}^{-1}\right]=-384.1+2.051 m_h+938.6 \dot{x}_{cm} -0.017 m_h^2-2.024 m_h \dot{x}_{cm}-756.8 \dot{x}_{cm}^2+ 0.017 m_h^2 \dot{x}_{cm}+0.114 m_h \dot{x}_{cm}^2+211.6 \dot{x}_{cm}^3$	0.74
$\xi_l[\%]=-0.27+0.02 m_h+4.98 \dot{x}_{cm}$	0.06	$\xi_l[\%]=-512.6+15.13 m_h+464.2 \dot{x}_{cm}- 0.121 m_h^2-11.58 m_h \dot{x}_{cm}-71.08 \dot{x}_{cm}^2+ 0.0002 m_h^3+0.06 m_h^2 \dot{x}_{cm}+1.3 m_h \dot{x}_{cm}^2$	0.28
$\dot{z}_{cm(0)}\left[\mathrm{ms}^{-1}\right]=-0.16 \dot{x}_{cm}-0.06$	0.35	$\dot{z}_{cm(0)}\left[\mathrm{ms}^{-1}\right]=0.4486 \dot{x}_{cm}-0.1457$	0.40

$\bar{h}=\frac{h-1.671}{0.078}, \bar{\dot{x}}_{cm}=\frac{\dot{x}_{cm}-1.195}{0.178}$

Table 3. Accelerations at mid-span of the structure caused by pedestrians' groups.

	RMS (m/s^2)			**Peak (m/s^2)**		
Test	Exp.	Model from [17]	This model	Exp.	Model from [17]	This model
3 people	0.10	0.09	0.11	0.39	0.33	0.34
5 people	0.15	0.18	0.16	0.71	0.59	0.64
7 people	0.16	0.22	0.20	0.59	0.69	0.72
10 people	0.17	0.23	0.22	0.65	0.82	0.80

The results in Table 3 show there is a slight improvement in the performance using the model proposed in this study, although the results using the linear model proposed in [17] are also quite close. Furthermore, it should be noted that just a few simulations may not be enough to assess the superior capability of the proposed predictive model. Thus, a lot more simulations are needed for comparison purposes. However, the strongest correlations found amount the model parameters in this study justify further use of this model.

6 Conclusions

An investigation to proposed parameters for a bipedal walking model was conducted in this study. Experimental GRF data from sixteen volunteers was utilized to correlate with numerical predictions in order to identify the biomechanical pedestrian parameters. By applying curve fitting to the empirical data, nonlinear regression equations were proposed to estimate the fundamental model parameters as functions of pedestrian mass, height and walking speed. The proposed nonlinear regression model produced values of R^2 higher than those obtained by linear regression. To verify the proposed numerical model, experimental tests on a prototype footbridge were employed. Some experimental situations of pedestrian groups walking on the footbridge were simulated numerically. The results suggest that the proposed regression model brings an improvement in the prediction over the linear regression model. However, this improvement was quite slight in the simulated tests. Consequently, more validations would be necessary to demonstrate the superior performance of this model.

Acknowledgements. This research was also supported by the Brazilian National Council for Scientific and Technological Development (CNPq) under Grant (141844/2016-7).

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] B. R. ELLIS, T. JI, and BRE. Human structure interaction in vertical vibrations. *Proceedings of the Institution of Civil Engineers Structures and Buildings*, vol. 122, n. 1, pp. 1–9, 1997.
[2] R. Sachse. *The Influence of Human Occupants on the Dynamic Properties of Slender Structures*. PhD thesis, Department of Civil and Structural Engineering University of Sheffield, 2002.
[3] E. Shahabpoor, A. Pavić, and V. Racic. Interaction of walking humans with structures in vertical direction: A literature review. *Shock and Vibration*, 2016.
[4] R. Sachse, A. Pavić, and P. Reynolds. Parametric study of modal properties of damped two degree of freedom crowd structure dynamic systems. *Journal of Sound and Vibration*, vol. 274, n. 3, pp. 461–480, 2004.
[5] F. Da Silva and R. Pimentel. Biodynamic walking model for vibration serviceability of footbridges in vertical direction. In *Proceedings of the 8th International Conference on Structural Dynamics (EURODYN)*, pp. 1090–1096, Leuven, Belgium, 2011.
[6] H. V. Dang and S. Živanović. Modelling pedestrian interaction with perceptibly vibrating footbridges. *FME Transactions*, vol. 41, n. 4, pp. 271–278, 2013.
[7] F. T. Da Silva, H. M. B. F. Brito, and R. L. Pimentel. Modeling of crowd load in vertical direction using biodynamic model for pedestrians crossing footbridges. *Canadian Journal of Civil Engineering*, vol. 40, n. 12, pp. 1196–1204, 2013.
[8] M. A. Toso, H. M. Gomes, da F. T. Silva, and R. L. Pimentel. Experimentally fitted biodynamic models for pedestrian structure interaction in walking situations. *Mechanical Systems and Signal Processing*, vol. 72-73, pp. 590–606, 2016.
[9] W. D. Varela, M. S. Pfeil, and da N. d. P. A. Costa. Experimental investigation on human walking loading parameters and biodynamic model. *Journal of Vibration Engineering and Technologies*, vol. 8, n. 6, pp. 883–892, 2020.
[10] M. Bocian, J. H. G. Macdonald, and J. F. Burn. Biomechanically inspired modeling of pedestrian-induced vertical self-excited forces. *Journal of Bridge Engineering*, vol. 18, n. 12, pp. 1336–1346, 2013.
[11] H. Dang. *Experimental and Numerical Modelling of Walking Locomotion on Vertically Vibrating Low-Frequency Structures*. PhD thesis, University of Warwick, 2014.
[12] J. Qin, S. Law, Q. Yang, and N. Yang. Pedestrian bridge dynamic interaction including human participation. *Journal of Sound and Vibration*, vol. 332, n. 4, pp. 1107 – 1124, 2013.
[13] J. W. Qin. Finite element analysis of pedestrian-bridge dynamic interaction. *Journal of Applied Mechanics*, vol. 81, n. 4, 2013.
[14] Y.-A. Gao, Q.-S. Yang, and J.-W. Qin. Bipedal crowd-structure interaction including social force effects. *International Journal of Structural Stability and Dynamics*, vol. 17, n. 07, pp. 1750079, 2017.
[15] B. Lin, Q. Zhang, F. Fan, and S. Shen. Reproducing vertical human walking loads on rigid level surfaces with a damped bipedal inverted pendulum. *Structures*, vol. 33, pp. 1789–1801, 2021.
[16] D. V. Ruiz, C. Magluta, and N. Roitman. Experimental verification of biomechanical model of bipedal walking to simulate vertical loads induced by humans. *Mechanical Systems and Signal Processing*, vol. 167, pp. 108513, 2022a.
[17] D. V. Ruiz, C. Magluta, and N. Roitman. Modeling human-induced loads through a calibrated bipedal walking model. *Structural Engineering International*, vol. 32, n. 1, pp. 92–102, 2022b.
[18] MATLAB. *version 9.12.0.2039608 (R2022a) Update 5*. The MathWorks Inc., Natick, Massachusetts, 2022.