Simple equation to account for the human-structure interaction effects on the modal damping of footbridges

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Abstract. Footbridges dynamic properties may be affected by the presence of pedestrians, especially the modal damping of the occupied structure which is greater than that of the isolated one. To account for this increase, particularly in lightweight and slender footbridges, which are susceptible to the human-structure dynamic interaction (HSI), the pedestrians are modelled as a mechanical system coupled to the footbridge model. In this paper, an equation is proposed to add the damping contribution provided by persons walking on footbridges to the isolated structure damping ratio. The presented equation is based on numerical results obtained from a computational tool especially developed to address the effects of HSI. A comparison of the damping ratio obtained with the proposed equation and the experimental measurements in a slender footbridge from the literature is favorable. The estimated damping ratio of the coupled system (footbridge plus pedestrians) could then be used to include the HSI effects to existing design methodologies based on the live load approach to simulate pedestrian forces.

Keywords: human-structure interaction, footbridges, damping ratio.

1 Introduction

Human-Structure Interaction (HSI) involves the dynamic interplay between humans and structures, where their actions mutually affect each other in a continuous feedback loop during their contact. HSI is influenced by human posture and activities, affecting structural response through various mechanisms. The most significant HSI effect observed in footbridges and floors is the increase in damping ratios of the coupled pedestrian-structure system, in relation to the isolated structure.

Several studies have highlighted the importance of accounting for HSI in predicting the dynamic response of structures under pedestrian-induced loading [1,2,3], for example studies on the Millennium Bridge, in London, and Solférino Passerelle, in Paris. However, current design methods often rely on simplified equivalent load models, which overlook the intricate interaction between pedestrians and structures. As a result, these approaches may lead to inaccurate assessments of vibration performance and overestimation of structural responses [4].

Recent literature [4,5] has suggested that modeling pedestrians as dynamic systems coupled to the single-degree-of-freedom model (SDoFM) of the structure may provide more realistic predictions. This approach results in a multi-degree-of-freedom (MDoFM) dynamic system that enables to account for pedestrian interaction on the footbridge. However, this method is not suitable for practical engineer purposes because (i) it may be computationally demanding due to the complex of the mathematical modelling resulted of the aforementioned coupled system and (ii) it requires extensive simulations to capture the variability in pedestrian walking and physical patterns.

In this context, this paper aims to propose a straightforward solution for this gap by introducing a simple equation to account for the increase in damping ratio due to the presence of pedestrians walking along footbridges. The estimated damping ratio of the coupled system (footbridge plus pedestrians) could then be used to include the HSI effects by applying existing design methodologies, as the proposed by codes and guides [6], which are based on the live load approach to simulate pedestrian forces.

The empirical equation presented herein is derived from a non-linear regression model (NLRM), obtained by fitting the estimated modal damping ratio of the coupled system determined for different footbridge-structures in a numerical tool, based on Monte Carlo simulations, especially developed with the purpose to address the effects of HSI. This equation was used to estimate the damping ratio of an existing in-service footbridge from the literature. The comparison of the simulated and the equivalent measurement damping ratio of this structure is favorable and shows the effective of the presented equation.

2. Matematical modelling of HSI

In this work, each pedestrian is modelling as a single-degree of free biodynamic model (BM). Among the BMs available in the literature, the proposal presented by Pfeil *et al*. [7] has, as novelty, a theoretical formulation based on the heel drop excitation, in contrast to those traditionally adopted on the literature [8,9], in which an external force is applied directly on the center of mass of the BM.

