Analysis of a Semi-Active Suspension Model with an Attached Inerter Device

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Abstract. This study employs computational numerical analysis to investigate a semi-active suspension system featuring an integrated inerter device within a 1/4 vehicle model. The study aims to assess suspension performance and the impact of the inerter on vehicle dynamics. Semi-active suspension offers adaptable parameter adjustments based on current conditions. Incorporating an inerter in suspension, a recent focus, adds inertia without increasing mass. A quarter car vehicle model was employed for analysis, representing vertical dynamics and passenger comfort. Numerical simulations encompassed a step road profile, comparing semi-active suspension versus passive suspension. Results, including body displacement, highlighted improved performance with the inerter. Vertical displacement and accelerations were significantly reduced, enhancing comfort. This underscores the inerter's potential to enhance semi-active suspension performance effectively.

Keywords: semi-active suspension, inerter device, quarter car vehicle model, computational simulation, suspension performance.

1 Introduction

The constant development of the automotive industry has driven the search for innovative technologies that enhance the comfort and performance of vehicles. In this context, suspension systems have been the subject of intensive research to meet the growing demands of consumers for a smoother and safer driving experience. Semi-active suspension with the coupled inerter device emerges as a promising solution, combining the simplicity of passive suspensions with the efficiency of active suspensions. This innovative device can adjust the damping coefficient through different technologies, such as magnetorheological or electrorheological fluids. Additionally, coupling the inerter to the semi-active suspension offers additional benefits, such as controlling the relative acceleration between masses, thereby improving the dynamic response of the system.

According to Yao et al. [1], the semi-active suspension system combines the advantages of both active and passive suspensions, offering favorable performance compared to passive suspensions. It provides a good balance between cost-effectiveness and safety without the need for high-power actuators or a substantial power source. In essence, the semi-active suspension system stands out as a cost-effective and secure alternative that achieves commendable performance levels when compared to active and passive suspension systems.

The semi-active suspension system utilizes the damper itself as the actuator. The control process involves various types of controllers, which can be classified into two-state "On-Off" controllers, multi-state controllers, or continuously variable controllers. "On-Off" controllers are typically depicted using open-loop block diagrams and operate as a straightforward switch-on and switch-off mechanism. Closed-loop systems employ dampers with multiple stages or continuous adjustment, offering numerous stages within a specified range. The damping variation is achieved through the integration of electromagnetic valves and/or magnetorheological fluids. [2-3].

The inception of the inerter by Smith [4] brought forth an innovative alternative for vibration suppression in vehicle suspensions. This mechanical two-terminal device has the ability to generate a resistance force that is proportional to the relative acceleration between the two terminals. The proportionality constant is referred to as inertance and is measured in kilograms (kg). Notably, this specific property does not arise from its mass but rather from a range of mechanisms that can constitute it, such as pinion and rack, ball screw, hydraulic, and electromagnetic systems [5].

The objective of this study is to integrate an inerter device into a semi-active on-off type suspension using the quarter-vehicle model, assessing its influence on the vertical dynamic performance of the vehicle. The relevant mathematical formulations were implemented into MATLAB routines, producing results in the time domain. By incorporating the inerter into the semi-active suspension system, we aim to investigate how this device affects the vehicle's vertical dynamics, considering factors such as ride comfort. The MATLAB simulations will enable us to analyze and compare the system's response with and without the inerter, providing valuable insights into the potential benefits of this innovative suspension configuration.

2 Mathematical Formulation

The modeling of a vehicular suspension system integrated with an inerter follows the ideal operating principle of the device introduced by Smith [4]. He defined it as a passive mechanical element with two terminals (or nodes) that can produce a resistance force proportional to the relative acceleration between these terminals. In this scenario, the constant of proportionality is termed "inertance," and it is measured in kilograms (kg). Thus, the equation governing the dynamic behavior of an ideal inerter, as outlined by Smith [4], can be expressed as follows:

$$F_b = b(\ddot{x}_1 - \ddot{x}_2) \quad (1)$$

Where Fb represents the resistance force generated by the device, $\ddot{x}_1$ and $\ddot{x}_2$ are the accelerations at each terminal, and b corresponds to inertance. The kinetic energy associated with an inerter is therefore given by:

$$\frac{1}{2}b(\dot{x}_1 - \dot{x}_2)^2 \quad (2)$$

Where $\dot{x}_1$ and $\dot{x}_2$ denote the velocities at the two terminals. Figure 1 illustrates the free-body diagram of an inerter, as visualized by the author, accompanied by the respective values.

