A smooth boundary extraction technique for topology optimization with binary design variables and a geometry trimming procedure

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Abstract. One important step for topology optimization methods is to obtain representative smooth boundaries of the design. This is a step mostly used as a post-processing tool to facilitate manufacturing. However, smooth boundaries become also relevant during optimization in problems where the information at the boundaries is crucial, e.g., in stress-based design or fluid-structure interaction problems. This work investigates a boundary smoothing procedure to be used at every iteration of the topology optimization procedure. The present technique is proposed for binary-based topology optimization methods, such as the BESO (Bi-directional Evolutionary Structural Optimization) or the TOBS (Topology Optimization of Binary Structures) algorithms. In this context, a nodal numerical filter is applied to the binary design, and a smooth level set boundary is extracted from the design. The idea is further discussed in the context of the TOBS-GT (TOBS with geometry trimming) method, where the optimization and finite element meshes are separated. In this method, the void regions are trimmed out of the analysis domain and a new finite element mesh is generated at every optimization step. In this procedure, smooth boundaries are fundamental to guarantee a reasonably cheap and convergent finite element mesh. The smoothing procedure has the potential to improve the performance of the resulting structural surface as well as its appearance. 3D Numerical examples are investigated to evaluate the smoother robustness.

Keywords: Topology optimization, Integer Linear Programming, Body-fitted mesh, Three-dimensional analysis

1 Introduction

Over the last few decades, topology optimization (TO) has gained ground in projects, replacing optimization tools such as shape or size. This growth is due to the versatility of the method for producing resistant parts and saving material. From this, TO has increasing application in several specialized and/or large production engineering fields, such as aerospace and automotive. Topology optimization methods can be classified into density methods such as Solid Isotropic Material with Penalization (SIMP) [1], binary methods such as Bi-directional Evolutionary Structural Optimization (BESO) [2] and Topology Optimization of Binary Structures (TOBS) [3], and boundary variation methods such as Level-set [4].

Despite the great development of TO over the years and the wide variety of methods, it is still necessary to advance from the point of view of the surface quality of the manufacturing solution. Generally, the topologies provided by the TO technique present irregularities on the surface in the form of a zig-zag or discontinuities [5], which require post-processing before fabrication. In order to solve this issue, several works in literature employed surface smoothing methods to produce ready-made solutions or closer to manufacturing conditions. Another point is the development of a body-fitted mesh technique combined with topology optimization methods, which has great potential for enabling integration with external mesh generation software.

As mentioned before, many works have proposed adopting smoothing strategies and body-fitted mesh into the topology optimization for various physical problems. Feppon et al. [6] implemented a framework using body-fitted mesh for shape and topology optimization of three-dimensional weakly-coupled fluid-thermal-mechanical systems. Kuci et al. [7] introduced a framework for topology optimization in electro-mechanical design problems. The authors used a level set function on a fixed mesh to define the design domain and generate a body-fitted mesh. Li et al. [8] proposed a new framework for the two- and three-dimensional topology optimization of the weakly-coupled fluid–structure system using the reaction–diffusion equation (RDE) for updating the level-set function. The framework presents two key ingredients: The body-fitted mesh approach and integration between FreeFEM as Finite Element Analysis (FEA) solver and PETSc for distributed linear algebra. Zhuang et al. [5] created a framework using a body-fitted triangular/tetrahedral mesh generation algorithm to yield smooth boundaries in the BESO method. The optimization problem is regularized by adding a diffusion term in the objective function.

Based on that, this work proposes a smoothing technique to be combined with the TOBS-GT method to solve 3D static structural problems. For this, it will be employed as an optimization method the TOBS algorithm implemented in MATLAB and COMSOL Multiphysics as external FEA solver. The compliance sensitivities are obtained via semi-automatic differentiation in COMSOL Multiphysics. The smoothing technique coupled with the TOBS-GT method is tested in the 3D cantilever beam and 3D bridge. The results show that the smoothing strategy provides clear and smooth boundaries.

