Energy balance for restrained steel columns in fire

Pedro Dias Simão[1,2], João Paulo C. Rodrigues[1,2]

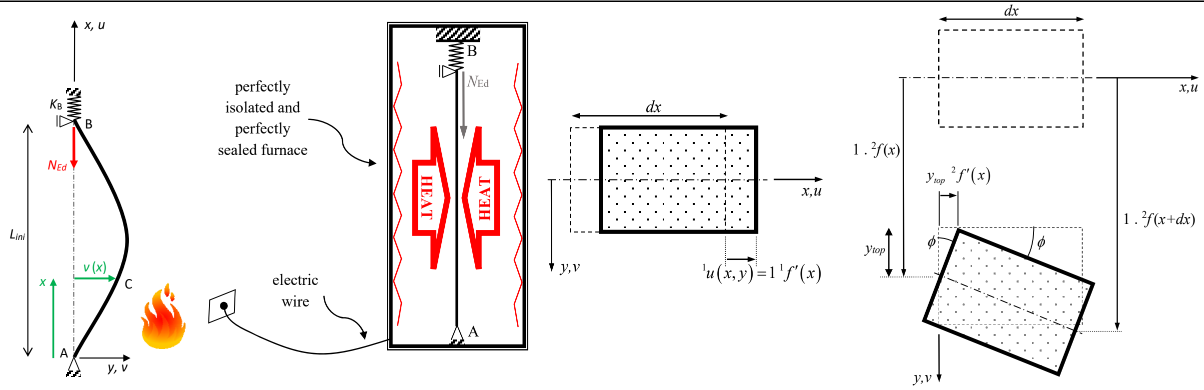
[1]*University of Coimbra, CERIS, Department of Civil Engineering*
Pólo II – Rua Luís Reis Santos, 3030-788 Coimbra, Portugal
pedro@dec.uc.pt, jpaulocr@dec.uc.pt
[2]*ITECONS, Rua Pedro Hispano, 3030-289 Coimbra, Portugal*

Abstract. The behavior of restrained steel columns in fire is analyzed in the paper, by quantifying the evolution of the mechanical and thermal energies from ignition until collapse. An appropriate structural model is adopted, to simulate real columns in buildings, and is analyzed by means of the voxels-based Rayleigh-Ritz method. This numerical method accounts for stability effects, spread of plasticity, degradation of the steel's mechanical properties with heating and non-uniform temperatures distributions at any stage of heating. The column's behavior in fire consists of thermal buckling followed by plastic collapse. From a Thermodynamics point of view, the heat given to the column is transformed into thermal and mechanical forms of energy stored in the column, and in energy conducted to the surrounding frame. The energy transformations during this process must respect all principles of Thermodynamics. To this end, a new energy balance expression is proposed, and the most relevant energy parts related to the mechanical problem are computed for an illustrative example. A new quantity – the thermal absorption capacity denoted by H – is proposed to quantify the resistance of columns in fire.

Keywords: energy balance, restrained steel columns in fire, voxels-based Rayleigh-Ritz analysis, Thermodynamics principles.