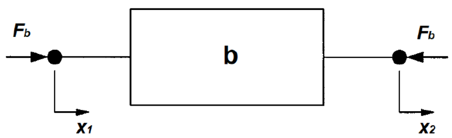

Figure 1 – Inerter free body diagram [4]

Equation (1), proposed by Smith [4], is then considered in constructing the quarter-vehicle model and its motion equations. Since it's an element attached to the suspension, in the model, the inerter is positioned between the sprung mass and the unsprung mass. However, its configuration can vary between series and parallel, depending on its arrangement in relation to the suspension spring and the damper. Smith and Wang [6] proposed the introduction of eight possible suspension layouts containing only one damper and one inerter. They highlight that these layouts are just a subset of numerous viable configurations in a model. In this work, the approach of Shen *et al*. [7] was chosen regarding the inclusion of the inerter in the model. This approach establishes an optimized configuration for an ISD (Inerter-based Suspension Damper). Figure 2 displays the aforementioned configuration, followed by its corresponding motion equations.

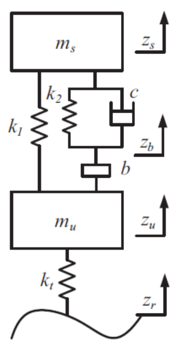

Figure 2 Quarter car model with inerter device [7]

$$m_s\,\ddot{z}_s + k_1(z_s - z_u) + \mathrm{F} = 0 \tag{3}$$
$$m_u\ddot{z_u} + k_t(z_u - z_r) - k_1(z_s - z_u) - \mathrm{F} = 0$$
$$F = b(\ddot{z}_b - \ddot{z}_u) = k_2(z_s - z_b) + c(\dot{z}_s - \dot{z}_b)$$

Where m_s represents the sprung mass, m_u is the unsprung mass, F_b denotes the resistance force produced by the inerter, b stands for the inertance of the inerter, c is the viscous damping coefficient of the damper, k_t is the equivalent tire stiffness, k_1 is the stiffness of the main suspension spring, k_2 is the stiffness of the auxiliary spring, z_s signifies the displacement of the body and chassis, z_b is the displacement of the inerter, z_u represents the displacement of the wheel-tire assembly, and z_r is the displacement relative to the road profile.

The ON/OFF Control, commonly referred to as bang-bang control, is a two-stage controller that rapidly toggles between two set limits based on data analysis. The algorithm compares the input to a target value. If the output surpasses the input, the actuator shifts to the off state (OFF). Conversely, if the output falls below the input, the actuator switches to the on state (ON). A visual representation of this controller can be found in the block diagram shown in Figure (3). This cost-effective controller finds widespread application in braking control systems, such as ABS (Anti-lock Braking System) [8].

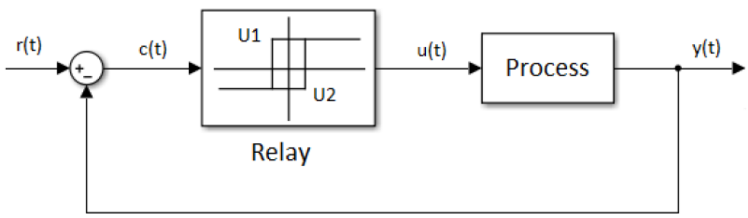

Figure 3 Block diagram of the 'On-Off' control [8]

As per Picado *et al.* [2], the two-stage semi-active suspension is notably straightforward, which is its key advantage. The damper, usually of the MR type, switches between two stages, each with higher and lower damping coefficients, depending on the suspension velocity applied. When the velocity affecting the damper aligns with the velocity of the sprung mass ms, the system employs the maximum damping stage. Conversely, in the opposite scenario, the minimum damping stage is utilized.

$$\begin{cases} c_{max}, IF\ \dot{x}_s(\dot{x}_s - \dot{x}_u) > 0 \\ c_{min}, IF\ \dot{x}_s(\dot{x}_s - \dot{x}_u) \leq 0 \end{cases} \frac{1}{2}b(\dot{x}_1 - \dot{x}_2)^2 \tag{4}$$

3 Numerical Example

Table 2 provides the characteristics of the passive ISD suspension model illustrated in Figure 2. It includes the values for parameters such as sprung and unsprung masses, tire stiffness, and auxiliary suspension spring stiffness. These parameters were chosen from the study of Costa [9]. This study introduces a parametric analysis employing the response map methodology, centered around a ¼-vehicle suspension model incorporating an inerter component. It assesses the effects of stiffness, damping, and inertance parameters on vibration amplitudes within the model's time and frequency domain response maps.