2 Solid Mechanics

Consider a structural domain Ω_s governed by the linear elasticity neglecting body forces and any acceleration, the governing equation for static structural analysis can be written as:

$$\nabla \cdot \sigma_s(\mathbf{u}_s) = \mathbf{f}_s \qquad \text{on } \Omega_s, \tag{1}$$

where $\sigma_s(\mathbf{u}_s)$ is the Cauchy stress tensor, $\mathbf{u}_s$ is the structural displacement field, and $\mathbf{f}_s$ denotes the vector with the loads applied on the structure. Dirichlet boundary conditions are applied on the boundary Γ_d of the structure as:

$$\mathbf{u}_s = \mathbf{u}_0 \qquad \text{on } \Gamma_d, \tag{2}$$

where $\mathbf{u}_0$ is the vector of constrained displacements. Neumann boundary conditions are applied by assembling $\mathbf{f}_p$ on the boundary portion Γ_p of the structure.

3 Topology Optimization Framework

3.1 Topology Optimization of Binary Structures

The TOBS method, proposed by Sivapuram and Picelli [3], employs binary design variables $\{0,1\}$. This methodology linearizes the objective and constraint functions associated with integer linear programming [9]. The method uses Taylor's series expansion and truncates it in the linear part to express the approximate structural mean compliance objective and volume constraint functions. Therefore, the approximate integer linear subproblem to be solved is given by:

$$\begin{aligned}
&\underset{\Delta x^k}{\text{Minimize}}\ \frac{\partial C(x^k)}{\partial x} \cdot \Delta x^k, \\
&\text{Subject to}\ \frac{\partial V_i(x^k)}{\partial x} \cdot \Delta x^k \leq \overline{V}_i - V_i(x^k) := \Delta V_i^k,\ i \in [1, N_g], \\
&\qquad\qquad ||\Delta x^k||_1 \leq \beta N_d, \\
&\qquad\qquad \Delta x_j^k \in \{-x_j^k, 1 - x_j^k\},\ j \in [1, N_d].
\end{aligned} \tag{3}$$

where $C(x)$ is the objective function (mean compliance), bounded by the constraints $V_i(x) \leq \bar{V}_i$ (volume). $i \in [1, N_g]$, where N_g and N_d are respectively the numbers of inequality constraints and elements in the vector of design variables. β is the truncation error parameter and $||\Delta x^k||_1$ is the truncation error. The term $V_i(x^k)$ is the value of the constraint g_i in the k_{th} optimization iteration. The ILP solver is used to find the optimal change Δx for the integer design variables x. After each iteration, the design variables are updated as $x_{k+1} = x_k + \Delta x_k$. The formulation of the binary optimization problem from Eq. (3) is related to compliance minimization of the structure subject to a given volume constraint. The optimization problem is expressed as:

$$\begin{aligned} \underset{x}{\text{Minimize}} \quad & C(x) \\ \text{Subject to} \quad & V_i(x) \leq \overline{V}_i, \quad i \in [1, N_g] \\ & x_j \in [0,1], \quad j \in [1, N_d] \end{aligned} \tag{4}$$

3.2 Sensitivity Analysis

The TOBS is a gradient-based optimization method, hence the gradients (sensitivities) of the objective and constraint functions are required. A general way of calculating the sensitivities of a **L** function is using the adjoint method (Haftka and Gürdal [10], Bendsoe and Sigmund [11]). The general formulation of the adjoint equation for a Lagrangian functional can be given by

$$\left(\frac{\partial \mathbf{R}}{\partial u}\right)^T \lambda = -\left(\frac{\partial f}{\partial u}\right)^T, \tag{5}$$

where λ corresponds to the vector of adjoint variables, u represents all state variables, f is the vector of objective function and $\mathbf{R}$ is the residual. Sensitivities can then be calculated by the following expression

$$\left(\frac{dL}{d\mathbf{x}}\right) = \left(\frac{\partial f}{\partial \mathbf{x}}\right)^T + \lambda^T \frac{\partial \mathbf{R}}{\partial \mathbf{x}}. \tag{6}$$

The structural mean compliance sensitivities are then calculated by the generic function, Eq. (6). The structural volume sensitivities with respect to the design variable x_j are expressed as

$$\frac{\partial V}{\partial x_j} = V(x), \tag{7}$$

where $V(x)$ is the volume fraction referring to the design variable x_j.