1 Introduction

The paper analyses the energy balance for restrained steel columns in fire. The structural model previously developed by Cabrita Neves [1] is adopted. It is depicted in Fig. 1-a) and pretends to simulate the real conditions of columns in buildings. Besides, it is easily reproducible at the laboratory, as observed in Fig. 1-b). This model accounts for both the restriction effect provoked by the surrounding frame, simulated by spring B that applies a restricting force N_{B} to the column, and a serviceability axial load N_{Ed}, applied before the ignition instant and that remains constant during fire. The behavior of restrained steel columns in fire is a complex nonlinear structural problem, since it encompasses a sequence of two nonlinear mechanical phenomena: thermal buckling followed by plastic collapse. Moreover, these phenomena occur while the steel's mechanical properties degrade with heating. The column's resistance in fire is defined by its collapse temperature θ_c, at which the column is no longer capable to bear the serviceability load N_{Ed} it was designed to. At the collapse temperature, the column is deeply in the plastic range and the restraining force N_{B} passes from compression to tension. The restraining force N_{B} is thus the axial force applied to the column by the surrounding frame, to oppose the column's thermal elongation. Spring B models solely the restriction effect provoked by the surrounding frame. For this reason, it is at rest at the onset of fire, at which the whole column is at ambient temperature $\theta_{amb} = 20°\mathrm{C}$. This means that $\theta = \theta_{amb} \Rightarrow N_{\mathrm{B}} = 0$. As the temperature rises, the column expands and a compressive restraining force N_{B} is generated. This force is linearly proportional to the longitudinal displacement u_{B} of section B, since spring B remains elastic during fire: $N_{\mathrm{B}} = K_{\mathrm{B}}(u_{\mathrm{B}} - u_{ref})$ (K_{B} is the spring's stiffness coefficient and u_{ref} is the axial displacement of point B at the ignition instant, caused solely by N_{Ed}). For the worst case scenario, fire occurs only in the column's compartment. Hence, the serviceability load N_{Ed} remains constant during fire. The column behavior is described by the plot $N_{\mathrm{B}} - \theta$ and collapse is defined by θ_c (henceforth named "collapse temperature"), at which the restraining force N_{B} returns to zero. Column's fire resistance is then defined as the time the member is able to resist to the effects of fire, starting at the ignition instant. For restrained steel columns this time lapse is obtained by measuring the time interval between the onset of fire and the instant the hottest fiber along the column reaches θ_c. The evolution of the temperatures distribution with time is usually provided by any appropriate fire curve, and consequently we can work solely in the temperatures domain.



a) The restrained column   b) The experimental apparatus   c) Mode 1: axial elongation   d) Mode 2: weak axis bending

Figure 1. The restrained column in fire and the adopted modes of deformation

The analysis of steel columns in fire presumes that the thermal and the mechanical problems are separable, in the sense that the displacements arising from the latter do not interfere with the temperatures distributions previously delivered by the former. For most works on structural fire resistance, the temperatures distributions arising from the thermal problem are merely an input data to the mechanical problem, and no further link between the two problems is sought thereafter. Although both problems – thermal and mechanical – coexist side by side, surprisingly until today no significant link was established between them in terms of energy balance. Theoretically, this link should be provided by the First Principle of Thermodynamics, that states “energy cannot be created nor destroyed during a process” [2]. The full application of this principle to columns in fire presumes that all energy transformations during this process are accurately quantified, which is rarely feasible. Even at the laboratory it is impracticable to measure accurately all energy transformations throughout a fire test of structural columns. Although it is probably impossible to apply this principle in full form, some parts of the corresponding equation, those with interest to the thermo-mechanical problem, can be exactly quantified: the thermal energy and the internal strain energy stored by the column, the work developed by external loads and/or at the supports and the electrical work input to the oven. Regrettably, in the literature that lays behind us on experimental testing of columns in fire we did not find any work that even attempted to measure these quantities during an experimental test of structural columns in fire.

The present work pretends to fulfil this gap by quantifying the energy quantities relevant to the mechanical problem during a fire, from ignition until collapse, and by seeking any link between them. Focus is placed on the experimental apparatus that reproduces real columns in fire, depicted in Fig. 1-b). The energy quantities involved in the problem encompass the internal strain energy U and the thermal energy T stored by the column, the work performed by the external loading W_N and the strain energy U_{B} stored by the restriction spring at point B. The thermal energy that passes through the column towards the neighboring frame is not of our concern here, because we deal with the resulting temperatures distributions. We presume that the steel column is a closed system, in the sense that it does not exchange any mass with its surroundings, and that it evolves by a succession of static equilibrium states, so that any dynamic effects are negligible. This complex process is analyzed here from the point of view of Structural Engineering, by computing consistently the energy balance during the fire process. The voxels-based Rayleigh-Ritz method, previously developed and validated in [3], is adopted here to describe the behavior of the restrained steel columns from ignition until collapse. In brief, the results arising from the structural analyzes performed by means of the voxels-based Rayleigh-Ritz method [3] are treated, to describe the behavior of restrained steel columns in fire from the point of view of energy balance. All computations are performed by means of the symbolic programing tool Wolfram Mathematica [4], and an illustrative example is presented.

