Elastic Buckling Load Prediction of Tapered Steel Columns Via Artificial Neural Networks

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Abstract. This study presents a novel approach for predicting the critical buckling load of slender, cylindrical, tapered steel towers commonly used in wind turbines and telecommunications equipment. These towers are prone to instability issues caused by buckling loads, which necessitates accurate evaluation. To overcome the limitations of existing instability load formulations and regulatory codes, we developed an artificial neural network (ANN) model. The ANN model utilizes a comprehensive database of 1,440 finite element models to accurately predict the critical buckling loads. An MLPRegressor model instantiated with the 'adam' solver and the 'tanh' activation function in the hidden layers demonstrated a significant alignment with the data, as the model accounted for approximately 97% of the variance in the dependent variable. Furthermore, the outcomes obtained from the ANN model closely aligned with the original values, surpassing the predictive precision of the classical shell and beam formulations, and offering insights into the complexities associated with transformed data.

Keywords: Eigenvalue, artificial neural network (ANN), machine learning, hollow columns, structural stability.

1 Introduction

Buckling is a structural failure mode involving the transition from one equilibrium state of deformation to another when a column undergoes to buckling, it can experience sudden, unstable deformation or collapse.

The classical approach to evaluate the buckling capacity of columns is given by Euler formula, the formulation works under the elastic regime and is derived for prismatic sections i.e., constant inertia. However, to improve the stability of the columns some authors, proposed the use of columns with variable inertia, according to Goel [1] there are two ways to evaluate the buckling capacity of nonuniform columns: using a continuum approach where the mathematical model is solved to find closed form solutions, on the other hand using computer solutions a numerical approach or approximated methods can be used.

Both approaches have their advantages and disadvantages, and the choice of approach depends on the specific application and the level of accuracy required. Usually solving mathematical models may not provide accurate predictions of the critical load for tapered cantilever columns, while numerical methods can be accurate but are computationally expensive and require accurate modelling of the columns.

When the case of hollow columns is taken into consideration, the problem becomes more complex, essentially when a cylindrical shell loaded in compression can fail by global i.e., Euler buckling, with a wavelength related to the length of the column, or local i.e., Shell buckling, with a wavelength related to the section width or thickness. The behaviour of tapered shell columns is even more complex as the deformation of the shell is not uniform and the buckling can be dependent on other geometric parameters, such as the slenderness factor proposed by Dick [2].

Some authors agreed that regulatory codes do not present a common understanding of which methodology should be used, to access the buckling capacity of nonuniform columns[1], [2].

Therefore, there is a need for more accurate and efficient methods for predicting the critical buckling load for tapered cylindrical shell columns, which is where the use of neural networks, can be a promising approach.

Recently Thai [3], provides a comprehensive overview of the current state of research in the application of machine learning (ML) techniques with successful applications in different areas of structural engineering. One of the presented ML techniques are the artificial neural networks (ANN), that mimic the behaviour of a nervous system and can be used to solve nonlinear and are one of the most popular ML algorithms. Previous studies conducted on the application of ANN for predicting the axial buckling mainly focus on uniform columns with different cross sections geometries as I [4]–[6], Y [7], and cylindrical columns [8], [9] ,while the nonuniform columns are analysed for Web-tapered I-section [4]. However, as the authors knowledge the application, of ANN techniques for estimating the elastic critical buckling load of Hollow Tapered Cylindrical Column are not studied yet.

The objective of this study is to develop an artificial neural network (ANN) model to more accurately predict the elastic critical buckling load of Hollow Tapered Cylindrical Columns fixed at one end and free on the other, i.e also known as tapered cantilever hollow columns under axial loads. A set of 1440 data of elastic critical buckling were generated using numerical analysis and used to develop the ANN model.

2 Database and parameters

2.1 Finite Element Model

This study expands on the previous authors investigations [2]. In this context, dataset generation relies on a numerical model employing Finite Element Method (FEM). The tapered hollow columns are modelled using shell elements in the software SAP2000. The material attributes remain constant, with steel S275 selected, featuring a Yield Strength of 275 MPa and an Elastic Modulus of 210 GPa. To determine meshing size, a sensitivity analysis is performed, leading to the selection of a maximum element size of (0.1 x 0.2) m.