In order to evaluate the derivatives of the structural compliance using the adjoint method via Eq. (6), the physical model should be interpolated with the design variables. For this, the classical SIMP material model is adopted which is expressed as:

$$E(x_j) = x_j^p E_0 \qquad \text{on} \ \ \Omega_s, \tag{8}$$

where E is the interpolated material property with respect to the design variable x_j, E_0 is Young's modulus of the solid element and p is the penalty exponent factor.

4 Numerical Implementation

The Topology Optimization of Binary Structures with Geometry Trimming (TOBS-GT) is a method proposed by Picelli et al. [12], which separates the optimization and FEA meshes using binary design variables. The optimization is carried out in MATLAB® via the TOBS implementation available at Picelli et al. [13]. The static structural analysis is solved using COMSOL Multiphysics®. In addition, this work proposes a smoothing technique to be applied at all optimization iterations. Based on Fig. 1, the smoothing contour procedure is to apply a nodal filter, to the distribution of design variables $\{0, 1\}$, which becomes a surface similar to a level-set function that represents the filtered design variables. After that, the smooth contour can be extracted by setting a value in the plane for the design variables ($x_j = 0.5$). In this work, the MATLAB function `contourf` was used in order to extract the contour points of the smooth contour.

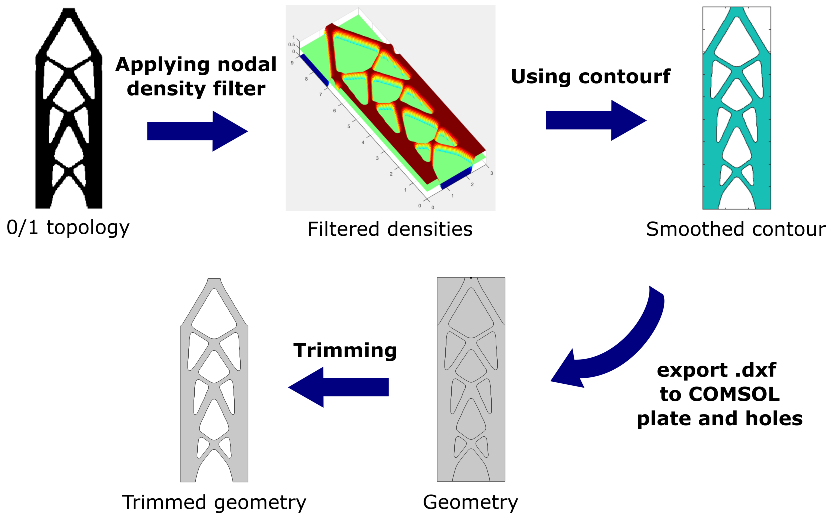

Figure 1. Illustration of the smoothing technique based on the level-set procedure.

4.1 Optimization Algorithm

A summary of the main steps for the TOBS-GT approach is presented below:

1. Define the TOBS parameters;
2. Create optimization grid and initialize design variables $\{0, 1\}$ in the TOBS module;
3. Generate a CAD geometry from contour and holes by reading the optimization grid variables;
4. Trim geometry with holes and create the solid topology in CAD;
5. Define the static structural problem and mesh the geometry created by the CAD model;
6. Solve the static structural governing equations;
7. Compute sensitivities through automatic differentiation in the grid points;
8. Extract the calculated sensitivities in a grid coincident with the optimization grid;
9. Filter the sensitivity field defined in the grid points;
10. Solve the ILP problem and update the design variables $\{0, 1\}$ in the optimization grid;
11. Update design variables to build a new $\{0, 1\}$ topology;
12. If converged, stop. If not, iterate from step 3.

