Implementation of the Euler-Rodrigues formula to define the initial configuration of offshore systems lines

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Abstract. During the construction of structural models for computational analysis, various pieces of information concerning the structure, including the initial spatial configuration, are provided. This configuration represents the spatial arrangement of the structure at the beginning of the simulation. In DOOLINES, an object-oriented framework that performs nonlinear dynamic analysis of mooring lines and offshore production systems in the time domain, the initial configurations of risers, hoses, and mooring lines are generated using the catenary equation, which considers only axial stiffness, despite the presence of other stiffnesses such as bending and torsional stiffness, as observed in risers. Because the initial configuration may differ from the relaxed configuration, it can lead to the development of internal loads, even at time zero of the simulation. Therefore, the algorithm must discern the difference between the initial and relaxed configurations and compute the corresponding loads. One possible strategy, known as the "assembly" approach, involves considering the line in its relaxed position and prescribing movement at one of the supports to bring it to its actual position in the initial configuration. However, this approach requires computational effort before the actual simulation of interest. In this study, we implement the Euler-Rodrigues formula of rigid body rotation to calculate the internal forces resulting from the initial configuration, thus replacing the "assembly" approach. Through analysis of reference examples, we demonstrate that the results obtained by the two strategies are similar. Consequently, due to its lower computational requirements, the Euler-Rodrigues formula serves as a suitable replacement for the "assembly" approach.

Keywords: Euler-Rodrigues formula, DOOLINES, Offshore systems.

1 Introduction

Rigid body rotations in space, as justified by Euler's theorem, can be described by at least three real parameters. In the literature, there are several methods that characterize rotation, among which the most well-known are axis angle, Euler angles, and quaternions. The Euler angles method is the most popular, according to Butzge [1], but it has some limitations, as shown by Biasi and Gattass [2].

The axis-angle method, as suggested by its name, parameterizes the rotation of a rigid body using an arbitrary unit axis and rotation angle. This method requires four parameters: the three-directional cosines of the rotation axis and angle. The Euler-Rodrigues formula was obtained from the formulation of this method and is widely used for the rotation of rigid bodies. Compared to the Euler angles method, its formulation is more complex and less intuitive; however, it does not have the same limitations.

According to Valdenebro [3], the Euler-Rodrigues formula is efficient and accurate from a computational perspective. It finds applications in various fields, such as robotics, engineering, physics, and computer graphics. S. Chen et al. [4] utilized the Euler-Rodrigues formula to estimate the position of cameras, which is crucial for robotic systems, including navigation, tracking, and camera calibration. Furthermore, Menin [5] demonstrated its application in obtaining a rotation matrix and pseudovector for the study of large rotations of spatial portals and triangular flat shells.

2 Euler-Rodrigues Formula

The principle of the Euler-Rodrigues formula can be better understood through Figure 1, in which a vector $\boldsymbol{v}$ in $\mathbb{R}^3$ is rotated at an angle θ around an arbitrary axis $\boldsymbol{e}$. This rotation generates the vector $\boldsymbol{v_r}$, and due to rigid body rotation, we have that $\|\boldsymbol{v}\| = \|\boldsymbol{v_r}\|$.

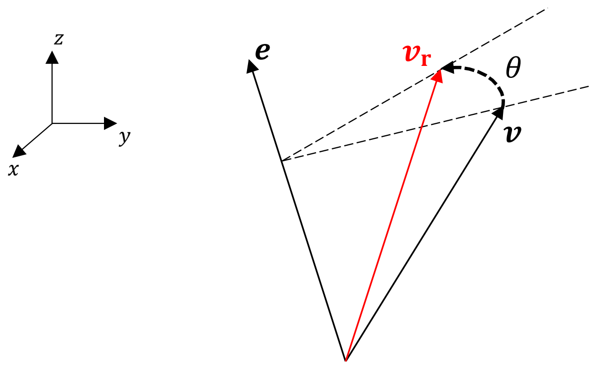

Figure 1: Rigid body rotation in space

We can then write the relationship between these two vectors as

$$\boldsymbol{v_r} = \boldsymbol{R_\theta} \boldsymbol{v} \tag{1}$$

where the matrix $\boldsymbol{R_\theta}$ is defined as the rotation matrix, obtained by

$$\boldsymbol{R_\theta} = \boldsymbol{I} + sin(\theta)\, \boldsymbol{S_e} + (1 - cos(\theta))[\boldsymbol{S_e}]^2. \tag{2}$$