The geometric parameters, including the slenderness factor (η), shell thickness (t), bottom radius (R) and column height (H), are employed in the FEM models. The predefined values for these parameters are presented in Tab. 1. The slenderness factor (η) is defined according to eq. (1) and is subsequently utilized to calculate the top column radius (r). In this calculation, I_t represents the second moment of inertia of the top section, while I_b represents the second moment of inertia of the bottom section.

$$\eta = 1 - \left(\frac{I_t}{I_b}\right)^{1/3}. \tag{1}$$

The elastic critical axial load was determined through an Eigenvalue buckling analysis. In this analysis, a unit load was applied at the free end (top) of the column. Notably, the analysis revealed two distinct buckling modes: shell and beam-type buckling. These modes are visually depicted in Fig. 1 for schematic clarity.

Table 1. Model geometric parameters

η	t (m)	R (m)	H (m)
0.0	0.003	0.25	5.0
0.1	0.005	0.50	10.0
0.3	0.010	0.75	15.0
0.5	0.020	1.00	20.0
0.7	0.030	1.50	30.0
0.9		2.00	40.0
			50.0
			80.0

(a) Shell buckling t = 0.003 m, Pcr = 1309.8 kN (b) Beam buckling t = 0.01m, Pcr = 5942.5 kN

Figure 1. Example of buckling modes for axial loading of thin-walled circular columns with a bottom radius (R) of 0.5 m, top column radius (r) of 0.3 m, and column height (H) of 10 m.

2.2 Dataset

The dataset used to train and test the ANN, are composed by 1440 samples, that after preprocessing to remove outliers and inconsistent values are reduced to 1402 samples. The input parameters are presented in Tab. 2, where the descriptive analysis of the parameters are shown and the output parameter are the Elastic buckling load Pcr.

Table 2. Descriptive statistics for the database

	Input					Output
	R(m)	n	t(m)	r(m)	H(m)	Pcr (MN)
mean	0.99	0.58	0.014	0.63	31.04	32.63
std	0.60	0.31	0.010	0.50	23.05	79.30
min	0.25	0.10	0.003	0.05	5.00	0.00
25%	0.50	0.30	0.005	0.23	15.00	0.61
50%	0.75	0.53	0.010	0.46	20.00	2.41
75%	1.50	0.90	0.020	0.90	40.00	17.04
max	2.00	1.00	0.030	2.00	80.00	528.76

The dataset needs to be separated in two data set, one for train and other for test the model, it is important that both data set are representative of the whole data. As the dataset contain information about the type of buckling that occurred (shell or beam), this parameter is used to stratify the data, to have a similar distribution of the output data in both data set, as shown in Tab. 3. The validation of the ANN model is computed using a stratified k-fold cross-validation (CV), where the data set is randomly portioned in K subsets by preserving the same percentage for each target class, the training dataset is then portioned in 4 folds stratified by type of buckling for inner validation and the validation portion is set to 20%.

Table 3. Dataset samples and proportions

				Proportion	
	All	Beam	Shell	Beam	Shell
Total	1402	862	540	61.5%	38.5%
Training	1000	615	385	61.5%	38.5%
Test	402	247	155	61.4%	38.6%

3 Developed ANN Model

An artificial neural network is a mathematical model built based on the human brain. These models consist of decision-making units arranged in layers and connected to form a network. A Multi-layer Perceptron (MLP) is a supervised learning algorithm capable of learning non-linear functions. MLPs are composed of an input layer, one or more hidden layers, and an output layer. The number of neurons in each layer and the number of hidden layers are hyperparameters that need to be defined through analysis and tuning.

In this study, the scikit-learn machine learning library in Python is used, specifically the MLPRegressor, to train the dataset using backpropagation.