5 Numerical Examples

This section presents the results obtained for 3D static structural problems. Two examples were explored: the cantilever beam and the bridge. In all cases, the optimization problem is compliance minimization subjected to a volume constraint, Eq. 4. A filter radius of 3 grid sizes is adopted. Material models are interpolated considering the penalization factor $p = 3$. The constraint relaxation parameter is $\varepsilon = 0.02$, and the truncation parameter is $\beta = 0.05$. In all the examples, the convergence is defined by averaging the changes in the compliance function over 6 consecutive iterations for a tolerance of $\tau = 0.001$. The numerical examples are performed in a computer with Intel(R) Core(TM) i9-12900KF CPU @ 3.20 GHz. The GPU is an NVIDIA GeForce GTX 710.

5.1 Cantilever beam

The design domain of the cantilever beam is illustrated in Fig. 2. The properties of the structure are assumed as Young's modulus of $E = 210$ GPa, Poisson's ratio of $\nu = 0.3$, and density of $\rho = 7850$ kg/m^3. A external load of $F = 1 \cdot 10^4$ N/m is considered. Due to the symmetry, only half of the design domain is analyzed using a mesh of $80\times40\times40$ regular hexahedral elements.

Figure 3 present the topology optimization results for the volume fraction of 30 %. As can be seen in Fig. 3(a) the surface of the 3D optimized solution is clear and smooth using the proposed method with body-fitted tetrahedral mesh produced in COMSOL Multiphysics. The number of tetrahedral elements in the optimized solution is 73,882

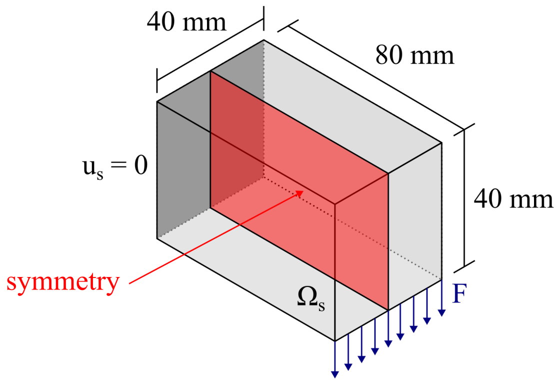

Figure 2. Design domain with dimensions and boundary conditions for the cantilever beam.

