Automated Approach For Multi-objective Optimization Of Steel Trusses Using Genetic Algorithms and Reliability

Marcio Maciel da Silva[1], Francisco de Assis das Neves[1], Hélio Luiz Simonetti[2], Marcilio Sousa da Rocha Freitas[1]

[1] *Graduate program in Civil Engineering, Federal University of Ouro Preto, Department of Civil Engineering Escola de Minas, Campus Universitário Morro do Cruzeiro, 35400-000, Ouro Preto, Minas Gerais, Brasil.*
marcio.maciel@aluno.ufop.edu.br, fassis@ufop.edu.br, marcilio@ufop.edu.br
[2] *Dept. of Mathematics, Federal Institute of Minas Gerais (IFMG)*
Rua Itaguaçu, 595, 32677-562, Minas Gerais/Betim, Brasil
helio.simonetti@ifmg.edu.br

Abstract: Optimization is a valuable tool in structural engineering, enabling the achieve of optimal solutions that meet design constraints and objectives. The application of multi-objective optimization in steel trusses can bring significant benefits in terms of structural efficiency and economy. In this article, an automated approach was developed to obtain parametric and multi-objective optimal settings for flat truss structures. The methodology was developed based on the utilization of the finite element method for structural analysis and the integration of evolutionary computing techniques, employing Genetic Algorithms (GAs) via MATLAB. Furthermore, to address the uncertainties inherent in the optimization process, a reliability analysis was conducted using the First-Order Reliability Method (FORM). This analysis took into consideration the variability of key parameters such as area, density, diameters, thicknesses and displacement, treating them as random variables. The results demonstrated good accuracy when compared to benchmark trusses from the literature, highlighting the potential of applying multi-objective optimization in steel trusses to improve efficiency and quality of these structures.

Keywords: Multi-objective Optimization, Plane Trusses, Genetic Algorithm, Finite Element Method, FORM.

1 Introduction

Optimization is a fundamental tool in structural engineering to find solutions that better meet the design's requirements and purpose. This involves defining an objective function to minimize or maximize, identifying the design variables that can be adjusted to achieve these objectives, Rao[1].

Within the scope of structural engineering, objective functions play an essential role as they are directly linked to the optimization of various aspects of the design. These functions comprehend volume reduction, minimizing material costs, decreasing maximum displacements, and most importantly, weight reduction. To achieve these goals, a variety of design variables must be taken into account, such as structural proportions, appropriate material selection, and geometry of structural elements. Each of these choices directly impacts the overall performance of the structure, making them vital for the design's success. During the manufacturing process of a product, performance requirements need to be evaluated. Therefore, in the optimization process, uncertainties in the structural system, such as external loads, material properties, manufacturing quality, and geometry, must be considered to mitigate performance degradation of the structure and/or have safer designs in the presence of these uncertain parameters, with better control over their lifespan. Therefore, deterministic optimal solutions may lead to structures with reduced levels of reliability, as stated by Beck and Gomes [2].

In this way, the application of models that consider the unpredictability of parameters in their analysis is an advantageous research avenue, aiming to construct safer and more reliable designs. In this context, Deterministic Optimization (DO) and Reliability-Based Design Optimization (RBDO) can be combined, with the main difference between them lying in the probabilistic constraints that must be simultaneously considered, as they both have the same objective. According to Kim et al. [3] and Yoon and Choi [4], probabilistic constraints in RBDO models are formulated with the purpose of constructing an approximate linear function to closely represent the nonlinear limit state function for the calculation of reliability and optimization index through appropriate transformations. Therefore, RBDO has emerged as an alternative to properly model the safety aspect under uncertainties in the optimization problem. With RBDO, it is possible to ensure that a minimum level of safety, predetermined by the designer, is maintained by the optimal structure. The objective of RBDO is to take into account the randomness of applied loads, material properties, and/or geometric parameters, among other variables that may be considered. The resulting optimal configurations depend on the desired levels of reliability. Kharmanda et al. [5], Silva et al. [6], and Nguyen et al. [7] propose reliability-based topology optimization algorithms for a single cycle.

