Simplified vehicle model validation through test track experimental data - An SUV case study analysis

Walter Paschoal[1,2], Igor Souza [2], Andre Murilo[1,2], Renan Ozelo[3], Rafael Silva[1,2].

[1]*Programa de pós-graduação em sistemas mecatrônicos, Universidade de Brasília*
Brasília, 70910-900, Federal District, Brazil
walterpaschoal@icloud.com, andremurilo@unb.br, rafael.rodrigues@unb.br
[2]*Automotive Research Center, Universidade de Brasília - Faculdade do Gama*
St. Leste Projeção A, Brasília, 72444-240, Federal District, Brazil
igor.sbssouza@hotmail.com
[3]*Research & Development, Pirelli Pneus Ltda*
Rua José Versolato, 101, São Bernardo do Campo, 09750-730, São Paulo, Brazil
renan.ozelo@pirelli.com

Abstract. The vehicle development process benefits from modeling as a tool for investigating ways to improve performance and efficiency. Vehicle modeling also enables virtualization of vehicle development, aiming to reduce costs through virtual simulation of use cases. However, the process of conceiving a vehicle requires a systematic and structured approach that encompasses validation. Complex models can result in increased development costs and reduced computational efficiency. In this study, a modified three-degree-of-freedom (bicycle) model is proposed to describe vehicle's lateral dynamics. As important as the vehicle model, a good representation of tire dynamic through tire models also play an essential role in representing general vehicle dynamics. In this work, tire force generation is described by Paceijka's Magic Formula. The reliability of the model proposed is ensured by comparing it with test track data from a real SUV vehicle, with estimated inertia data. The model output exhibited correlation with the actual data.

Keywords: Vehicle dynamics, Tire model, Bicycle model, Vehicle model validation

1 Introduction

The number of victims involved in automotive accidents has increased over the years in some markets, such as the Brazilian one [1]. This has led to new safety regulations which imply in active vehicle safety systems regulation and development, such as the stability control system (ESC). It acts on the available traction forces, and it depends on the vehicle and tire model as the system's reference. The lateral motion modeling is utterly important to controllers acting on the road-vehicle interaction, seeking to avoid loss of control of the vehicle during critical maneuvers. Therefore, there is a growing interest among researchers and engineers to approach and validate vehicle models which are simple but effective in describing vehicular dynamics.

Vehicle models possess the capability to anticipate alterations in vehicle attributes resulting from project modifications, thereby attaining specified performance objectives. The vehicle development procedure derives advantages from this modeling as it serves as a mechanism for probing avenues to enhance performance and efficiency. Furthermore, vehicle modeling facilitates the virtualization of the vehicle development process, aiming to reduce costs through the simulated emulation of usage scenarios. It is possible to describe lateral dynamics through the single-track (bicycle) model Gillespie [2] and Milliken and Milliken [3] . It consists of a representation of the vehicle by collapsing the left and right wheels into one. The bicycle model represents a good approximation of the vehicle handling dynamics [4].

One of the simplest vehicle dynamics models is the single-track (bicycle) model. Its origins date back to 1940, but its relevance persists through today [5]. Due to the model's success to approximate real vehicle's behavior while being relatively simple, authors aim to verify the bicycle model's accuracy for its utilization across diverse applications. Martinez et al. [6] introduce an modified bicycle vehicle model that integrates dynamic contact friction, based on the Lu-Gre friction model. The model proposed finds applicability in simulating challenging driving conditions, including scenarios involving full sliding. The article by Ren et al. [7] focuses on the development of an effective bicycle model that can accurately represent a wide range of vehicle operations. The author compares the Paceijka's Magic Formula tire model with a developed piece-wise linear tire model that well represents tire forces. The simulation outcomes indicate a close match between the trajectory of the proposed bicycle model and the CarSim output for ramp steer and sine-with-dwell maneuvers. In [8], authors propose an equation representing lateral jerk, which is the rate of change of lateral acceleration, was created and incorporated into a predictive control formulation based on a kinematic bicycle model. Their modeling approach was validated in a dynamic simulator (Dynacar) and compared to an instrumented vehicle (Renault Twizy) ensuring reasonable correlation. The work conducted by Arikan et al. [9] rely on experimental data to estimate physical parameters of the linear bicycle model, which generated information on the handling dynamics of the vehicle tested. The model was validated using data that was not utilized for identification purposes. Authors in Polack et al. [10] examined the reliability of employing a kinematic bicycle model for trajectory planning objectives through a comparison of its outcomes with those generated by a sophisticated, realistic vehicle model. The authors found out that if the lateral acceleration of the vehicle is limited to a certain threshold, the bicycle model shows consistent correlation, allowing its use in low-level controllers.