2 The voxels-based Rayleigh-Ritz method – brief overview

The voxels-based Rayleigh-Ritz method [3] is adopted here to analyze the structural behavior of the column depicted in Fig. 1-a) (for full details on the methodology we suggest the reading of [3]). In the context of the

Generalized Beam Theory (GBT), here adapted to the Bernoulli bending theory, the column's displacements are aptly described by two modes of deformation: axial elongation ($k = 1$) and weak axis bending ($k = 2$), both depicted in Figures 1-c) and 1-d). The displacement of a generic point (x, y) is given by:

$$\vec{u}(x,y)=\begin{Bmatrix} u(x,y) \\ v(x,y) \end{Bmatrix}=\begin{Bmatrix} \sum_{k=1}^{2} {}^{k}u(y)\cdot {}^{k}f'(x) \\ \sum_{k=1}^{2} {}^{k}v(y)\cdot {}^{k}f(x) \end{Bmatrix} \tag{1}$$

where the unitary displacements patterns ${}^{k}u(y), k = 1,2$ are given by:

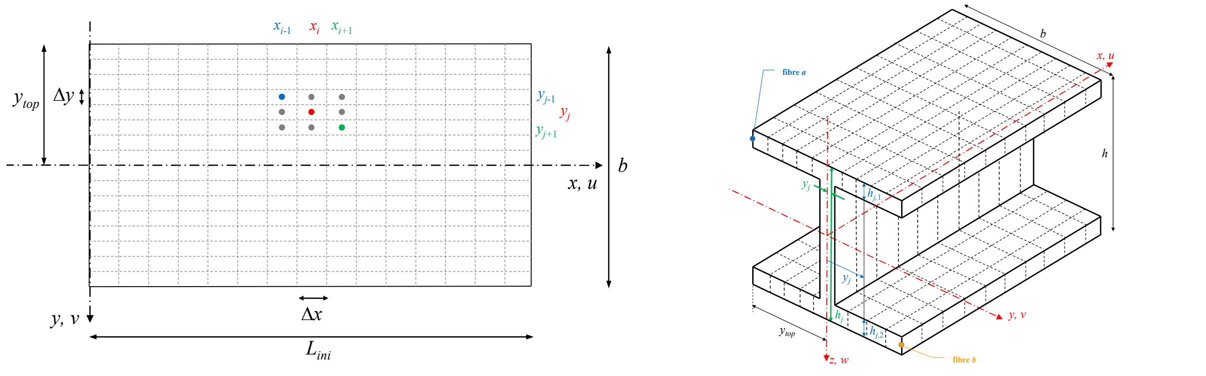
$$\text{mode 1:}\begin{cases} {}^{1}u(y)=1 \\ {}^{1}v(y)=0 \end{cases};\ \text{mode 2:}\begin{cases} {}^{2}u(y)=-y \\ {}^{2}v(y)=1 \end{cases} \tag{2}$$

${}^{k}f(x), k = 1,2$ are the modal amplitude functions and $'$ denotes differentiation with respect to x. These functions are approximated by means of the Rayleigh-Ritz method as follows:

$${}^{k}f(x)\approx\sum_{m=1}^{n_{coord,k}} {}^{k}a_m\ {}^{k}\varphi_m(x) \tag{3}$$

${}^{k}\varphi_m$ are polynomial coordinate functions that arise from the relevant modal boundary conditions by means of a sequential procedure developed in [5], and ${}^{k}a_m$ are the generalized coordinates that become the unknowns of the problem (after this discrete rendering process, a global numbering is applicable to the generalized coordinates ${}^{i}a, i = 1, \dots, n_{coord}$ and subscript $_m$ may be suppressed). From expression (1) we can derive the mechanical strains along the member. On the other hand, for any instant t during fire, we can calculate the temperatures distributions from $\theta_{ref,t}$, that is $\theta(x, y, t) = \theta(x, y, \theta_{ref,t})$. These temperatures distributions define the thermal strains along the column at any stage of heating. We assume a temperatures-sensitive elastic-perfectly plastic constitutive law for steel, and the resulting equilibrium system is solved for all stages of heating, from the ambient temperature $\theta_{amb} = 20°C$ until θ_c. The voxels-based Rayleigh-Ritz method divides the 2D column in small and uniform parallelepipeds (see Fig. 2): n_{Long} along the longitudinal direction and n_{Trans} along the cross section plane. For a generic voxel (i, j), its volume is $V_{ij} = \Delta x\ \Delta y\ h_j$, where h_j is the voxel's width (see Fig. 2-b)) and the coordinates of its geometric center are inferred from the voxel's ordering (i, j) in the grid:

$$\begin{cases} x_i=\Delta x(i-0.5) \\ y_j=-y_{top}+\Delta y(j-0.5) \end{cases}\quad i=1,\dots,n_{Long},\ j=1,\dots,n_{Transv}\ ,\ \Delta x=\frac{L_{ini}}{n_{Long}}\ \text{and}\ \Delta y=\frac{b}{n_{Transv}} \tag{4}$$



a) The voxels The voxels coordinates along the action plane

b) The width h_i and h_j of voxels with coordinates x_i and y_j, and fibres a and b