The purpose of this article is to analyze issues related to steel trusses using a multi-objective optimization approach to obtain an optimal solution that balances the structure's weight with its displacement, resulting in a low-cost and safe design for the project under consideration. To achieve this goal, the Finite Element Method (FEM) was employed for structural analysis. Thus, the evolutionary computation technique, Genetic Algorithms (GAs), in conjunction with MATLAB software, was used to perform the structural optimization. This approach allowed the optimization of conflicting functions, weight and displacement of the structure, seeking a balance between them.

Additionally, the Reliability Index Approach (RIA) is frequently used to estimate probabilistic constraints with acceptable computations. The probabilistic constraints of RIA are formulated in terms of the reliability index. The genetic algorithm has been applied in truss optimization problems, as described by Lee [8], Lage [9], and Mattos [10]. However, so far, none of these cases address multi-objective optimization. Therefore, implementing a multi-objective optimization approach can be an innovative proposal, bringing several benefits, such as achieving a better trade-off between structural weight reduction and mechanical strength enhancement of the truss. This would allow exploring these conflicting objectives, resulting in solutions that offer an optimal balance between these two performance criteria.

2 Multi-objective Optimization

In contrast to single-objective optimization, a solution for a multi-objective problem is more of a concept than a definition. Typically, there is no single global solution, and often it is necessary to determine a set of points that fit within a predetermined definition for an optimum. In general, a multi-objective problem (MOP) can be expressed as follows:

$$\begin{aligned} &\text{Minimize: } \mathrm{f}(x) = [f_1(x), f_2(x), f_3(x), \dots, f_k(x)]^T \\ &\text{Subject to: } \mathrm{h}_i(x) = 0, \quad \mathrm{i} = 1,2,\dots,\mathrm{n} \\ &\qquad \mathrm{g}_j(x) \le 0, \; \mathrm{j} = 1,2,\dots,\mathrm{m} \\ &\qquad \mathrm{x}_{\mathrm{lj}} < x_t < x_{uj}, \; \mathrm{t} = 1,2,\dots,\mathrm{p} \end{aligned} \tag{1}$$

where k is the number of objective functions , *m* is the number of inequality constraints, *n* is the number of equality constraints, *p* is the number of design variables , $x \in R^N$ is the vector of design variables, and $f(x) \in R^k$ is the vector of objective functions where $f_i(x): R^N \to R$. The feasible design space is defined as $X = \{x \,/\, g_j(x) \le 0, j = 1,2,\dots,m\}$. The feasible criterion is defined as the space $f(x) = \{f(x)/x \in X\}$.

In MO problems, it is known that finding an x* that minimizes several objective functions simultaneously is an extremely difficult task. A way to determine a solution that partially satisfies these equations present in the MO is contained in the definition of Pareto Optimality, Pareto [11] , which is defined as follows:

Definition 2 - Pareto optimality *– A point, $x^* \in X$ is Pareto optimal if there is no other point $x \in X$, such that $F(x) \le F(x^*)$ and $F_i(x) \le F_i(x^*)$ for at least one function. Where i is the number of functions to be optimized.*

All Pareto optimal points lie on the boundary of the feasible criterion space. Often, algorithms provide solutions that may not be Pareto optimal but can satisfy other criteria, making them meaningful for many practical applications. For example, the weak Pareto optimality is defined as follows:

Definition 3 – Weak Pareto Optimality – *A point $x^* \in X$ is a weak Pareto optimal if there is no other point $x \in X$, tal que $F(x) \le F(x^*)$.*

There are several strategies for solving multi-objective optimization (MO) problems. Some of these strategies include the Hierarchical Optimization Method, the Negotiation Method, the Goal Programming Method, the Global Criterion Method, and the Weighted Objectives Method. In this article, the Weighted Objectives Method will used for the objective optimization procedure, which involves assigning weights to different objectives and transforming the multi-objective problem into a single-objective optimization problem, where the objective function is a weighted combination of the original objectives.