Although the single track model shows good correlation to real data as shown by the works, the vehicle's performance is significantly influenced by the forces generated between the tires and the road surface. Therefore, when simulating the dynamic response of a vehicle, the precision of simulation outcomes is affected by the accuracy of the calculated longitudinal and lateral tire forces from the tire model [7]. One of the key authorities on tire dynamics is Pacejka, credited with developing the renowned Magic Formula, an empirical tire model built upon curve fitting using experimentally collected tire data. The "Magic Formula" is used to describe tire forces due to its effectiveness of providing a real approximation of generation representation [11] and [12] . This valuable tool developed by Paceijka, which will be approached in the next chapter, will also be used in this paper.

By means of empirical vehicular data through a sport utility vehicle (SUV) track test, it becomes conceivable to translate and encapsulate the vehicle lateral dynamics, within the realm of a simplified lateral dynamics framework. This work proposes to fulfill such objective by applying a modified three-degree-of-freedom model. The work is divided as it follows: the second chapter seeks to provide the reader with theoretical background; the third chapter aims to expose the proposed methodology; the fourth chapter is composed of results and discussion and the last chapter, conclusions, intents to conclude the work and to indicate future works.

2 Theoretical review

This section of the paper aims to offer a theoretical overview to furnish the reader with a conceptual foundation regarding the topics addressed in this study, specially regarding vehicle and tire models.

2.1 Vehicle model

Vehicular dynamic modeling can be categorized into three aspects: longitudinal dynamics, encompassing acceleration and braking; vertical dynamics, involving road excitation frequency responses and comfort considerations; and lateral dynamics, encompassing steering and vehicle maneuverability [2]. This study primarily concentrates on lateral dynamics, aiming to validate the efficacy of a modified single-track model that evaluates the lateral load transfer by comparing it to data obtained from a real vehicle test.

In this study, the lateral dynamics will be approached through a modified three degree-of-freedom (DOF) single-track (bicycle model), based on the analytical mathematical formulation of Pacejka [13]. This model simplifies the vehicle's representation by merging the left and right wheels into a single unit for the calculation of the three DOFs, as shown in the Fig. 1.

The Figure 1 shows the variables that compose the model: a is the distance from the center of gravity to the front axle, while b denotes the distance to the rear axle. h_{CG} represents the height of the center of gravity, while h_{RCf} and h_{RCr} correspond to the heights of the roll axis (front and rear). Additionally, h represents the distance from the roll axis to the center of gravity, δ_f signifies the steering angle, u stands for the longitudinal velocity, and v represents the lateral velocity. The variable β represents the side-slip angle, r denotes the yaw rate, p signifies the roll rate, and ϕ represents the roll angle.

Furthermore, ω_f and ω_r indicate the angular velocity of the front and rear wheels, while F_{xf} and F_{xr} represent the longitudinal forces exerted by the front and rear tires, respectively. Additionally, F_{yf} and F_{yr} stand for the lateral forces generated by the front and rear tires.