Figure 2. The 2D voxels ordering and the voxels width h_i for weak axis bending of H cross sections

Any quantity is supposedly constant along any voxel, and is measured at the voxel's geometric center. This turns the integration of this quantity along the voxel very easy: it is given simply by the value the quantity assumes at the voxel's geometric center (x_i, y_j) times the voxel's volume V_{ij}. The quantity may vary from one voxel to another one, which enables the modeling of plastic zones and general temperatures distributions while using a small number of generalized coordinates. This method of analysis delivers, at any stage of heating t, the generalized coordinates ${}^{k}a$ and the reference temperature $\theta_{ref,t}$, from which we can compute all stresses, displacements and temperatures along the member – this is the starting point for the energy balance analysis.

3 The energy balance for restrained columns in fire

3.1 Introduction

Having analyzed the column at any stage of heating, that is from the ignition instant t_0 until collapse t_c, the distributions of stresses, strains and temperatures along the member are known. Consequently, the task ahead of us now is to interpret these results, by quantifying the evolution of the thermal and strain energies during this process. To this end, consider the ideal experimental setting depicted in Fig. 1-b), that aims to simulate a real restrained steel column in fire. This laboratory apparatus comprises an electric furnace that contains a restrained steel column to be tested. The furnace runs perfectly, in the sense that:

i) it is perfectly insulated – no heat is transferred from inside the furnace's chamber towards the exterior;
ii) it is perfectly sealed – no matter can leave the interior of the furnace's chamber;
iii) it runs with perfect efficiency – it turns into heat, inside the furnace's chamber, all electric work that enters the system through the electric wire.

The elastic spring at section B aims solely to restrict the column's thermal elongation and is supposedly perfectly isolated. Hence, it does not heat and the spring's stiffness K_{B} does not degrade during heating. Moreover, spring B shall be at rest when the oven's switch button is turned on. The serviceability axial load N_{Ed} supposedly arises from a gravity potential and is applied at the top section of the column [AB]. It simulates the resulting axial force applied to the column at the ignition instant, caused by the weight of goods stored in the building and by the building's self-weight. The magnitude of N_{Ed} is surely much smaller than the column's design load for buckling N_{bRd} [6], and remains constant during the test or during the fire. In these ideal conditions, the only energy interaction with the exterior concerns:

i) the electric work $W_{electric}$ that crosses the furnace's boundary, to be fully transformed into heat Q_{oven} inside the furnace's chamber – note that the furnace runs with perfect efficiency;
ii) the work performed by N_{Ed} as the column deforms.

The First Principle of Thermodynamics states that "energy cannot be created nor destroyed during a process" [2]. The application of this principle to the experimental apparatus depicted in Fig. 1-b) generates the following energy balance equation at any instant t of heating:

$$\Delta W_{electric} = \Delta Q_{oven} = \Delta T_{air} + \Delta K_{air} + \Delta T_{[\mathrm{AB}]} + \Delta U_{[\mathrm{AB}]} + \Delta U_{\mathrm{B}} - \Delta W_{N} \tag{5}$$

$\Delta W_{electric}$ denotes the electric work input, ΔT_{air} and ΔK_{air} designate the variation of thermal and kinetic energies for the air inside the furnace's chamber, $\Delta T_{[\mathrm{AB}]}$ and $\Delta U_{[\mathrm{AB}]}$ are the variation of the thermal and internal strain energies stored in the column [AB], ΔU_{B} is the variation of the internal strain energy for the spring B and ΔW_N is the net amount of work performed by the serviceability load N_{Ed} during fire. When a correspondence with real fires is pretended, $\Delta W_{electric}$ shall be replaced by the energy released by the heat source. Equation (5) expresses the energy fluxes that occur during the fire test. It states that the column heats and deforms at the same time. This simultaneity means that the experimental apparatus depicted in Fig. 1-b) turns the electric work $\Delta W_{electric}$ into both thermal energy ΔT_{column} and strain energy ΔU_{column} at the same time. Therefore, none of these energies can be transformed one to the other, because both increase simultaneously and both arise from the same energy source.

