Improving the vibration control performance of metamaterial structures by the inclusion of nonlinear local resonators

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Abstract. The use of periodicity has become an exciting solution for structural noise and vibration reduction in many engineering applications. Acoustic metamaterials are structures built using repetitive assemblies of identical elements to explore either Bragg-scattering or internal resonance to control mechanical waves. They present frequency bands in which waves do not freely propagate, named bandgaps, allowing acoustic and vibration attenuation at various frequency ranges. If properly designed and implemented, nonlinear stiffness can result in resonance frequency shifts that broaden the attenuation frequency band and better isolate subsystems that could be sensitive to higher excitation levels. This work investigates the effects of geometrically nonlinear local resonators on the low-frequency bandgap formation of a metamaterial beam. Attention is paid to the realization of lightweight nonlinear resonators via additive manufacturing of compliant mechanisms as the nonlinear resonant unit. The proposed model is validated through simulations and experimental analysis. This investigation contributes to understanding improvements provided by nonlinear elements for vibration control of a metastructure.

Keywords: Metamaterial, Bandgap, Transmissibility, Local Resonator, Nonlinearity

1 Introduction

Vibration and noise have become major concerns for engineering applications Song et al. [1]. Besides unusual exceptions, such as musical instruments and vibrating conveyors, in which function depends on vibration, it is an unwanted byproduct that can cause occupational hazards, discomfort, and fatigue, reducing the efficiency or even causing health issues and accidents. From machine parts to large structures, continuous exposure to vibration can lead to excessive wear of bearings, the formation of cracks, the loosening of fasteners, the development of cracks, and mechanical failure. These situations increase the need for frequent maintenance, causing the operational cost to be increased Mead [2].

Most recently, vibration reduction has been achieved by using the concept of periodicity, first related to the electromagnetic field Shamonina and Solymar [3]. Structures built under this framework are called periodic structures or metamaterials and involve the repetitive assembling of unit cell elements Mead [4]. This idea has already been applied, for example, in aircraft panels, railway tracks and truss structures Narisetti [5]. Depending on the design of the unit cell, periodicity can be used to achieve frequency regions in which waves are highly attenuated, granting periodic structures an effect similar to band rejection filtering Brillouin [6], often referred to as *bandgaps*.

In practice, the bandgaps provided by periodic structures allow them to serve as filters, resonators, or waveguides. For linear periodic structures, the bandgap regions depend mostly on geometry, inclusions, and elastic properties and are likely to occur at frequencies higher than those often encountered in structural vibration problems. In the low-frequency range, internal resonant metamaterials are more useful but bury a relationship between added mass and effectiveness which is detrimental for mass-critical applications.

More recently, nonlinear metamaterials have attracted increasing attention due to their unusual properties, *e.g.*, amplitude-dependent bandgaps, which can be used to overcome the bandwidth limitation dependency on the mass ration of the resonator Yu et al. [7]. Nonlinear periodic structures may exhibit additional exciting properties, which enable enhanced wave transmission properties Wang et al. [8], without the necessity of adding extra mass, which increases the total mass of the structure.

In this work, attention is paid to investigating the effects of local resonators (LR) on the bandgaps formation to enhance the wave propagation properties of a metamaterial beam. Numerical and experimental validations are performed, from the single-cell to the full-system level, contributing to understanding improvements provided by nonlinear elements for vibration control of a periodic structure.

2 Problem statement

The schematic diagram of the metamaterial structure considered in this work is presented in Figure 1. The complete structure is built upon n repeated elements named unit-cell, and highlighted by the elements in the blue rectangle. The metamaterial consists of a homogeneous steel beam with repeated local resonators attached, where E_b, ρ_b, and $L = 2l$ are the Young's modulus, density and length of the beam unit cell. The equivalent stiffness and mass of the resonator are represented by k_{eq} and m, respectively. The complex displacement amplitudes at each node point and the external applied force are represented as W, θ and F_L, where the subscript L and R indicates the left and right end of the beam. The total length of the beam with n unit cells is $L_T = nL$.

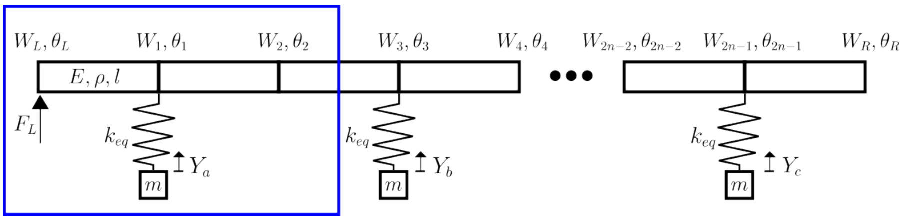


Figure 1. Schematic diagram of a metamaterial beam attached to local resonators. The unit cell is defined under the blue rectangle area