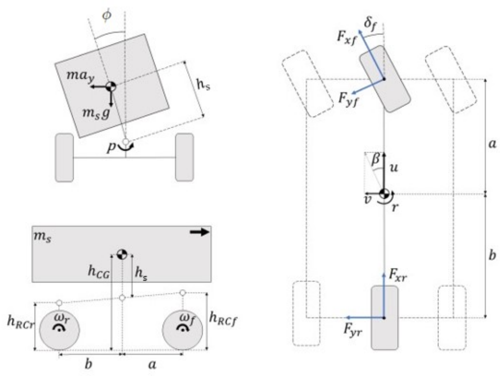

Figure 1. Representation of bicycle model and roll motion ([14])

To obtain the equations of motion for the three-degree-of-freedom model, Lagrange's equations can be employed. Authors such as Ulsoy et al. [15] and Pacejka [13], have utilized Lagrange's equations from an energy-based standpoint. Deriving the equations (which are explained in depth in the reference Pacejka [13]) for the variables shown, the dynamic equations of motion are found:

$$\begin{cases} \sum F_y = m(\dot{v} + ru) - m_s(r^2 + p^2)h_s \sin\phi + m_s\dot{p}h_s \cos\phi \\ \sum M_z = \dot{p}I_{xz} + \dot{r}I_z + m_s h_s p^2 \sin\phi + m_s r h_s^2 p \sin(2\phi) - m_s\dot{u}h_s \sin\phi \\ \sum M_x = m_s h_s(\dot{v} + ru)\cos\phi + \dot{p}I_x + \dot{r}I_{xz} + m_s r^2 h_s^2 \sin\phi\cos\phi \end{cases} \tag{1}$$

The external forces acting on the vehicle constitute the forces generated at the tires and the reaction of the suspended mass for roll dynamics, forming the equations of external forces:

$$\begin{cases} \sum F_y = F_{yf}\cos\delta_f + F_{yr} \\ \sum M_z = aF_{yf}\cos\delta_f - bF_{yr} \\ \sum M_x = (m_s g h_s - K_\phi)\phi - C_\phi\dot{\phi} \end{cases} \tag{2}$$

Through the equations of external forces and the equations of motion dynamics, it becomes feasible to derive the state variables. These variables will then be utilized as inputs for calculating slip angles, a distinction from the conventional bicycle model as the calculation will be applied to all four wheels in the model developed within this study.

$$\begin{array}{ll} \alpha_{FL} = \dfrac{V_y + r\cdot a}{V_x - r\cdot t_F/2} - \delta_{RF} + toe_{FL} & \alpha_{FR} = \dfrac{V_y + r\cdot a}{V_x + r\cdot t_F/2} - \delta_{RF} + toe_{FR} \\ \alpha_{RL} = \dfrac{V_y - r\cdot b}{V_x - r\cdot t_R/2} + toe_{RL} & \alpha_{RL} = \dfrac{V_y - r\cdot b}{V_x + r\cdot t_R/2} + toe_{RR} \end{array} \tag{3}$$

As the four wheels are considered, the front lateral force is $F_{yf} = (F_{y_{fl}} + F_{y_{fr}})cos\delta_f$ and the lateral force for the rear axle is $F_{yr} = (F_{y_{rl}} + F_{y_{rr}})$. In order to evaluate the four wheel vertical loads and its effects in the tire force generation, it is possible to calculate the lateral load transfer using the equation $\Delta W = \frac{WA_y h_{CG}}{t}$, from Milliken and Milliken [3].