Moreover, the theoretical-numerical simulation obtained with the human-model of Pfeil *et al*. [7] show an excellent agreement with the data experimentally collected at a prototype walkway structure built on Laboratory of Structures of COPPE/UFRJ while a person walked. For these reasons, this BM, which the fitted equations for estimating its modal parameters is presented in Eq. (1), is adopted in this paper to simulate pedestrians walking.

$$
\begin{gathered}
m_p[kg] = 0.874M[kg] - 9.142f_p[Hz] + 12.940 \\
k_p[N/m] = 360.300m_p[kg] - 1282.500 \\
\xi_p[\%] = -20.818f_{md}[Hz] + 87.513
\end{gathered}
\tag{1}
$$

where M and f_p are the mass and the walking frequency of the pedestrian, respectively; m_p, k_p and ξ_p are the modal mass, stiffness and damping ratio of the BM, respectively. f_{md} is the damped natural frequency of the BM, which is iteratively determined by: $f_{md} = f_m\sqrt{1-\xi_p^2}$, whereas $f_m = (1/2\pi)\sqrt{k_p/m_p}$ is natural frequency of the BM.

The mathematical model proposed by Pfeil *et al.* [5] deserves acknowledgement for its alignment with the theoretical formulation of the BM adopted herein. This theoretic model has been adapted in this work to consider multiple individuals. This adjustment results in a system of coupled differential equations that combines the single-degree-of-freedom (SDoF) model of footbridge with the SDoF models representing each person on the structure. The multi-degree of freedom model (MDoFM) relating this integrated dynamic system is presented, in a general form, as depicted in Eq. (2). Moreover, Figure 1 illustrates the representation of the pedestrians as BMs walking along the footbridge.

$$
\begin{aligned}
&\begin{bmatrix} m_s & 0 & \cdots & 0 \\ 0 & m_{p_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & m_{p_{N_p}} \end{bmatrix}\ddot{\boldsymbol{Y}} + \begin{bmatrix} -\left(c_s + \sum_{i=1}^{N_p} \Phi_{p_i}^2 c_{p_i}\right) & \Phi_{p_1} c_{p_1} & \cdots & \Phi_{p_N} c_{p_{N_p}} \\ \Phi_{p_1} c_{p_1} & -c_{p_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & c_{p_{N_p}} \end{bmatrix}\dot{\boldsymbol{Y}} \\
&+ \begin{bmatrix} -\left(k_s + \sum_{i=1}^{N_p} \Phi_{p_i}^2 k_{p_i}\right) & \Phi_{p_1} k_{p_1} & \cdots & \Phi_{p_N} k_{p_{N_p}} \\ \Phi_{p_1} k_{p_1} & -k_{p_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & k_{p_{N_p}} \end{bmatrix}\boldsymbol{Y} = \begin{bmatrix} F(t) \\ 0 \\ \vdots \\ 0 \end{bmatrix}
\end{aligned}
\tag{2}
$$

where m_s, c_s and k_s are, respectively, the modal mass, stiffness and damping constant of the SDoF model of the footbridge-one; Φ_{p_n} is the amplitude of the modal coordinate of the structure corresponding to the instantaneous position of the N-th pedestrian; $F(t)$ is the modal force induced by the pedestrians.

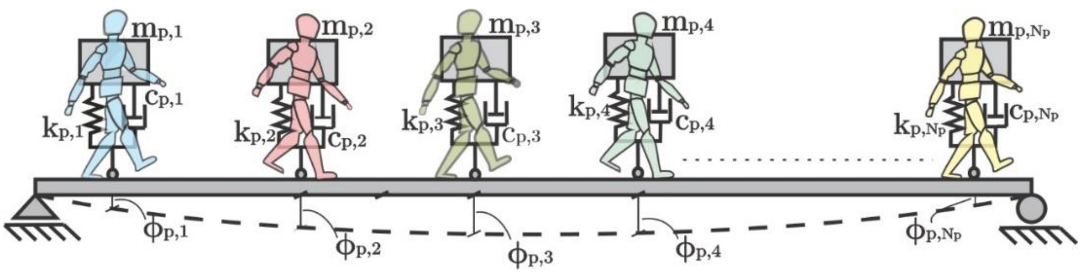

Figure 1: Pedestrians walking along the footbridge.