3 Reliability Analysis: First-Order Reliability Method (FORM)

The concept of structural safety is related to the ability of a particular structure to withstand the various actions imposed on it during its service life while satisfying the functional conditions for its construction, Haldar [12]. Thus, the safety of a structure can be defined based on the probability of the occurrence of one of the failure states throughout its life. For a given design rule, the basic random variables are defined by their joint probability distribution associated with some expected parameters; the vector of random variables is denoted here as **X**, with realizations written as x. To assess the probability of failure with respect to a chosen failure scenario, a limit state function G(**X**) is defined by the condition for the proper functioning of the structure. In Fig. 1, the boundary between the failure state G(**X**) < 0 and the safety state G(**X**) > 0 is known as the limit state surface G(**X**) = 0. The probability of failure is then calculated by:

$$P_f = P_r[G_i(\boldsymbol{X}) \leq 0] = \int_{G_i(\boldsymbol{X})\leq 0} \dots \int_{G_i(\boldsymbol{X})\leq 0} f_X(x)dx \tag{2}$$

Where $f_{\boldsymbol{X}}(x)$ is the joint Probability Density Function (PDF) of the random vector **X**. The integral presented by equation 2 does not have an analytical solution for most practical cases. Additionally, in typical engineering problems, the majority of random variables are correlated, and their distributions are complex, making exact multidimensional integration of the probability of failure impossible. The solution to the problem can be obtained by adopting simulation methods or approximate methods, where the probability of failure is obtained through reliability indexes.

Lee and Chen [13] reviewed several analytical methods for approximately solving this integral. Among these approaches, FORM is particularly favorable due to its simplicity and efficiency. The idea of FORM is to transform the integral in the original random space into a measurable reliability index, which is interpreted as the minimum distance from the origin to the limit state function in the normalized space (**U**-space). The limit state function, G(**X**), is described as a function of the variables **U**, i.e., G(**U**). Then, we search for the point **U***, called the design point, whose distance to the origin is minimum (Most Probable Point - MPP), as shown in fig.1. The distance between **U*** and the origin represents the system's reliability index, β, and the probability of failure can be obtained through the following equation:

$$P_f = \Phi(-\beta)\ and\ \beta^t = -\Phi^{-1}(P_f^t) \tag{3}$$

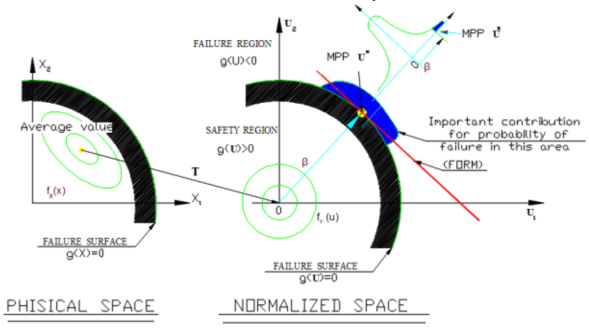


Figure 1 - First Order Reliability Method (FORM) - Adapted from Choi et al. [14]

where CDF is the standard cumulative distribution function, β_i is the target reliability index for the i-th constraint, and β_i^* is the reliability index evaluated by the distance from the origin to the MPP in the normalized space. The vector X is transformed into an independent and normalized vector **U** (zero mean, unit variance), expressed as $U = \frac{(X-\mu)}{\sigma}$, where μ and σ are the vectors of mean values and standard deviations associated with X, respectively. In the case of a normal distribution, a normalized vector **U** is given by:

$$U = \frac{(X - \mu_x)}{\sigma_x} \tag{4}$$

where μ and σ are the vectors of mean values and standard deviations associated with **X**, respectively. Based on the transformation above, the constraint function is defined as follows:

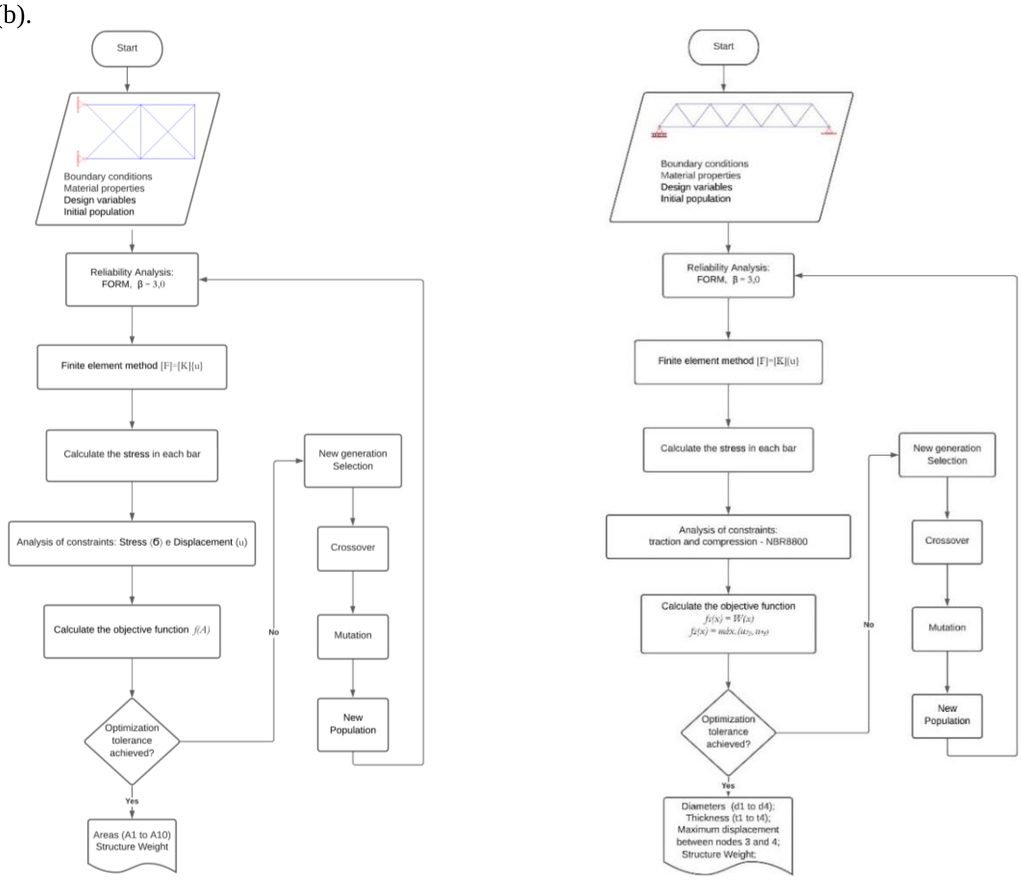
$$G_i(\boldsymbol{X}) = G_i(T^{-1}(\boldsymbol{U})) = G_i(\boldsymbol{U}) \tag{5}$$

where $G_i(\boldsymbol{U})$ is the i-th constraint in normalized space.

In this paper, the FORM (First-Order Reliability Method) was adopted, which is a method that employs Taylor series expansion, approximating the failure function iteratively by linearizing it at each point until the results converge to the design point. The design point is located at the minimum distance from the origin to the failure function. This method is based on the work of Hasofer and Lind [15].

4 Numerical Examples

The optimization for the 10-bar truss was developed as shown in fig.2 (a). After validating the results of the implemented program with the literature, it generated a 19-bar truss by increasing the number of bars, which in turn increased the number of functions to be optimized, thus creating a multi-objective optimization, as depicted in fig.2 (b).



(a) **(b)**

Figure 2 - Flowchart: (a) Plane Truss 10 bars; (b) Plane Truss 19 bars.

4.1 Plane Truss with 10 bars

The steel structures were dimensioned using the Finite Element Method (FEM), and the formulation of the method based on displacement is founded on the principle of virtual displacements, Bathe [16]. The optimization was implemented using GAs. Initially, a 10-bar cantilever truss was analyzed to validate the results, which is a well-known benchmark in the literature. The parametric optimization was validated by comparing the results with those of Lee [8]. The material properties of the bars are as follows: density = 2767.99kg/m³; elastic modulus = 68947.57MPa; and lengths L1 = 9.144m and L2 = 9.144m. The applied loads are P = 444.78x10³N at nodes 2 and 4, as shown in fig. 3.

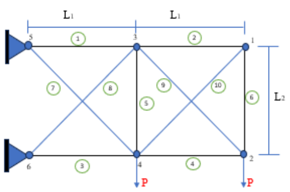


Figure 3 – Plane Truss 10 bars.