2.2 Tire model

Equally significant is the task of modeling tire force generation, governed by tire models. The selection of appropriate tire models holds paramount importance in accurately depicting forces, a prerequisite for numerous applications such as electronic active safety systems (e.g., ESC). The objective is to mirror real-world forces as closely as possible, thereby ensuring optimal functionality across diverse scenarios. The Magic Formula, is an empirical tire model that is widely recognized as the most extensively used model, renowned for its strong alignment with experimental data. The mathematical formulation of the Magic Formula (Equation 4) was derived from Pacejka [16].

$$F_y = D_y \sin C_y \arctan(B_y\alpha - E_y(B_y\alpha - \arctan(B_y\alpha))) + S_{Vy} \tag{4}$$

((a)) Instrumented cabin ((b)) GPS sensor on top of the car

Figure 3. Instrumented car for the track test

The terms B_y, C_y, D_y, E_y and S_{vy} represent constants, which have physical meanings related to tire parameters. The term B_y represents the rigidity parameter, C_y is the shape parameter, D_y is the peak curve parameter and their mathematical product equals to the rigidity at the zero lateral displacement point. The parameter E_y represents the curvature factor. The last parameter, S_{vy} is the vertical fit that indicates inherent residual forces of tire's construction. It is important to address that these coefficients won't be specified here since it envolves industrial secret knowledge and it was a condition to proceed with the study.

3 Methodology

In this section the methods and proceedings to perform the experiment, develop and validate the vehicle model will be detailed. The objective is to implement the vehicle and tire model in the MATLAB/Simulink environment, based in the theory discussed in the previous section (2). Additionally, we aimed to validate the developed model by assessing its correlation with the experimental data.

3.1 Experimental setup

The experiment was conducted in the *Circuito Panamericano*, a Pirelli test facility sited in Brazil. The vehicle, an SUV (Tiguan 2.0 TSI Four Motion R-line), was instrumented and the driver performed a lap around the circuit. The case study consisted of a fast lap on a wet circuit, a challenging scenario that rigorously assesses vehicle performance and driver skill. This high-stakes lap, featuring rapid accelerations, decelerations, lateral movements, and intricate combinations, serves as a test for the vehicle model's validation under demanding conditions. The steering wheel angle and speed profiles that compose the maneuvers performed are shown in Figure 2.

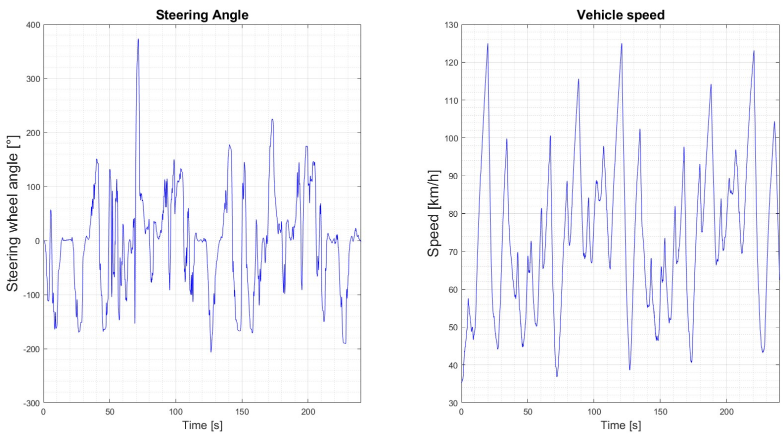

Figure 2. Steering wheel angle and speed profile

The vehicle was instrumented with GPS, a steering wheel sensor, that captures the steering wheel torque and angle and an inertial platform that logs the accelerations in three axles - longitudinal, lateral and vertical - and rotational velocities around the same three axles of the vehicle. With this acquisition data system, it is possible to obtain the necessary variables for the proposed model inputs and for comparison and validation. The Figure 3 presents the instrumentation installed in the car.

3.2 Data treatment

To enhance data analysis, a data filtering process was employed, and this study opted to utilize the Savitzky-Golay filter due to its characteristic of minimizing signal delay. Savitzky and Golay introduced a data smoothing technique based on local least-squares polynomial approximation. This filter effectively diminishes noise while preserving the shape and amplitude of waveform peaks [17]. The variables chosen to be treated were Yaw rate, Roll rate and Lateral acceleration.