3 Proposed methodology

First the damping ratio of the coupled dynamic system comprising the structure with the pedestrians is determined for different dynamic systems, as reported in section 3.1. With the estimated damping ratio, the proposal equation is obtained with the determined DataSet by fiting an equation using non-linear regression model (NLRM), as described in 3.2.

3.1 Determining the damping ratio of the coupled system

To determine the modal damping ratio ($\xi_{o,s}$) of the structure in the presence of pedestrians walking, a numerical-theoretical methodology is developed herein. The step-by-step of the developed numerical tool is described, as follows:
(i) input the parameters of the SDoF model of the isolated strucutre;
(ii) determine the number of pedestrians (N_p) walking along the footbridge;
(iii) generate random samples for the mass (M) and natural frequency (f_p) of each person, following the established probability distribution of these parameters;
(iv) calculate the modal dynamic properties of the BM according to Eq. (1);
(v) define the position (x_p) of each person along the footbridge by sampling;
(vi) apply an initial perturbation on the system;
(vi) solve the system of nonlinear differential equations presented in Eq. (2), using numerical integration methods on the time domain;
(vii) calculate the damping ratio (ξ_{o,s_i}) of the couple dynamic system for each sample;
(viii) repeat (Monte Carlo samples) steps (i)-(vii) and determine the mean value of the modal damping ratio ($\xi_{o,s}$).
To exemplify the proposed numerical-theoretical methodology, an existing footbridge is evaluated herein. This structure is a steel box girder, in which a series of experimental campaign under normal in-service conditions was carried out [1]. Table 1 presents the modal parameters of the footbridge and the data of the pedestrian traffic, while Figure 2 shows the cumulative mean for the damping ratio of this system ($\xi_{o,s}$), for different samples, determined according to the step-by-step presented above for the traffic indicated.

Table 1: Modal parameters of the structure and the pedestrians traffic [1].

Parameters of the structure one and pedestrian traffic	Values
f_s (Hz)	2,00
m_s (ton)	58000
ξ_s (%)	0,26
N_p	78

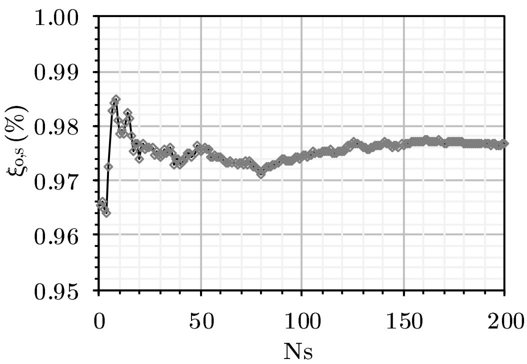

Figure 2: Cumulative mean of the modal damping ratio ($\xi_{o,s}$) of the coupled dynamic system.

3.2 Non-linear Regression algorithm

A Non-Linear Regression algorithm (NLRA) is used to determine the curve of adjustment for the modal damping ratio of different footbridge dynamic systems subject to persons walking. It involves minimizing the least-square error (LSE) between the predicted values ($\hat{y}$ = $\hat{f}(\boldsymbol{x},\boldsymbol{\theta})$), which are determined by the developed equation, and the DataSet ($\mathbf{y}$), that contains the structural parameters and the equivalent damping ratio of the coupled dynamic system, determined according to the methodology described in section 3.1:

$$LSE(\boldsymbol{\theta}) = \sum_{t=1}^{N_s} \left[\boldsymbol{y}(t) - \hat{f}(\boldsymbol{x}(t), \boldsymbol{\theta})\right]^2 \tag{3}$$

where: N_s is the number of samples; $\boldsymbol{x}(t)$ are the parameters of the structure and crowd density; $\boldsymbol{\theta}$ is the vector of parameters to be determined on the adjustment of the fitted curve.