The design variables to be optimized in order to minimize the weight of the structure are the cross-sectional areas of the bars (A1 to A10). The objective function is given by equation (6), subject to stress and displacement constraints, as well as lateral constraints with a minimum limit of $6.45\text{x}10^{-5}$m².

$$\text{Minimize: } f(A) = W(A) = \gamma \sum_{i=1}^{n} L_i A_i$$
$$\text{Subject to: } g_1(A) = \sigma(A) - \sigma_{máx} \leq 0\,,\ \sigma_{máx} \pm 172.36MPa \qquad (6)$$
$$g_2(A) = \mu(A) - \mu_{min} \leq 0\,, \mu_{min} \pm 0.0508m$$

4.2 Plane Truss with 19 bars

The material properties of the bars in the 19-bar truss are as follows: density = 7850 kg/m³; elastic modulus = 210000 MPa; yield strength $f_y = 355$MPa; ultimate tensile strength 510 MPa. The bars have a length (L) of 6.0m and a height (h) of 3.984m. The applied loads are $P = 10^6$N at nodes 7, 8, 9, 10, and 11, as shown in Fig. 4. The design variables are the external diameter and thickness, divided into 4 groups according to the bar identification in Fig. 4.

The design variables to be optimized for minimizing the weight of the structure and the maximum vertical displacement between nodes 3 and 4 are the external diameters (d1, d2, d3, d4) and the thicknesses (t1, t2, t3, t4). These variables must satisfy lateral constraints with diameters ranging from 88.9x10^{-3}m to 508.0x10^{-3}m and thicknesses from 3.2x10^{-3}m to 25.0x10^{-3}m. The objective function, equation 7, along with the constraints from NBR8800:2008 [17], are applied:

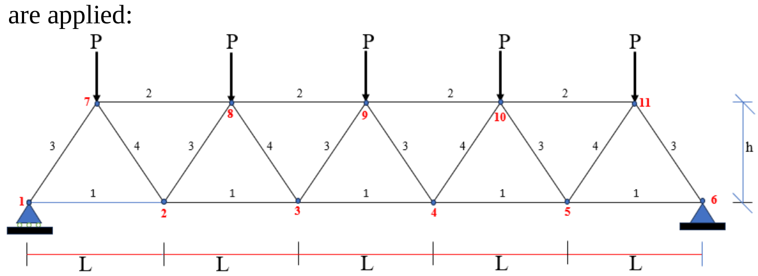


Figure 4 - Geometry, loads, nodes and element

$$\text{Minimize: } f_1(x) = W(x) = \gamma \sum_{i=1}^{n} L_i A_i\,, x = d_1, d_2, d_3, d_4, t_1, t_2\,, t_3, t_4$$
$$f_2(x) = máx\,(u_{3y}, u_{4y}), x = u_{3y}, u_{4y}$$
$$f(x) = w_1 f_1(x) + w_2 f_2(x) \qquad with \quad w_1 + w_2 = 1 \qquad (7)$$
$$\text{Subject to: } g_1(x) = N_{c,Sd} - N_{c,Rd} \leq 0$$
$$g_2(x) = N_{t,Sd} - N_{t,Rd} \leq 0$$

For compressive strength, according to NBR8800, equations (8) and (9) are as follows: $N_c = \frac{\pi^2 EI}{kL^2}$, $\lambda_0 = \sqrt{\frac{QA_g f_y}{N_c}}$ and $N_{c,Rd} = \frac{\chi A_g f_y}{\gamma_{al}}$.

Thus, $\lambda_0 \leq 1.5$, there is:

$$\chi = 0.658^{\lambda_0{}^2} \qquad (8)$$

Thus, $\lambda_0 > 1.5$, there is:

$$\chi = \frac{0.877}{\lambda_0{}^2} \qquad (9)$$

where $N_{c,Rd}$ is the design compressive resistance, $N_{c,Sd}$ is the design compressive force, N_e is the elastic axial buckling force; λ_0 is the reduced slenderness index; Q is the total reduction factor associated with local buckling; A_g is the gross cross-sectional area of the bar; χ is the reduction factor associated with compressive strength, f_y is the yield strength of the steel.

For tensile strength, according to NBR8800:2008, equation (10) is as follows:

$$N_{t,Rd} = \frac{A_g f_y}{\gamma_{al}} \qquad (10)$$

where: $N_{t,Rd}$ is the axial tensile strength of calculation, $N_{c,Sd}$, is the axial tensile force requesting calculation.

In the reliability analysis, weighted variables were considered for the two trusses: the 10-bar truss, which involves the area and density variables, and the other with 19 bars, considering the diameters, thicknesses and displacement. All of these variables follow a normal probability distribution, employing a reliability index of β=3.0, corresponding to a probability of failure of 0.0013.