3.3 Tire and Vehicle model

A hybrid model composed of three subsystems: Driver, Vehicle and Tires was developed. The Vehicle subsystem contains the equations 1 and 2 to evaluate the 3 degrees of freedom (Lateral velocity, yaw rate and roll rate) to output the vertical load and slip angle of each wheel to the Tires subsystem (the vertical loads are calculated and each tire slip angle are defined by equation 3). And to do so, the Vehicle subsystem has the inputs of steering wheel angle and longitudinal speed from the Driver and the lateral forces from the Tire subsystems.

In order to provide the model proposed with the data from the test, some assumptions had to be made. Since some of the vehicle data specification were not available, it was necessary to estimate the inertia data and the vehicle's center of gravity. The estimation of the center of gravity (CG) height for the VW Tiguan was carried out using the Static Stability Factor (SSF) method, employing data sourced from NHTSA's 2013 model. The Static Stability Factor (SSF) is a metric that quantifies the static stability of vehicles by considering factors related to the vehicle's geometry and center of gravity [18] [19]. The equation for the SSF is defined by $SSF = \frac{b}{2h_{CG}}$, where b is the wheelbase and h_{CG} is the center of gravity height. The inertia calculations were performed based on a parallelepiped (formulation available in [20]), taking into consideration the left/right weight distribution of the vehicle. The relevant data has been incorporated within the modularized code structure.

3.4 Validation procedure

The validation phase consist of two parts, the visual comparison of the model and experimental data signals, followed by a correlation calculation between these signal. It is valuable to understand if the model is capable and at which level represents the main dynamics of the vehicle. For the correlation, the R^2 technique will be applied for the following variables: Lateral acceleration, Yaw rate and Roll rate, where $R^2 = \frac{\textit{Sum of Squared Residuals}}{\textit{Total Sum of Squares}}$.

4 Results and Vehicle model validation

This sections will present and discuss the results obtained in the experimental data filtering and the model validation process.

4.1 Filtering results

Figure 4 presents the visual comparison between the original experimental signals of Yaw rate, Roll rate and Lateral acceleration and the resultant filtered signals. The filtered signals were capable of capturing the main characteristics of the original signals, with minimum delay, fulfilling the main objective of the data treatment phase.

4.2 Model comparison and validation

Through the visual analysis of Figure 5, globally the model was capable to represent the main dynamics of the real vehicle, with one region of inconsistency, in the 50-60 seconds zone. At this point, the lateral acceleration and the yaw rate signal were considerably off the real vehicle trace. Besides that, the model has shown its potential to be a valuable tool in modeling and development of vehicle systems by this qualitative aspect.

Further analysis of Figure 5 shows that proposed model (green line) is capable of represent the main dynamics of the reference car (black line) as the green line follows the black one for the variables highlighted in Figure 5: lateral acceleration, yaw rate and roll rate. Similar to the work conducted by Matute et al. [8], where the authors validated the model through tests with a real vehicle, the shape and amplitude of waveform peaks are mostly preserved in the proposed model. The parts of the curve that are not accurately tracked by the model could be attributed to the omission of certain vehicular dynamics in the simpler 3DOF bicycle model, as well as the