The minimization of the LSE is performed by deriving Eq. (3) and using the iterative optimization Levenberg-Marquardt (LM) technique [10]. For each iteraction, the algorithm calculates the predicted values ($\boldsymbol{y}$), based on the current parameter estimates ($\boldsymbol{\theta}$), and computes the gradient ($\nabla = \partial\hat{\boldsymbol{f}}/\partial\boldsymbol{\theta}$) of the error function with respect to the parameters.

4 RESULTS OF DAMPING RATIO OF THE COUPLED SYSTEMS OBTAINED BY SIMULATION

Figure 3 shows the modal damping ratio of a footbridge system from the literature [11] for different crowd pedestrians densities evaluated with the methodology proposed in section 3.1. The structure is a simple supported beam with a length of 50 m and a width of 2 m. The modal mass (m_s), natural frequency (f_s) and damping ratio (ξ_s) of the SDoF model related to the flexural bending mode of this footbridge one are, respectively, 25.0 t, 2.0 Hz and 0.5%.

From this Figure it is possible to verify that the modal damping ratio of the coupled dynamic system increases with the number of pedestrians walking along the structure. For this case-example, this increase seems to follow an almost linear relationship (R^2=0.97) with the crowd density. Nevertheless, this behavior should be investigated for other case-examples, as performed amidst of the section 5, with the simulations conducted with the aim to fit the proposed equation. Literature [1], for example, suggests that there is no obvious correlation between the number of pedestrians and the increase of the damping ratio of the system.

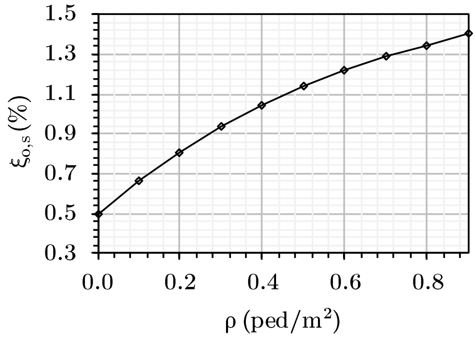

Figure 3: Damping ratio ($\xi_{s,o}$) of the coupled dynamic system under different crowd densities (ρ).

5 PROPOSED EQUATION

A set of numerical simulations were performed to attain the modal damping ratio of the coupled system composed of the footbridge plus pedestrians, according to the step-by-step presented in 3.1. The empirical equation was fitted with the non-linear regression model (NLRM) described in 3.2.

Different footbridge dynamic systems were evaluated under various crowd densities (0.1 ped/m² - 0.9 ped/m²). The range of values for the sampled parameters of the structure are presented in Table 2. It encompass dynamic systems characterized as highly flexible and low damping structures, in which the effects of HSI are pronunced, and as heavy mass and high damping structures, where the effects of HSI may be neglected.

The fitted equation is presented in Eq. (4). This equation was derived based on the interplay of the three most crucial parameters that impact the damping ratio contributed by individuals. This relationship was confirmed through Pearson correlation analysis conducted among the variables involved. The DataSet used to fit the presented equation was obtained from a total of 2000 simulations and the the outliers were removed using the distance matrix technique. For practical engineering purposes, the ratio (μ) between the total mass of pedestrians and the modal mass of the structure was limited to be less than 0.9.

Table 2: Range of values for the sampled parameters of the footbridge dynamic system.

Parameter of the isolated footbridge	Distribution	Range of values
m_s (ton)	Uniform	U [1 – 80]
ξ_s (%)	Uniform	U [0.1 – 5.0]
f_s (Hz)	Normal	N [2.0; 0.4]
L (m)	Uniform	U [20 – 80]

$$\xi_{o,s}(\%) = \theta_1 \xi_s(\%)^{\theta_2} + \theta_3 f_e(Hz)^{\theta_4} + \theta_5 \mu^{\theta_6} \tag{4}$$

where $\theta_1 - \theta_6$ are the parameters of the fitted equation, determined according to the NLRM, and presented in Table 3:

Table 3: Parameters (θ) of the fitted equation.