5 Results and Discussion:

The input parameters for the Genetic Algorithms (GAs) in MATLAB for the 10-bar truss were a population of 800 individuals over 20 generations, with the design variable set to an initial search value of an area equal to $6.45\text{x}10^{-5}\text{m}^2$. From the obtained results, a high similarity was observed between the found areas and the total weight of the truss when compared to the author Lee [8], demonstrating the satisfactory implementation of the optimization. Furthermore, the optimization resulted in lighter truss structures, as shown in tab.1.

Table 1 - Area values and optimized structural weight - Plane Truss 10 bars.

Areas ($\text{x}10^{-5}$ m²)	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	Weight (kg)
Lee, 2004	1945.1	6.581	1465.2	985.2	6.581	35.10	486.5	1390.9	1383.8	6.452	2294.2
Author	2000.0	6.452	1624.5	929.7	6.452	6.452	535.5	1347,1	1310.3	6.452	2290.6

Figure 5 shows the convergence of the objective function with 6 generations in Genetic Algorithms, reaching the global optimum.

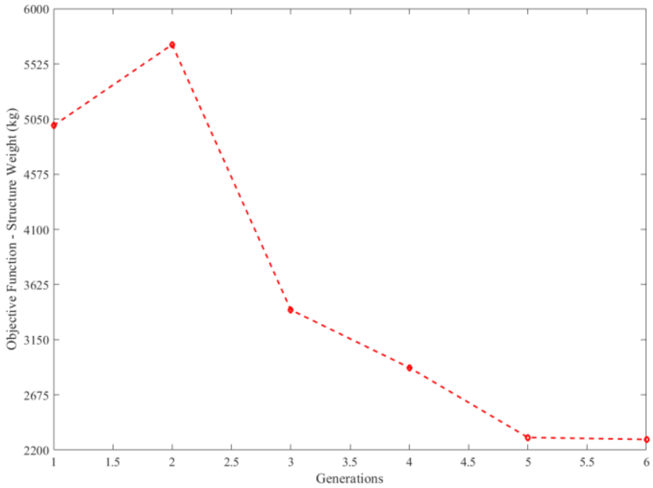


Figure 5 - Objective Function by Generations - 10-bar Truss

In the 19-bar truss, the Pareto frontier graph (fig. 6) reveals the weight-displacement trade-off. The lowest weight (10517.4 kg) corresponds to the highest displacement ($28.24\ \text{x}10^{-3}$m), and the lowest displacement ($18.39\ \text{x}10^{-3}$m) is associated with a weight of 12027.6 kg.

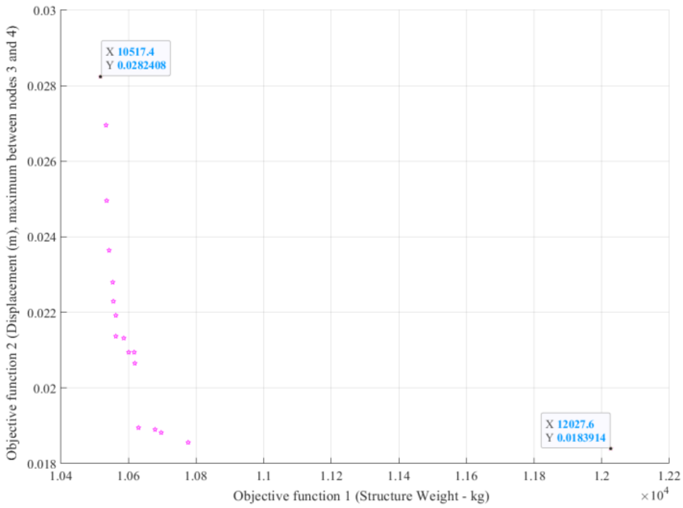


Figure 6 - Pareto Frontier - 19-bar Truss

6 Conclusion

Based on numerical simulations and reliability analysis, it can be concluded that the optimization of the 10-bar truss using genetic algorithms is fast and effective, achieving the specified tolerance in just 6 generations. The obtained weight is satisfactory and accurate, being lighter than Lee's proposal [8]. The bar areas also align well with the literature, and the reliability analysis validates the structural safety. In the 10-bar truss, the bars with connections at the ends have the largest areas, ensuring resistance, while others are more relevant for stability. Multi-objective optimization in the 19-bar truss shows that reducing weight has a low impact on displacement, staying within specified safety limits. Multi-objective optimization with reliability is an engineering technique, enabling more efficient designs. The graphical representation of the Pareto frontier facilitates decision-making to select the best design option for the flat structural truss to be developed.