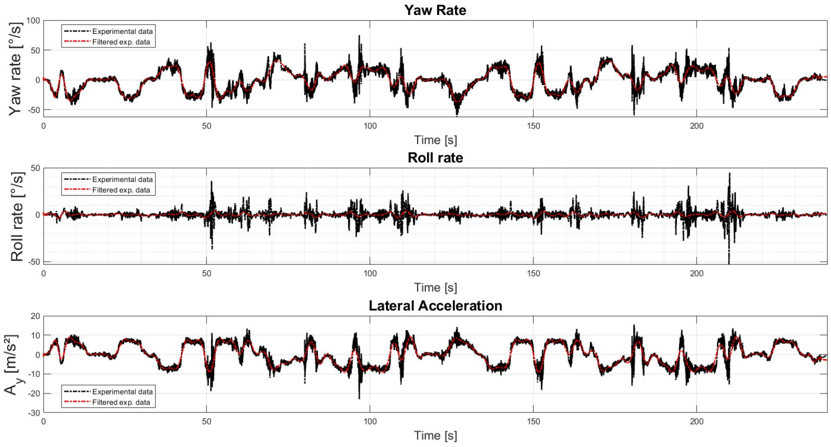

Figure 4. Filter results of Yaw rate, Roll rate and Lateral acceleration

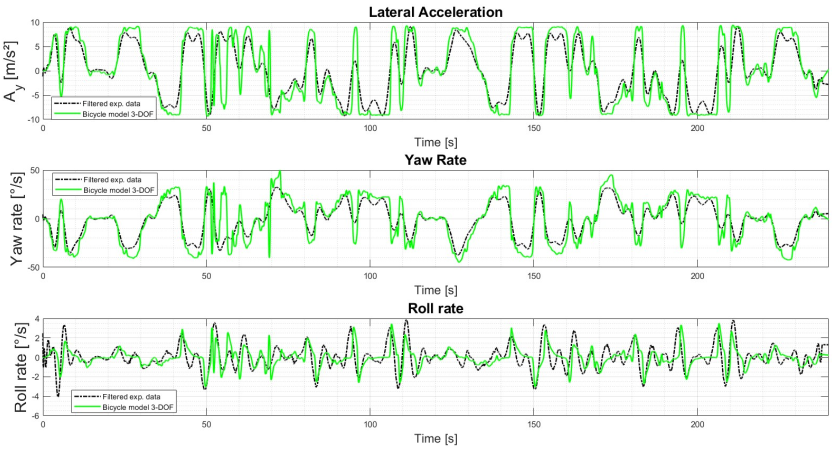

Figure 5. Comparison between model and experimental data

estimated inertia and h_{CG} data which might differ from the actual data which wasn't available.

Variable	r^2
Roll rate	27.5%
Yaw rate	85.5%
Lateral acceleration	84.8%

Table 1. R^2 correlation between model and experimental data

The correlation of roll rate, yaw rate, and lateral acceleration using the r^2 technique, presented in Table 1, points out a concern about the roll rate variable, that could be a filtering or from an unknown source. It demands more investigation and further research in different conditions to validate this quantitative metric. It is important to mention that improved positioning of the inertial platform, as well as an investigation into the calculations used to report the measured values to the estimated center of gravity, could enhance the presented results, especially those related to the roll rate variable. As for the other two variables analyzed, a signal fitting close to 85% was achieved.

5 Conclusions

Despite being one of the simplest vehicle dynamics models, many authors rely on the bicycle model to describe vehicular dynamics due to its effectiveness. The model proposed by this paper was implemented as a modularized code structure, composed of three subsystems: Driver, Vehicle and Tires. The goal was to validate the model with inputs from a real track test (wet fast lap) with an instrumented vehicle.

A qualitative analysis of the proposed experiment revealed a satisfactory level of correlation between the model and the experimental data, as the model accurately represented the lateral dynamics for the majority of the signal length. Future studies will need to confirm whether the quantitative metric obtained is suitable for applications like ESC controls. In future research, performing ISO testing will be essential to establish a comprehensive model validation procedure, ensuring that the model is tested across various scenarios. Beyond that, it is possible to conclude that more investigation has to be done to understand why there are some inconsistencies and to define the limitations of the model for the proposed case. Despite the necessary further research and validation, the model has potential to serve as a valuable tool in the automotive systems development industry.

Acknowledgements. The authors acknowledge the financial support provided by Fundação de Apoio da UFMG (Fundep) - Rota 2030 - Linha V and Fundação de Apoio a Pesquisa do Distrito Federal (FAP-DF). The help of all the *Circuito Panamericano* and experimental team from Pirelli was much appreciated by the authors and fundamental to this work's develpoment.