Equation (2) is known as the Euler-Rodrigues formula, where the matrix $\boldsymbol{S_e}$ is an antisymmetric matrix determined by

$$\boldsymbol{S_e} = \begin{bmatrix} 0 & -c_z & c_y \\ c_z & 0 & -c_x \\ -c_y & c_x & 0 \end{bmatrix} \tag{3}$$

where c_x ,c_y, c_z are the direction cosines of the rotation axis $\boldsymbol{e}$. If we calculate the trace of the matrix $\boldsymbol{R_\theta}$, we can obtain the following result:

$$tr(\boldsymbol{R_\theta}) = r_{11} + r_{22} + r_{33}. \tag{4}$$

where r_{ii} are the elements of the main diagonal of $\boldsymbol{R_\theta}$. By finding these elements from eq. (2), we have that:

$$tr(\boldsymbol{R_\theta}) = (1 - cos(\theta))\left({c_x}^2 + {c_y}^2 + {c_z}^2\right) + 3\, cos(\theta). \tag{5}$$

Once $\left({c_x}^2 + {c_y}^2 + {c_z}^2\right) = 1$, isolating θ in eq. (5) we obtain:

$$\theta = cos^{-1}\left(\frac{tr(\boldsymbol{R_\theta}) - 1}{2}\right). \tag{6}$$

Furthermore, it is possible to show that:

$$\boldsymbol{e} = \frac{1}{2\sin(\theta)} \begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix} \tag{7}$$

With eqs. (6) and (7) it is then possible to obtain the rotation angle θ and the axis $\boldsymbol{e}$ given the rotation matrix $\boldsymbol{R_\theta}$.

3 Methodology

The Euler-Rodrigues formula is employed in the DOOLINES (Dynamics Of Offshore LINES) framework, an object-oriented platform for nonlinear dynamic analysis of mooring lines and offshore production systems in the time domain (SILVEIRA et al. [6]). This formula is used to rotate the nodal axes at the initial moment of the simulation, where the rotation axis corresponds to the binormal vector of the elements, and the rotation angle is determined from the curvature between the elements at the node.

In this study, a computational model will be simulated, implementing the Euler-Rodrigues formula, both with and without using the "assembly" approach, in the beam corotational (BeamCR) and spatial truss (BTruss) elements present within the DOOLINES framework. Subsequently, the obtained results will be compared to validate the implementation of the Euler-Rodrigues formula, aiming to consider the necessary adjustments in the internal efforts of the elements when modeled with an initial configuration different from the relaxed configuration, as an alternative to the "assembly" strategy utilized thus far.

4 Results

The simulated model is a flat beam belonging to the xz plane, supported by two second-genre restraints at its ends, preventing translations in the x and z directions. The beam is 9 meters long with a span of 6 meters, axial stiffness (EA) of 22,500 kN, and flexural stiffness (EI) of 15 kN·m². It has a mass of 0.1265 t/m. The beam was discretized into 30 elements, and the static equilibrium configuration is determined using the explicit integration method of dynamic relaxation, with a total simulation time of 120 s.

In the captions of the graphs represented in the figures of this section, the suffixes **_A** and **_ER** are present, indicating the use of the “assembly” technique and the Euler-Rodrigues formula, respectively. The absence of either suffix indicates that neither techniques is used.