According to Burkov [10] machine learning models perform better in normally distributed data. However, in Fig. 2, it can be observed that our feature input parameters are not normally distributed. Since the parameters are not continuous, no transformation is applied. However, the output parameter Pcr is highly skewed. To obtain a distribution closer to normal, the logarithm of the Pcr values is taken, resulting in the log_Pcr parameter shown in Fig. 2, which exhibits a closer-to-normal distribution.

To improve the accuracy of the ANN model, both the input and output parameters are normalized to a range of [-1,1], as shown in eq. (2). Here, X represents the data test sample, X_N is the normalized data sample, and X_{min} and X_{max} are the minimum and maximum values of the parameters under consideration.

$$X_N = 2\,\frac{(X-X_{min})}{(X_{max}-X_{min})} - 1\,. \qquad (2)$$

In this study, the evaluation of the ANN and ANFIS models involved the use of two important metrics: the coefficient of determination (R^2) and the root mean square error (RMSE). These metrics, represented by equations 3 and 4 respectively, were employed as benchmarks to assess the performance of the models.

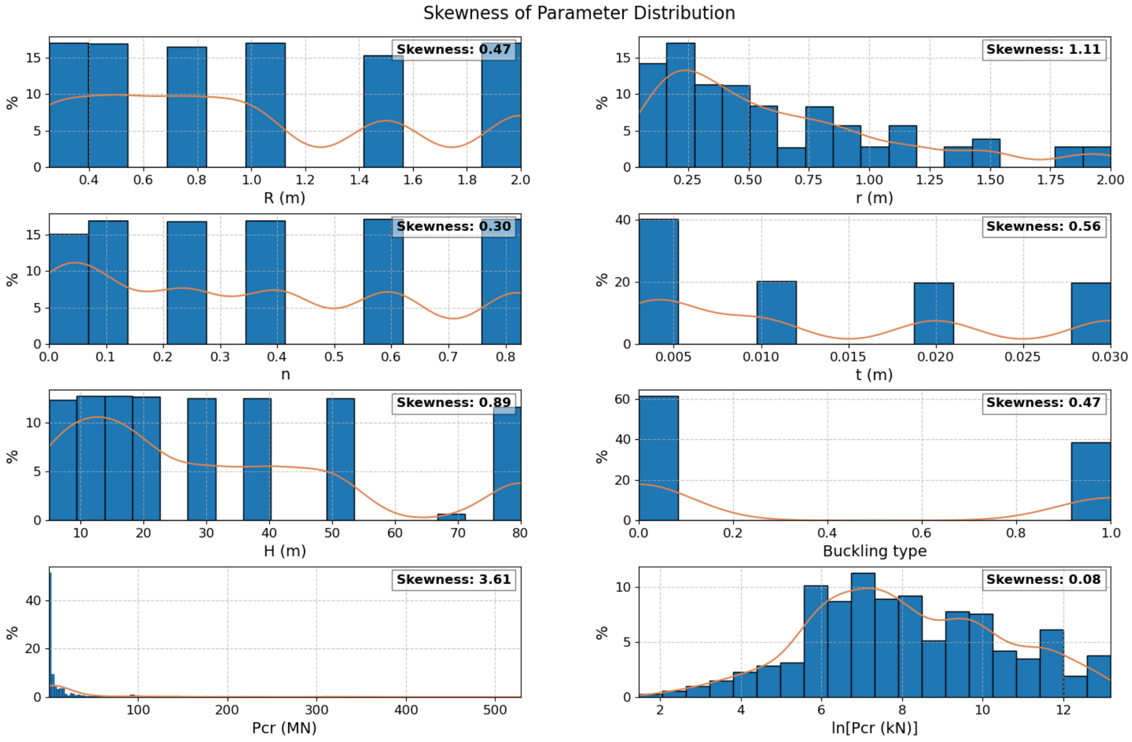

Figure 2. Parameter distribution

$$R^2 = 1 - \left(\frac{\sum_{i=1}^{n}(t_i - o_i)^2}{\sum_{i=1}^{n} o_i{}^2}\right), \quad (3)$$

$$RMSE = \left(\sqrt{\frac{1}{n}\sum_{i=1}^{n}(t_i - o_i)^2}\right), \quad (4)$$

where t_i is the target value of ith the sample, o_i is the output value of ith sample, and n is the number of samples.