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] C. N. D. TRANSPORTE. Painel cnt de acidentes rodoviários, 2021.
[2] T. D. Gillespie. *Fundamentals of Vehicle Dynamics*. SAE International, 1992.
[3] W. F. Milliken and D. L. Milliken. *Race Car Vehicle Dynamics*. SAE International, 1995.
[4] G. Mastinu and M. Plöchl. *Road and off-road vehicle system dynamics handbook*. CRC Press, Taylor amp; Francis Group, 2014.
[5] E. Kutluay and H. Winner. Validation of vehicle dynamics simulation models – a review. *Vehicle System Dynamics*, vol. 52, n. 2, pp. 186–200, 2014.
[6] J.-J. Martinez, J.-C. Avila, and de C. C. Wit. A new bicycle vehicle model with dynamic contact friction. *IFAC Proceedings Volumes*, vol. 37, n. 22, pp. 625–630, 2004.
[7] H. Ren, T. Shim, J. Ryu, and S. Chen. Development of effective bicycle model for wide ranges of vehicle operations. *SAE Technical Paper*, , n. 2014-01-0841, 2014.
[8] J. Matute, M. Marcano, S. Diaz, and J. Pérez. Experimental validation of a kinematic bicycle model predictive control with lateral acceleration consideration. volume 52, 2019.
[9] K. Arikan, Y. S. Unlusoy, O. Celebi, and I. Korkmaz. Identification of linear bicycle model of an automobile using experimental data, 2006.
[10] P. Polack, F. Altché, B. Novel, and de A. La Fortelle. The kinematic bicycle model: A consistent model for planning feasible trajectories for autonomous vehicles? pp. 812–818, 2017.
[11] Z. R. M. Júnior, A. Murilo, and R. V. Lopes. Vehicle stability upper-level-controller based on parameterized model predictive control. *IEEE Access*, vol. 10, pp. 21048–21065, 2022.
[12] J. D. Setiawan, M. Safarudin, and A. Singh. Modeling, simulation and validation of 14 dof full vehicle model. In *International Conference on Instrumentation, Communication, Information Technology, and Biomedical Engineering 2009*, pp. 1–6, 2009.
[13] H. B. Pacejka. *Tyre and Vehicle Dynamics*. Butterworth-Heinemann, 2nd edition, 2006a.
[14] L. A. Torres. Análise e validação em simulador automotivo de modelo dinâmico veicular para aplicação em sistema de controle de estabilidade. Trabalho de Conclusão de Curso (TCC), Universidade de Brasília, Curso de Engenharia Automotiva. Orientador: André Murilo de Almeida Pinto, 2021.
[15] A. G. Ulsoy, M. Çakmakci, and H. Peng. *Automotive Control Systems*. Cambridge university press, 2012.
[16] H. Pacejka. Using the pac2002 tire model. Accessed 2/22/7, 2006b.
[17] R. W. Schafer. What is a savitzky-golay filter? [lecture notes]. *IEEE Signal Processing Magazine*, vol. 28, n. 4, pp. 111–117, 2011.
[18] G. J. Heydinger, R. A. Bixel, W. R. Garrott, M. Pyne, J. G. Howe, and D. A. Guenther. Measured vehicle inertial parameters-nhtsa's data through november 1998. *SAE transactions*, vol. 108, n. 6, pp. 2462–2485, 1999.
[19] M. C. Walz. Trends in the static stability factor of passenger cars, light trucks, and vans. Technical Report HS-809 868, 2005.
[20] E. Edlerman, H. Agalarian, V. Balabanov, D. Roychowdhury, P. Gurfil, M. Laterza, D. Risselada, H. Shlagman, A. Warshavsky, and I. Kronhaus. Development of the orbit and attitude control hardware of the drivesat cubesat, 2018.