2.1 Dynamic stiffness matrix equation

According to Fahy and Walker [9], by considering the proper boundary conditions along with the general solution of the dynamic equilibrium equation, the dynamic stiffness matrix (DSM) of the unit-cell assembly involving two beam elements and one mass-spring subsystem representing the LR can be built considering a finite element procedure Zienkiewicz and Taylor [10], i.e.,

$$\begin{Bmatrix} F_L \\ M_L \\ F_A \\ M_A \\ F_{Ya} \\ F_R \\ M_R \end{Bmatrix} = H \begin{bmatrix} -K_{11} & -P & K_{12} & V & 0 & 0 & 0 \\ -P & Q_{11} & -V & Q_{12} & 0 & 0 & 0 \\ K_{12} & -V & -2K_{11} + \frac{DL}{H} & 0 & \frac{-D_L}{H} & K_{12} & V \\ V & Q_{12} & 0 & 2Q_{11} & 0 & -V & Q_{12} \\ 0 & 0 & \frac{-D_L}{H} & 0 & \frac{(D_L+D_M)}{H} & 0 & 0 \\ 0 & 0 & K_{12} & -V & 0 & -K_{11} & P \\ 0 & 0 & V & Q_{12} & P & 0 & Q_{11} \end{bmatrix} \begin{Bmatrix} W_L \\ \theta_L \\ W_A \\ \theta_A \\ Y_A \\ W_R \\ \theta_R \end{Bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ +FNL_a \\ -FNL_a \\ 0 \\ 0 \end{bmatrix} \quad (1)$$

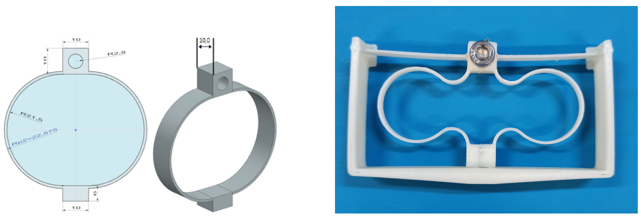
where, D_M, D_L and FNL_a are the dynamic stiffness of the mass, the dynamic stiffness of the linear term the resonator spring force, and the nonlinear force as defined in Santo et al. [11].

2.2 Linear LR design

Here, the frequency of interest is between 80 Hz and 90 Hz, where the 5^{th} bending natural frequency of the hoist beam is located. Also, the design goal set for the LR metamaterial solution is to limit to around 23% the mass of the hoist structure, which corresponds to 87g of the total added mass. The resonant units are built using a ring-shape design and are fabricated in ABS by the FDM additive manufacturing technology using a 3D printer, and its design is highlighted in Fig. 2(a).

2.3 Nonlinear LR design

By leveraging nonlinear elements to provide frequency band broadening or more efficient energy absorption, a geometrically nonlinear design is proposed. and for the resonant elements. Such nonlinear systems can provide resonant frequency shifts, which could result in better isolation and more efficient attenuation of the host structure [12, 13]. In this work, a new LR design based on the quasi-zero stiffness (QZT) devices presented in Gatti et al. [13], Shaw et al. [14] will be considered as the LR initial model. Furthermore, to reduce the effect of hysteresis caused by the friction of the joints and hence increase the performance, the nonlinear device is devised as a compliant mechanism Kochmann and Bertoldi [15], Ling et al. [16], Deng et al. [17], Kuppens et al. [18], which reduce the part count and manufacturing price. In addition, the use of compliant mechanisms can enhance the effects of the nonlinear incursion, in particular, due to the significant reduction in weight over their rigid-body counterparts. The nonlinear LR device inspired in Fan et al. [19], which is composed of three principal components fabricated in ABS using FDM additive manufacturing technology: (a) the internal spring made of a couple of semicircular arches and a horizontal beam; (b) external stiffer walls, where the lateral beams are pre-tensioned, and (c) added mass. Since the horizontal beam will exhibit negative stiffness under lateral load generated by the external pre-tensioned lateral walls, and the internal semicircular arches springs will exhibit positive stiffness under vertical load, the quasi-zero stiffness property can be achieved by rearranging these elements in a proper manner. The full assembly of the LR and the professional 3D printer used to fabricate the resonators is shown in Fig. 2(b).