3.1 Artificial Neural Network architecture

During the training phase, an MLPRegressor model is instantiated with the 'adam' solver and the 'tanh' activation function in the hidden layers. To ensure reproducibility, a fixed random state of 5 is set. The model is trained with an initial learning rate of 0.01, a maximum of 500 iterations, and training halts if no improvement is observed for 10 consecutive iterations.

Hyperparameter search covers batch sizes (16 to 200) and hidden layers (1-2 layers, 1-20 neurons in first, 1-2 neurons in second). GridSearchCV optimizes using RMSE and R^2 as scoring metrics.

Through systematic testing, the optimal architecture is determined to consist of two hidden layers, with 11 neurons in the first layer and 2 neurons in the second layer, and a batch size of 16.

4 Results and discussion

Residual Analysis and the visualization of Predicted and Actual Values are essential for assessing the performance of the Artificial Neural Network (ANN) model. Fig. 3 provides a comprehensive understanding of the model's performance by comparing its projected values with the actual values.

Within the test dataset, the metrics demonstrate a significant alignment with the data, as the model accounts for approximately 97.0% of the variance in the dependent variable. This highlights the robust proficiency of the model in capturing the underlying data patterns. The Mean Absolute Error (MAE) and Root Mean Squared Error (RMSE) are -0.058 and -0.073, respectively, further validating the accuracy of the model within the test set.