However, real tests cannot be made in pressure pots. Therefore, some hot air must leave the experimental apparatus towards the exterior, carrying with it mass and energy. Moreover, no furnace is perfectly insulated nor perfectly sealed, and no furnace runs with perfect efficiency neither. Consequently, for the quantities we can usually measure at the laboratory during a fire test within a reasonable accuracy, the very best we can assert is:

$$\Delta W_{electric} > \Delta Q_{oven} > \Delta T_{[\mathrm{AB}]} + \Delta U_{[\mathrm{AB}]} + \Delta U_{\mathrm{B}} - \Delta W_{N_{\mathrm{Ed}}} \tag{6}$$

Consequently, we will focus on the quantities listed in the last member of expression (5) that we can quantify exactly during fire, to analyze their evolution from ignition until collapse. In [3,7] we proved that the energy parts referring solely to the mechanical problem are related by:

$$\Delta U_{[\mathrm{AB}]} + \Delta U_{\mathrm{B}} - \Delta W_{N_{\mathrm{Ed}}} = 0 \tag{7}$$

We wrote expressions (5) to (7) in variational form. Applying variations to the global energy balance of the system, in order to derive the mechanical equilibrium equations, means that, by any appropriate form, we shall derive the general expression of energy balance with respect to the generalized coordinates ${}^{k}a$ that describe the column's kinematics. This means that expression (7) can be derived from the global expression (5) because, in mathematical terms, $\Delta W_{electric}$, ΔT_{air} and ΔK_{air} do not depend on the generalized coordinates ${}^{k}a$, and so their derivatives with respect to these coordinates are zero. In contrast, the strain energy $\Delta U_{[\mathrm{AB}]}$ depends both on the generalized coordinates ${}^{k}a$ and on the temperature θ_{ref}, and acts as the unifying link between the thermal and the mechanical problems. In summary, the task ahead of us now is to compute the evolution of the energy parts in expression (5) by quantifying them exactly for validated cases [3] from ignition to collapse, in the context of Thermodynamics.

3.2 Computing the column's energy parts from the results of analysis

The internal strain energy density [8] stored at instant t by a generic voxel (i, j) is given in general by [3]:

$$\bar{U}_{ij,t} = \int_{0}^{t} \sigma_{x,ij,t} \, d\varepsilon_{x,Mec,ij,t} \tag{8}$$

Between instants t and $t+1$, this internal strain energy density is incremented by:

$$\Delta\bar{U}_{ij,t} = \frac{\sigma_{x,ij,t+1} + \sigma_{x,ij,t}}{2} \left(\varepsilon_{x,Mec,ij,t+1} - \varepsilon_{x,Mec,ij,t} \right) \tag{9}$$

The total increment of energy at instant t is given by the sum, for all voxels (i, j) that compose the column, of this strain energy density times each voxel's volume V_{ij}:

$$\Delta U_{t} = \sum_{i=1}^{n_{Long}} \sum_{j=1}^{n_{Transv}} \Delta\bar{U}_{ij,t} \, V_{ij} = \sum_{i=1}^{n_{Long}} \sum_{j=1}^{n_{Transv}} \Delta\bar{U}_{ij,t} \, \Delta x \, \Delta y \, h_{j} \tag{10}$$

The total energy stored at the collapse instant t_c is simply the sum, for all instants t between ignition (t_0) and collapse (t_c), of ΔU_t: $U_{[\mathrm{AB}],c} = \sum_{t=t_0}^{t_c} \Delta U_t$. The column stores the strain energy $U_{[\mathrm{AB}]}$ and thermal energy $T_{[\mathrm{AB}]}$ simultaneously, and steel is an incompressible solid material from a Thermodynamics point of view. This means that the steel's unit mass ρ_a remains constant with temperature [9]. Besides, both the steel's unit mass ρ_a and specific heat $c_a(\theta)$ are supposedly independent of the stresses applied to the column. Hence, the thermal energy density stored by voxel (i, j) at instant t of heating is [2,8]:

$$\bar{T}_{ij,t} = \int_{0}^{t} \rho_{a} \, c_{a}\left(\theta_{ij,t}\right) d\theta_{ij,t} \tag{11}$$