Acknowledgments: This study was financed in part by the Coordination for the Improvement of Higher Education Personnel - Brazil (CAPES) - Finance Code 001, CNPq (National Council of Scientific and Technological Development) under Grant Number 315632/2020-8, FAPEMIG under Grant number TEC-PPM-00692-18 and Federal University of Ouro Preto (UFOP).

Statement of Authorship: The authors confirm that they are solely responsible for the authorship of this work and that all the material included herein as part of this work is owned (and authored) by the authors, or they have obtained permission from the owners for its inclusion here.

References

[1] S. S. RAO, Engineering optimization: theory and practice, John Wiley & Sons, 2019.

[2] A. Beck and W. Gomes, "A Comparison of Deterministic, Reliability-Based and Risk-Based Design Optimization," Probabilistic Engineering Mechanics, 2012.

[3] S. R. Kim, J. Y. Park, W. G. Lee, J. S. Yu and S. Y. Han, "Reliability-based topology optimization based on evolutionary structural optimization.," Int J Mech Syst Sci Eng, pp. 1(3), 135-139, 2007.

[4] B. Youn, K. Choi and Y. Park, "Hybrid analysis method for reliability-based design optimization," Journal of Mechanical Design, pp. 125, pp. 221-32, 2003.

[5] G. KHARMANDA, N. OLHOFF and A. EL-HAMI, "Optimum values of structural safety factors for a predefined reliability level with extension to multiple limit states," Structural and Multidisciplinary Optimization, pp. v. 27, p. 421-434, 2004.

[6] M. Silva, D. A. Tortorelli, J. A. Norato, C. Ha and H.-R. Bae, "Component and system reliability-based topology optimization using a single-loop method.”," Struct. Multidiscip. Optim., pp. 41(1), 87–106, 2010.

[7] T. H. Nguyen, J. Song and G. H. Paulino, "Single-loop system reliability-based topology optimization considering statistical dependence between limit-states," Struct. Multidiscip. Optim., pp. 44(5), 593–611, 2011.

[8] K. S. Lee and Z. W. Geem, "A new structural optimization method based on the harmony search algorithm," Computers and Structures, p. 18, 2004.

[9] C. M. Lage, F. d. A. Neves and M. Freitas, "Integration methodology of different softwares for constrained tubular truss size optimization problems," Revista Internacional de Métodos Numéricos para Cálculo y Diseño en Ingeniería, p. 10, 2019.

[10] L. E. G. Mattos and R. K. Tessari, "Comparison of different optimization methods applied to steel trusses structures," CILAMCE, p. 7, 2021.

[11] V. PARETO, "L'ofelimità nei cicli non chiusi," Giornale degli economisti, pp. v. 33, p. 15-30, 1906.

[12] A. HALDAR and S. MAHADEVAN, Probability, reliability, and statistical methods in engineering design, John Wiley & Sons Incorporated, 2000.

[13] S. a. C. Lee, " A comparative study of uncertainty propagation methods for black-box-type problems.," Structural and Multidisciplinary Optimization, pp. 37(3): 239-253., 2009.

[14] K. Choi, X. Yu and K. Chang, "A Mixed Design Approach for Probabilistic Structural Durability," Sixth AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, Bellevue, WA, September 4-6, 1996.

[15] A. M. HASOFER and N. C. LIND, "Exact and invariant second-moment code format," Journal of the Engineering Mechanics division, pp. v. 100, n. 1, p. 111-121, 1974.

[16] K. J. BATHE, "Finite Element Procedures," Watertown, MA, pp. 2. ed. , p.1043, 2014.

[17] ASSOCIAÇÃO BRASILEIRA DE NORMAS TÉCNICAS, Projeto de estruturas de aço e de estruturas mistas de aço e concreto de edifícios, Rio de Janeiro: ABNT NR8800, 2008.