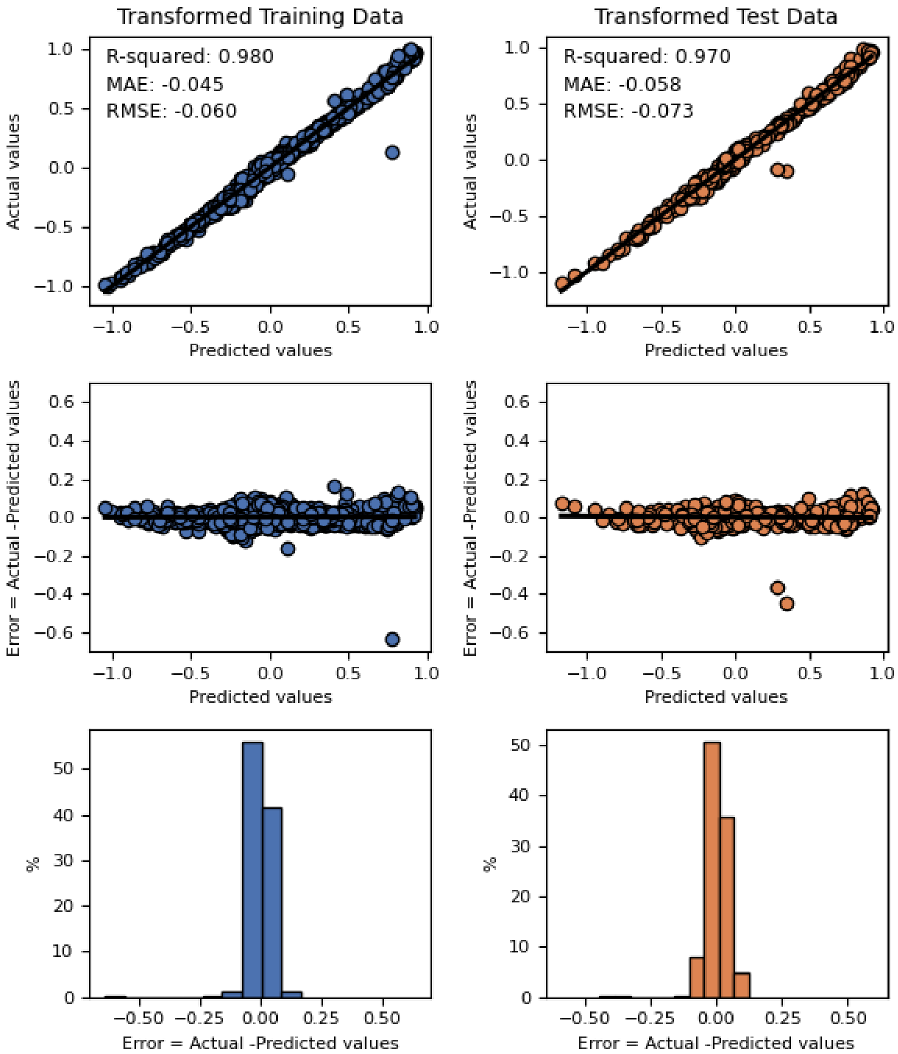

Figure 3. Comparison of Predicted and Actual Values, Residual Plot, and Histogram (with a bin width of 0.05) for Evaluating the Predictive Capability of the ANN Model.

However, it becomes evident that there are outliers present in both the training and test datasets. Analysing the distribution of prediction errors (actual - predicted) reveals that majority of these errors fall within a narrow range of approximately (-0.1 to 0.1) for the normalized log data. This tight distribution suggests that the model tends to make predictions that are consistently close to the actual values.

The initial dataset's comparison of predicted and actual values is shown in Fig. 4. Logarithmic transformation compresses the dataset, notably in higher Pcr values. This leads to more pronounced disparities when reverting predictions to the original scale, especially for larger values. While errors cluster around zero, higher values show greater divergence between original and predicted values. These underscores complexities introduced by data transformation, especially for higher magnitudes.

Importantly to notice, that the model was trained generally without segregating data by buckling behaviours (shell and beam). This could notably impact accurate prediction for higher values. Unique characteristics of these buckling behaviours demand tailored modelling for improved accuracy.

The outcomes obtained from the ANN model were compared to the predictions derived from the classical theory of shell buckling, represented by eq. 5. Similarly, the classical theory for column buckling, characterized by the Euler formula stated in eq. 6, was utilized. The calculations were performed using averaged values obtained from the bottom and top sections of the structure. Here E is modulus of elasticity, R radius of shell, t the thickness of shell and ν Poisson's ratio, I is the inertia of the cross-section, the effective length factor (k) dependent on the support conditions of the structure, for an ideal perfectly cantilevered column k = 2.0 and the height (H) of the column.