Between instants t and $t+1$ the thermal energy density $\bar{T}_{ij}$ stored at a generic voxel (i, j) is incremented by:

$$\Delta\bar{T}_{ij,t} = \rho_{a} \frac{c_{a}\left(\theta_{ij,t+1}\right) + c_{a}\left(\theta_{ij,t}\right)}{2} \left(\theta_{ij,t+1} - \theta_{ij,t} \right) \tag{12}$$

and the total increment of thermal energy along the whole column is given by:

$$\Delta T_{t} = \sum_{i=1}^{n_{Long}} \sum_{j=1}^{n_{Transv}} \Delta\bar{T}_{ij,t} \, \Delta x \, \Delta y \, h_{j} \tag{13}$$

The total thermal energy stored by the column [AB] at collapse is thus given by $T_{[\mathrm{AB}],c} = \sum_{t=t_0}^{t_c} \Delta T_t$.

At step t, the longitudinal end displacement u_{B} is given by:

$$u_{\mathrm{B},t} = {}^{1}u \; {}^{1}f_{t}'\left(L_{ini}\right) \approx 1 \times \sum_{m=1}^{n_{coord,1}} {}^{1}a_{m,t} \; {}^{1}\varphi_{m}'\left(L_{ini}\right) \tag{14}$$

Due to this displacement, the serviceability load N_{Ed} produces an amount of work between instants t and $t+1$:

$$\Delta W_{N,t} = N_{\mathrm{Ed}}\left(u_{\mathrm{B},t+1} - u_{\mathrm{B},t}\right) \tag{15}$$

and the total work performed from ignition until collapse is merely given by $W_{N,c} = \sum_{t=t_0}^{t_c} \Delta W_{N,t}$.

Above we mentioned that, at step t, spring B applies a compressive force to the column equal to $N_{\mathrm{B},t} = K_{\mathrm{B}}\left(u_{\mathrm{B},t} - u_{ref}\right)$. Between instants t and $t+1$ the strain energy U_{B} stored at spring B is thus increased by:

$$\Delta U_{\mathrm{B},t} = \frac{N_{\mathrm{B},t+1} + N_{\mathrm{B},t}}{2}\left(u_{\mathrm{B},t+1} - u_{\mathrm{B},t}\right) \tag{16}$$

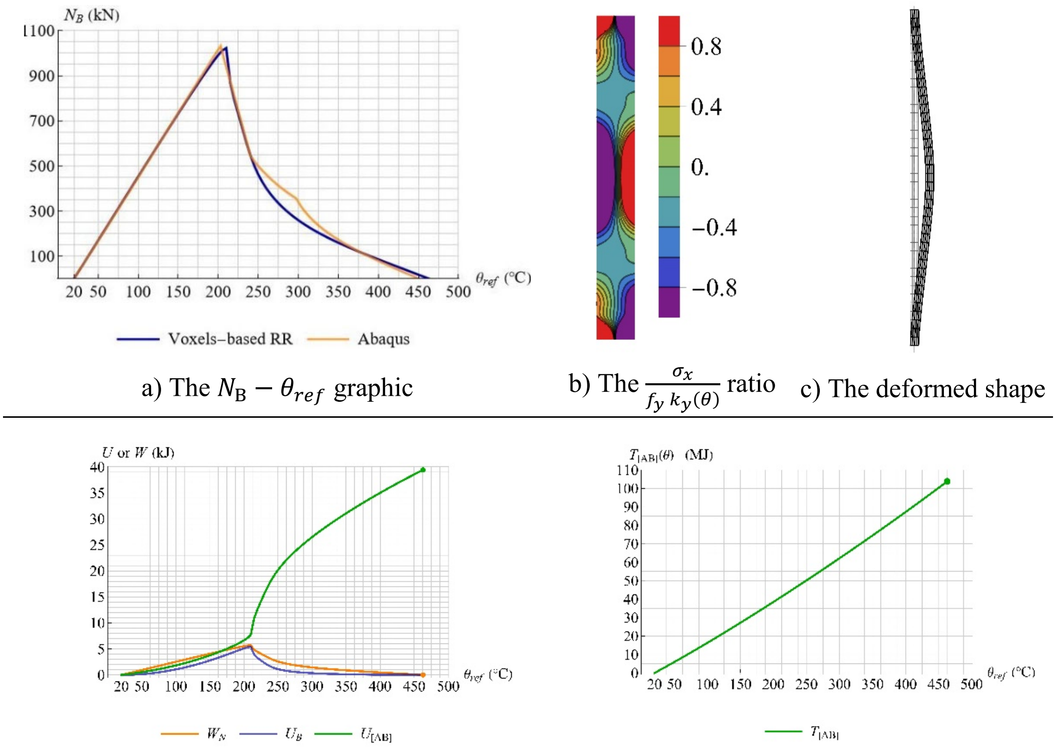
The total energy stored until collapse is $U_{\mathrm{B},c} = \sum_{t=t_0}^{t_c} \Delta U_{\mathrm{B},t}$. Now we are ready to apply these formulae to real cases of columns in fire, and to analyze the evolution of each energy part from ignition until collapse.