Table 1 - Properties of the ¼-vehicle model for the ISD suspension.

Properties	Values
Sprung mass m_s (kg)	350
Unsprung mass m_u (kg)	50
Tire stiffness k_t (kN/m)	200
Auxiliary spring stiffness k_2 (kN/m)	10
Main spring stiffness k_1 (kN/m)	42
Damping coefficient c (Ns/m)	944
Inertance b (kg)	220
Minimum damping coefficient $c_{\min}$ (Ns/m)	800
Maximum damping coefficient $c_{\max}$ (Ns/m)	1500

In this section, the analyzed model was subjected to a step input, $z(t)$, given by:

$$z(t) = \begin{cases} 0 \; t \leq 0 \\ 0{,}02 \; t > 0 \end{cases}$$

The expression $z(t)$ aims to simulate a hypothetical road profile with an amplitude of 0.02m (2cm).

In Figure 3, the time history displacement of the sprung mass is shown for two scenarios: a conventional passive suspension and an ISD suspension utilizing the optimized parameters presented by Costa [9], applying an ON/OFF semiactive control strategy. Notably, the ISD configuration exhibits improved performance, characterized by reduced peak amplitudes. The reduction was about 8,5% compared to the passive suspension.

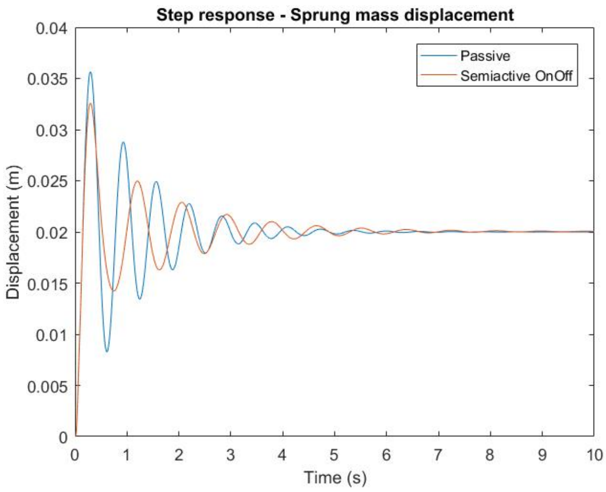

Figure 3 – Time history displacement of the sprung mass

In order to assess the impact of inertance value on the semi-active on-off suspension, an inertance value of $b = 500$ kg was selected. Numerical analyses performed revealed that employing inertance values lower than those specified in Table 1 led to a worse suspension performance. Figure 4 depicts the evolution over time of the displacement of the sprung mass for the mentioned case.

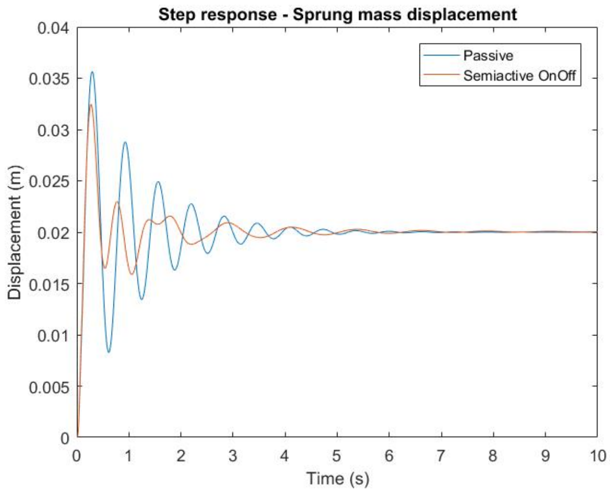

Figure 4 – Time history displacement of the sprung mass (b = 500 kg)

From the evolution of the response, it can be inferred that in addition to the reduction in peak response, it now also dampens more rapidly. Subsequent analyses demonstrated that further increasing the inertance value did not result in significant changes in the response.

4 Conclusions

This study examined the effectiveness of employing the semi-active on-off control strategy for a 1/4-vehicle model with an inerter-integrated suspension. The obtained results, under a step-type road profile, representing a broad-band excitation, demonstrated that the on-off control reduced the response peak and hastened the damping of the system. For future research, it is advisable to explore random road profiles and semi-active control strategies with continuous parameter variation.

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