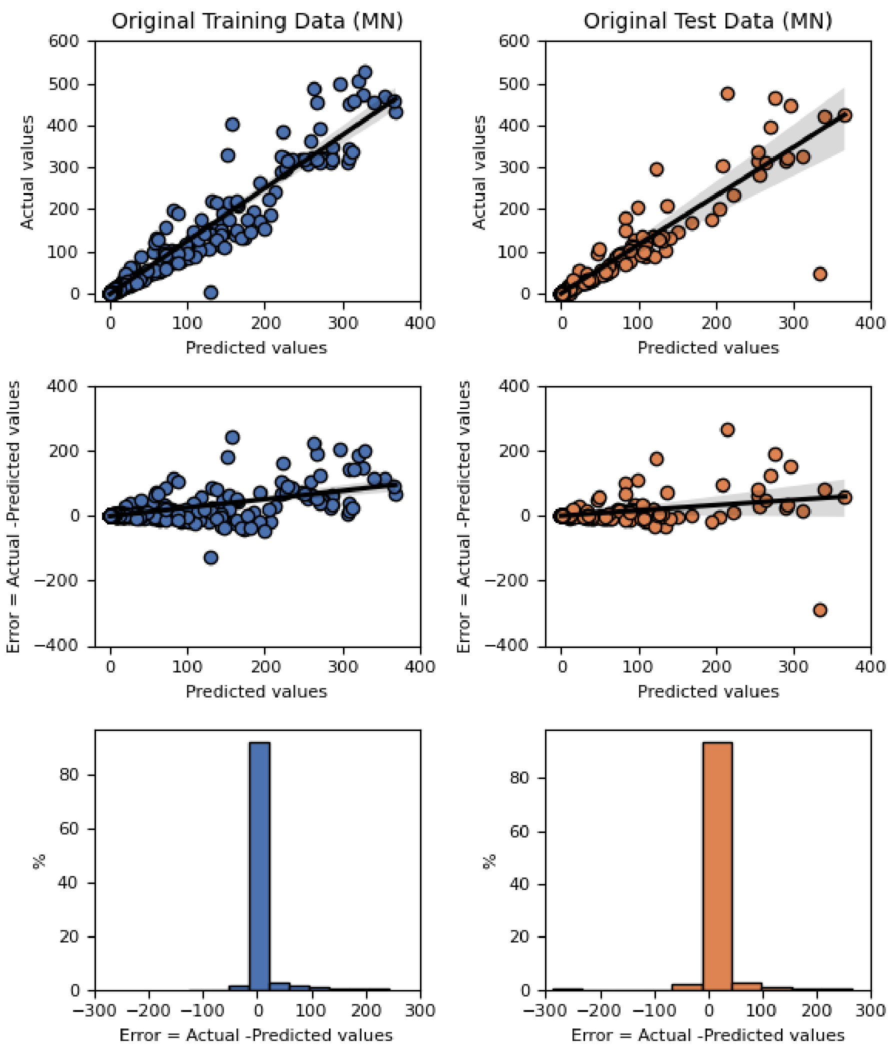

Figure 4. Comparison of Predicted and Actual Values, Residual Plot, and Histogram from original data

$$\sigma_{cl}{}^{2} = \frac{E}{\sqrt{3(1-v^2)}}\left(\frac{t}{R}\right), \tag{5}$$

$$P_{Ecr} = \frac{\pi^2 EI}{(kH)^2}, \tag{6}$$

As depicted in Fig. 5, the outcomes obtained from the ANN model closely align with the original values, surpassing the predictive precision of the classical shell and beam formulations.

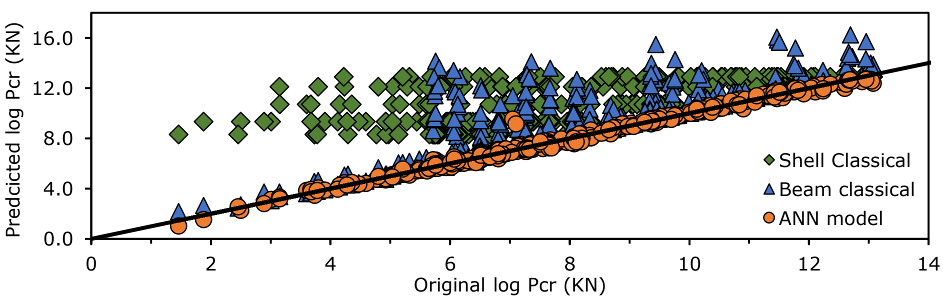

Figure 5. Comparison of Predicted and Actual Values for the test dataset, from classical beam and shell formulation and ANN model.

Although the classical theory for column buckling follows a similar trend, it consistently overestimates the true value. The tendency toward overestimation in the classical theory for column and shell buckling is not surprising, as these theories are known as upper bounds theories.

5 Conclusions

In conclusion, the results of the ANN model demonstrate its predictive capabilities, as evidenced by its alignment with approximately 97% of the variance in the test set. The presence of outliers and the complexities associated with transformed data highlight the significance of meticulous data preprocessing and outlier detection techniques. Furthermore, the model's difficulty in distinguishing between different buckling types offers valuable insights for further refining the model, with the potential to enhance its applicability to specific buckling scenarios.

When comparing the outcomes of the ANN model with classical theories, it becomes apparent that the ANN model exhibits superior predictive precision. This contrasts with the classical theory, which consistently overestimates values. Overall, these findings provide valuable insights into the performance of the ANN model and its potential for accurately predicting buckling loads for tapered hollow columns.

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