4 Illustrative example and conclusion

Consider that the column depicted in Fig. 1-b) is made up of S355 steel grade and has a standard HEB 200 cross section. The column is clamped at both ends, bends around the weak axis and its initial length is $L_{ini} = 8.51301$ m, to which corresponds a reduced slenderness ratio $\bar{\lambda} = 1.1$ [6]. The applied serviceability load is $N_{\mathrm{Ed}} = 537.040$ kN (40% of $N_{\mathrm{b,Rd}}$), and the adopted spring stiffness is $K_{\mathrm{B}} = 9.63044 \times 10^4$ kN/m (50% of the column's axial stiffness EA/L_{ini}, where E is the Young's modulus and A is the cross section area). At any stage of heating, we suppose that the temperature is constant along the yy axis but may vary along the xx axis, to simulate real temperatures distributions. For the voxel at position (i,j) and for a generic stage of heating t, related to any $\theta_{ref,t}$ between θ_{amb} and θ_c, its temperature $\theta_{ij,t}$ is:

$$\theta\left(x_i, y_j, t\right) = \theta_{ij,t} = \theta_{amb} + \Delta\theta_{ij,t} = \theta_{amb} + \frac{4\left(L_{ini} - x_i\right)x_i + 0.5\left(L_{ini} - 2x_i\right)^2}{L_{ini}^2}\left(\theta_{ref,t} - \theta_{amb}\right) \tag{17}$$

This function imposes that, at any stage of heating t, the temperatures increase $\Delta\theta_{ij,t}$ at the mid-height cross section is twice the corresponding increase at the edge sections, that a parabolic variation occurs between these three sections, and that the reference temperature $\theta_{ref,t}$ is equal to the temperature at the mid-height cross section. For validation, Fig. 3-a) compares the $N_{\mathrm{B}} - \theta_{ref}$ graphic for both the voxels-based Rayleigh-Ritz method [3] and for a FEM analysis using Abaqus [10], and a perfect agreement is observed. The collapse temperature is computed when N_{B} returns to zero, yielding $\theta_c = 462.916$ °C. For $\theta_{ref} = \theta_c$, Fig. 3-b) depicts the ratio $\sigma_x/\left(f_y\, k_y(\theta)\right)$ (σ_x is the normal longitudinal stress, f_y is the steel's yield stress and $k_y(\theta)$ represents the degradation of f_y with temperature [3,9]), and three plastic zones are clearly observed, related to the typical three-hinges plastic mechanism [7]. Fig. 3-c) represents the column's deformed shape at collapse. To compute all the energy parts, we adopted the values listed in the code of practice [6,9] for all thermal and mechanical properties. Fig. 3-d) depicts the evolution of the mechanical forms of energy. Both U_{B} and $W_{N_{\mathrm{Ed}}}$ reach their maximum value at the thermal buckling temperature and decrease afterwards, being both null at collapse. In contrast, $U_{[\mathrm{AB}]}$ always increases with heating until a maximum value at collapse, and its slope increases abruptly when the thermal temperature is reached. This means that, at the thermal buckling state, the structural model reaches its maximum capacity of transferring mechanical energy to its surroundings. Afterwards, the surrounding frame sends this energy back to the column. The thermal energy $T_{[\mathrm{AB}]}$ stored by the column is depicted in Fig. 3-e) and increases always, reaching its maximum value at collapse. The most important observation is that the thermal energy $T_{[\mathrm{AB}]}$ is always of much higher magnitude than all mechanical forms of energy involved in the problem. This means that the major part of heat Q given to the column during fire is stored by the column as thermal energy, and only a residual part is transformed into mechanical forms of energy. This agrees with the Second Principle of Thermodynamics, that states, in the Kelvin-Planck form, "it is impossible for any device that operates on a cycle to receive heat from a single reservoir and produce a net amount of work" [2]. Applying this principle to the problem under analysis means that only a small portion of heat Q arising from the column's surroundings can be transformed into mechanical forms of energy, such as strain energy or work. The remaining part must be transformed into thermal energy stored by the column. Both expressions (5) and (6) respect this principle, and respect the simultaneous increase of $T_{[\mathrm{AB}]}$ and $U_{[\mathrm{AB}]}$ as well. This suggests the definition of a new quantity to characterize the problem: the thermal absorption capacity $H_{[\mathrm{AB}]} = T_{[\mathrm{AB}]} + U_{[\mathrm{AB}]} \approx T_{[\mathrm{AB}]}$. At collapse, we found that $T_{[\mathrm{AB}],c} = 0.9996\, H_{[\mathrm{AB}],c}$. The development of energy based criteria to assess the resistance of restrained steel columns in fire conditions requires now the computation of $H_{[\mathrm{AB}]}$, or of the corresponding average density $\overline{H}_{[\mathrm{AB}]} = H_{[\mathrm{AB}]}/(A\ L_{ini})$, for a large number of cases, and to search for any trend $H_{[\mathrm{AB}]}$ may follow as a function of the most relevant properties, in order to develop energy based criteria to assess the resistance of restrained steel columns in fire.