Parameters of the Reg. model	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
Values	0.586	1.097	0.017	5.019	2.303	0.849

Figure 4 presents the predicted ($\hat{y}$) and the simulated (y) values for the modal damping ratio of the coupled dynamic system ($\xi_{o,s}$). Moreover, this Figure presents the metrics (Mean Square Error and Determination Coefficient) for the error of the determined damping determined with the fitted curve (estimated) and with the DataSet (simulated).

To demonstrate the effectiveness of the proposed equation (Eq. 4) against experimental results, the modal damping ratio ($\xi_{o,s}$) of the steel box girder structure (described in Table 1) under pedestrian walking is assessed. Figure 5 presents a comparison of the damping ratio of the coupled dynamic system based on (i) the application of the fitted equation, (ii) the utilization of the cumulative mean of simulation values as detailed in section 3.1, and (iii) the assumed inherent damping of the footbridge that was established to align experimental measurement amplitudes with the dynamic response achieved when modeling pedestrians as live loads [1].

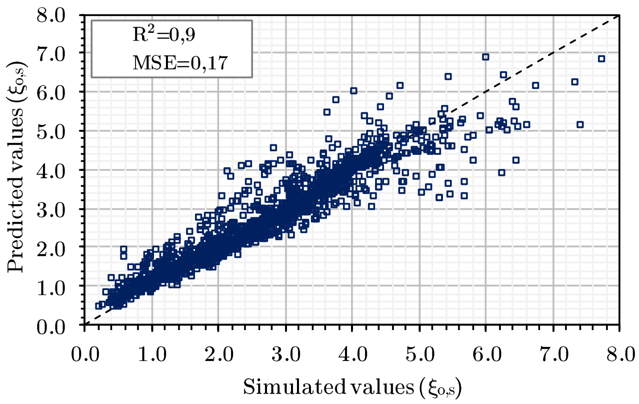

Figure 4: Predicted ($\hat{y}$) and simulated (y) values for the modal damping ratio of the coupled dynamic system.

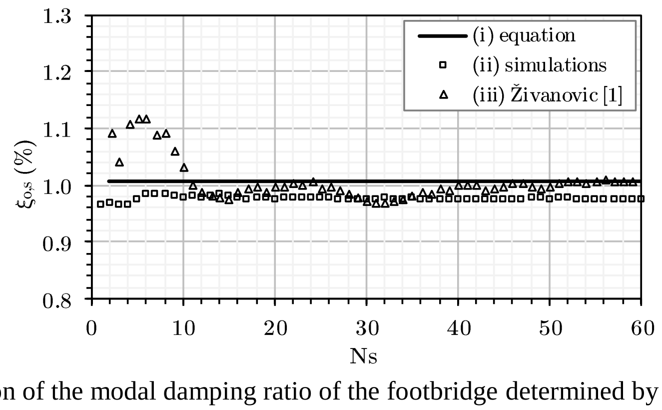

Figure 5: Comparison of the modal damping ratio of the footbridge determined by the fitted equation, the simulations and with the ficticious inherent damping obtained in [1].

6 Conclusions

This paper aims to propose a practical equation for estimating increased damping ratio on footbridges subjected to pedestrians walking. This equation was determined from a set of Monte Carlo samples in different dynamic systems, relating to footbridge-structures, and crowd densities. A non-linear regression algorithm was used to fit the numerical DataSet obtained from the simulations by minimizing the least-square error (LSE).
The comparison of the predicted and estimated values for the modal damping ratio of the coupled dynamic system shows the robustness of the fitted equation (R^2=0.90), which depends on the damping ratio of the isolated structure, the natural frequency of the isolated structure and the ratio of the pedestrian's mass and the modal mass of the footbridge.
Moreover, a case example was presented, where the modal damping ratio of an existing in-service footbridge was determined with the fitted equation. The comparison of the measured equivalent damping ratio of this structure and the estimated one shows a difference less than 1,0%.

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