Figure 2 shows the curves of axial force variation along the beam without using Euler-Rodrigues. It can be observed that only the curve of the beam element without “assembly” deviates significantly from the others, indicating that the non-use of “assembly” in the beam element leads to different results for the forces, as expected.

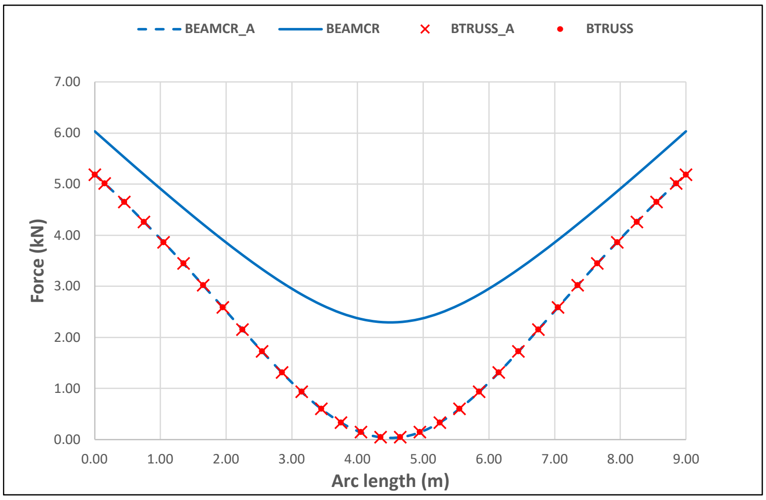

Figure 2. Comparison of the distribution of forces along the element, without using the Euler-Rodrigues formula

The assembly of the truss model does not exhibit significant differences in the results. This behavior is expected since, for this element, the undeformed configuration has a straight axis. For any deformed configuration passed to the BTruss, the bending forces are calculated based on the deviation between two elements.

Figure 3 shows the distribution of axial forces along the beam, comparing both alternatives. It can be observed that the use of the Euler-Rodrigues formula yielded a result close to that obtained with the "assembly". This approximation is confirmed in Figure 4, which displays the curve of the percentage error between the axial forces along the beam. It is noticed that the error between the two methodologies reaches a maximum of 3.8%.

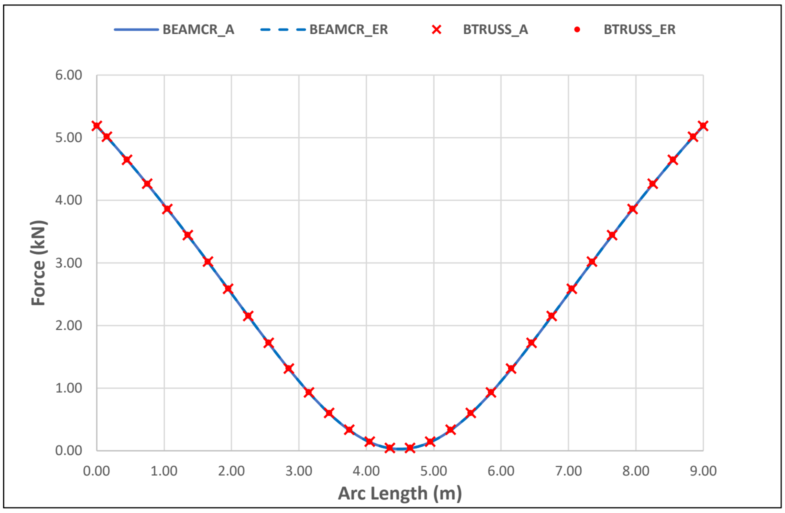

Figure 3 .Comparison of the distribution of forces along the element, without using the Euler-Rodrigues formula