a) The $N_{\mathrm{B}} - \theta_{ref}$ graphic b) The $\frac{\sigma_x}{f_y\, k_y(\theta)}$ ratio c) The deformed shape

d) The energies U_{B} and $U_{[\mathrm{AB}]}$ and work $W_{N_{\mathrm{Ed}}}$ e) The thermal energy $T_{[\mathrm{AB}]}(\theta_{ref})$

Figure 3. The results from the analysis and the plot of the energy parts against the reference temperature θ_{ref}

Acknowledgements. This work was supported by “Fundação para a Ciência e a Tecnologia” (FCT-Portugal) under project grants UIDB/04625/2020, PCIF/MOS/0129/2018 and PCIF/AGT/109/2018.

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] I.Cabrita Neves, “The critical temperature of steel columns with restrained thermal elongation”. *Fire Safety Journal*, vol. 24, p. 211–227, 1995.
[2] Y.A. Çengel, M.A. Boles and M. Kanoglu, “*Thermodynamics: An Engineering approach*” (9th Ed.). McGraw Hill, 2020.
[3] P.D. Simão, J.P.C. Rodrigues and H.D. Fernandes, “A voxels-based Rayleigh-Ritz method for the post-buckling elasto-plastic analysis of restrained columns in fire”. *Journal of Constructional Steel Research*, vol. 201, paper n. 107736, 2023.
[4] Wolfram Research, “*Wolfram Mathematica*” (version 13). Wolfram Corporation, 2023
[5] P.D. Simão, A.M. Girão Coelho and F.S.K. Bijlaard, “Influence of splices on the buckling of columns”. *International Journal of Non-Linear Mechanics*, vol. 47, p. 806–822, 2012.
[6] European Committee for Standardization (CEN), “*Eurocode 3: Design of Steel Structures – Part 1-1: General rules and rules for buildings* (EN 1993-1-1)”. CEN, 2005.
[7] P.D. Simão and J.P.C. Rodrigues, “Direct evaluation of fire resistance for restrained columns in frames using consistent rigid-plastic models and energy formulations”. *Engineering Structures*, vol. 276, paper n. 115344, 2023.
[8] J.N. Reddy, “*Energy and variational methods in applied mechanics*”. John Wiley & Sons, 1984.
[9] European Committee for Standardization (CEN), “*Eurocode 3: Design of Steel Structures – Part 1-2: General rules – Structural fire design* (EN 1993-1-2)”. CEN, 2005.
[10] Simulia Corp., “Abaqus/CAE Standard User’s Manual” (version 6.14-1). Dassault Systems – Simulia Corp., 2014.