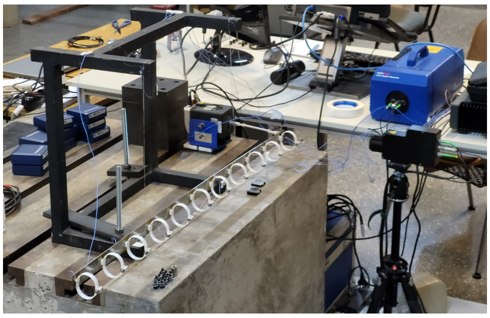
(a) (b)

Figure 2. The conceptual design of: (a) linear ring-shape LR, (b) geometrically nonlinear LR

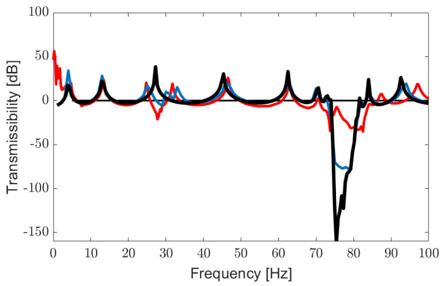
3 Linear and nonlinear metamaterial solution

In the first case, the linear locally resonant metamaterial is composed of 11 ring-shaped resonators, made of ABS, periodically distributed along the host steel beam, as shown in Fig. 3(a). A FE model of the complete metamaterial structure was built utilizing the commercial FE software NX Nastran Siemens [20]. The FE model of the metamaterial beam is utilized to predict their numerical resonance frequencies. From the unit-cell point of view, the design of the upper part, added mass and the flexible part of the numerical model consists of 100, 476, and 1864 linear CHEXA8 solid elements, respectively. The base of the resonator consists of 270 TETRA10 elements. The host beam is built as 223 Thin Shell CQUAD8 elements. The unit cell is repeated 11 times to complete the whole structure. Table 1 shows the parameters used for the analytical and numerical validations. The comparison of the experimental validation and the results by solving the DSM equation and the numerical solution is shown in Fig. 3(b).

For the second case, the experimental assembly used in the investigation of the effects on the bandgap of a metamaterial beam when nonlinear LR is considered is presented in Figure 4(a). In this case, an array of 11 equally spaced nonlinear LRs is attached to the hoist beam. The total suspended mass is considered as an array composed of 11 internal springs and added masses with 17.985g and 4.25g, respectively. In the experimental results shown in Fig. 4(b), it is possible to observe the appearance of two bandgaps around 27 Hz to 32 Hz and 70 Hz to 80



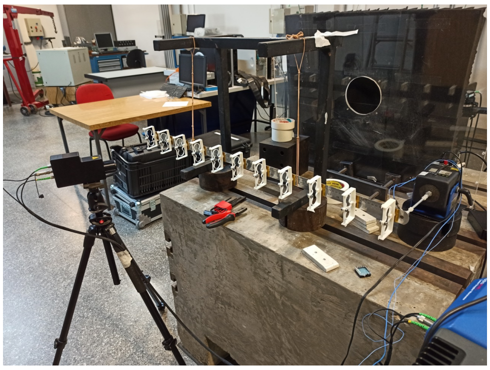
(a)



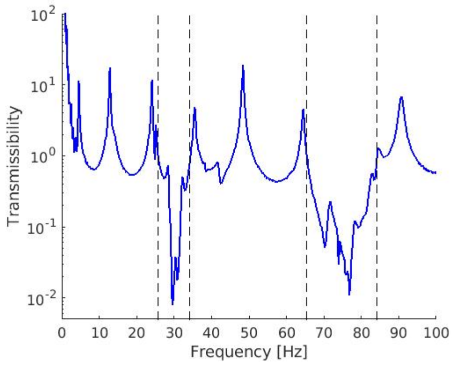
(b)

Figure 3. Linear metamaterial structure: (a) Experimental assembly, and (b) the magnitude of the Transmissibility in dB: experimental (red), fem (blue), DSM equation (black)

Hz. For this case, the suspended mass represents roughly 19% of the metastructure total mass, which represents a reduction of 4% in mass when compared to the ring-shaped linear resonators.



(a)



(b)

Figure 4. Metamaterial structure: (a) Experimental assembly, (b) metamaterial structure FE model

Table 1. Parameters used for the analytical and numerical validations

Constitutive relation	Nomenclature	Value
Half of beam unit-cell length	l	0.047 m
Beam total length	L_T	1.05 m
Beam cross-section area	A	4.6 x 10^{-5} m^2
Beam Young's modulus	E_B	2.1693 x 10^{11} N/m^2
Beam density	ρ_B	7860 kg/m^3
ABS Resonator Young's modulus	E_R	2 x 10^9 N/m^2
ABS Resonator density	ρ_R	0.8
Added mass	m	5.03 g

4 Conclusions

This work proposes the use of geometrically nonlinear local resonators as a solution to improve the effects on the the low-frequency bandgap of metamaterial structures. Attention was paid to the realization of lightweight linear and nonlinear resonant units via additive manufacturing. In summary, the results shown that the nonlinear resonators improves the bandgaps of the metastructures with the use of less added mass. It is expected that this investigation contributes to understanding the role of nonlinear elements in the context of periodic structures. Further investigation should be carried out, considering numerical and experimental analysis, in order to enhance the wave attenuation properties provided by the LR. A more precise prototype is being developed using an injection moulding process fabrication, aiming at the reduction of the variability of the LRs and to enlarge the quasi-zero stiffness region, which should increase the attenuation frequency band.

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