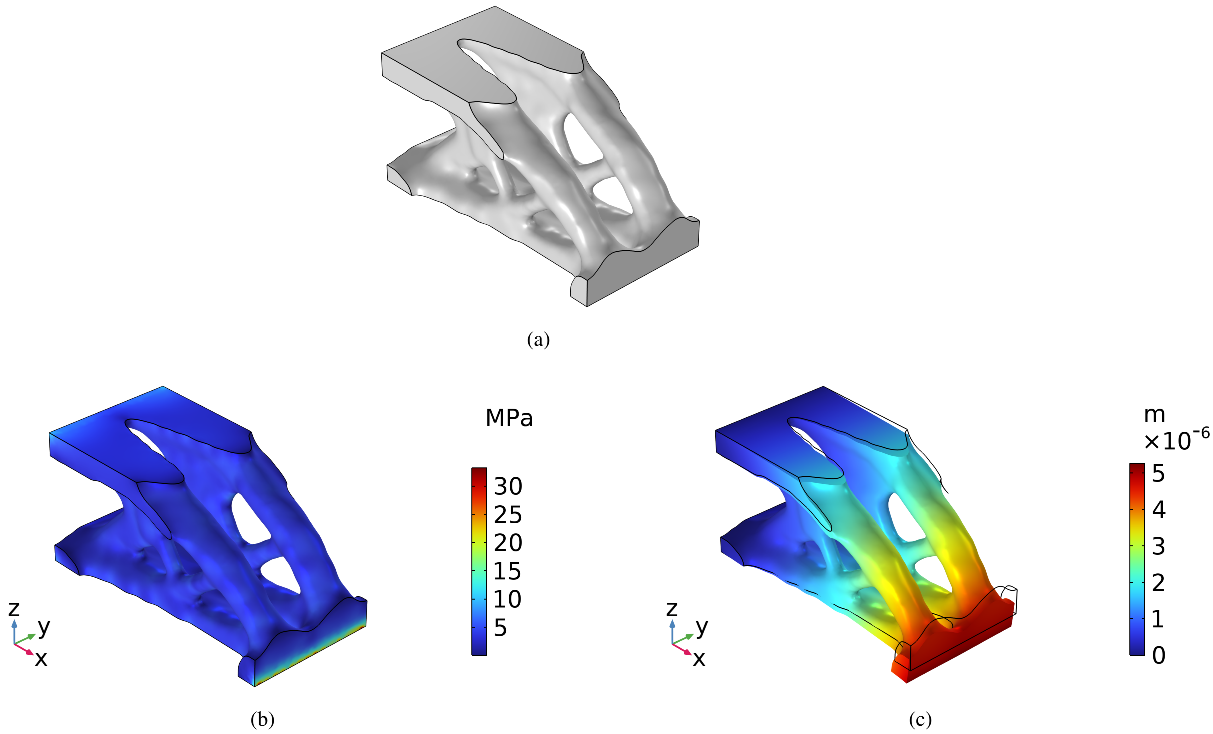

Figure 3. Topology optimization results for the cantilever beam example: (a) Final topology; (b) von Mises stress; (c) Displacement field; and (d) Compliance and volume history.

for the whole design domain. The number of elements is even lower than the number of elements in the optimization grid. The optimization is converged in 85 iterations with its objective function value as 9.8331×10^{-4} N·m. The main advantage of the proposed technique is saving computational costs in mesh generation. In the body-fitted approach, the mesh generated for the jagged geometry is higher than the mesh generated for the smoothed contour. Figures 3(b) and 3(c) present the finite element solution for displacement and von Mises stress field.

5.2 The Bridge

The design domain of the bridge is illustrated in Fig. 4. The properties of the structure are assumed as Young's modulus of E = 70 GPa, Poisson's ratio of ν = 0.33, and density of ρ = 2700 kg/m^3. A external load of $F = 1 \cdot 10^3$ N/m is considered. Due to the symmetry, only half of the design domain is analyzed using a mesh of $100 \times 20 \times 60$ regular hexahedral elements.

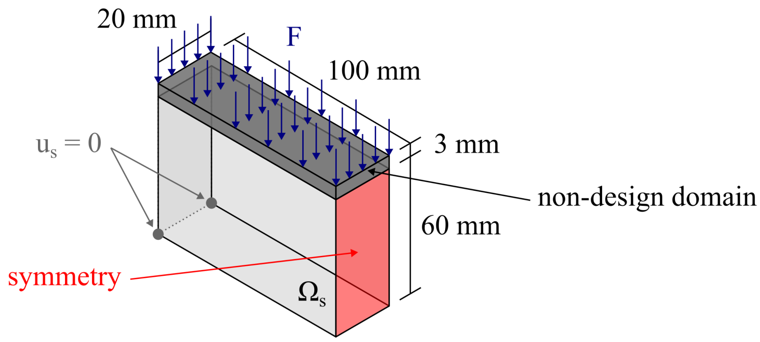

Figure 4. Design domain with dimensions and boundary conditions for the bridge.