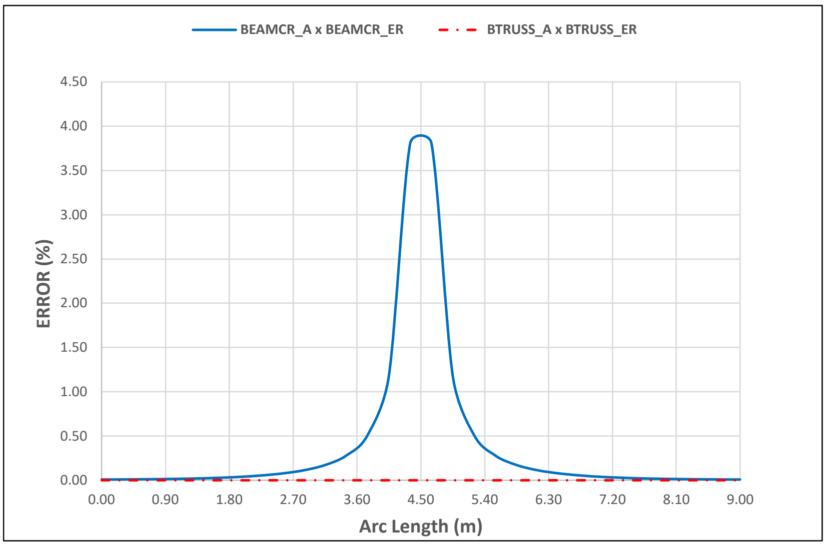

Figure 4. Percentage error between the results of forces along the beam using "assembly" and Euler-Rodrigues formula

On the other hand, when comparing the results obtained by either of the two methodologies with not using any ("assembly" or Euler-Rodrigues formula), this error can exceed 5,400% (see Figure 5), highlighting the importance of calculating the forces resulting from the initial position.

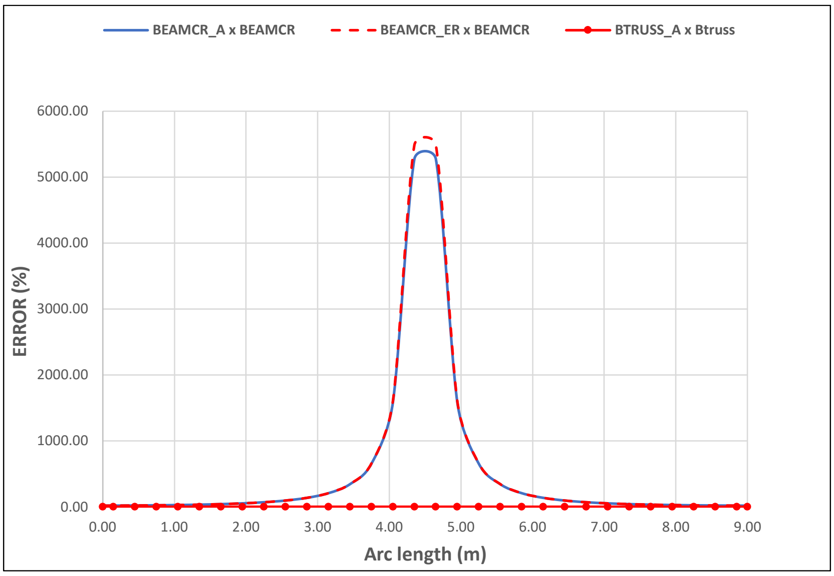

Figure 5. Percentage error between using any alternative (“assembly” or Euler-Rodrigues formula) and not using them

5 Conclusions

As observed, not using these methodologies (“assembly” or Euler-Rodrigues) in the beam corotational element (BeamCR) results in significantly different outcomes. Therefore, it becomes evident that disregarding these initial forces generated due to the initial position can lead to a considerable error in the final result, potentially compromising the analysis.

Furthermore, it was noticed that the results obtained with “assembly” are very close to those obtained with the Euler-Rodrigues formula, allowing this support prescription strategy to be replaced by the formula in the DOOLINES algorithm. This eliminates the need to allocate simulation time for arranging the element in the desired configuration, thus reducing computational costs.

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