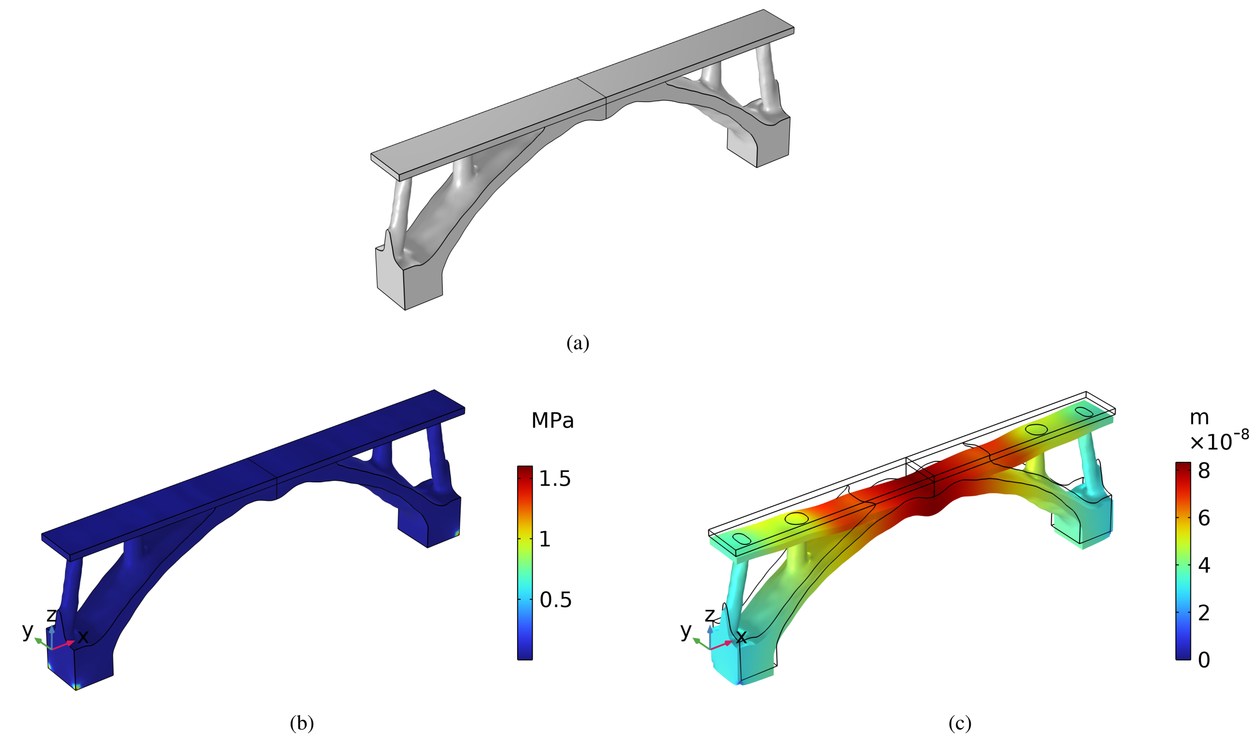

Figure 5. Topology optimization results for the bridge example: (a) Final topology; (b) von Mises stress; (c) Displacement field; and (d) Compliance and volume history.

Figure 5 present the topology optimization results for the volume fraction of 30 %. The final topology shown in Fig. 5(a) demonstrates that the method works well for dealing with geometries containing a non-design domain. In this case, the number of tetrahedral elements in the optimized solution is 26,246 for the whole design domain. The optimization is converged in 45 iterations with its objective function value as 5.9131×10^{-8} N·m. Figures 5(b) and 5(c) present the finite element solution for displacement and von Mises stress field.

6 Conclusions

A smoothing technique is proposed to provide smoothed topologies in all iterations of the optimization process. Two 3D static structural examples were explored: the cantilever beam and the bridge. The optimization problem was compliance minimization subjected to a volume. The TOBS-GT method was employed in order to obtain body-fitted meshes and to use FEA external solver. The results ensure that the smoothing technique works efficiently providing clear and smooth boundaries.

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