Model Mixing with Frequency Based Substructuring: 4 DoF Half-Vehicle Analysis

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Abstract. This paper reviews the system equivalent model mixing (SEMM) technique. This method is applied to obtain hybrid dynamic models, primarily by using a numerical model in order to expand the degree of freedom space of an experimental model. The formulation of this method is based on coupling and decoupling of substructures, by means of the Lagrange multiplier frequency-based substructuring (LM-FBS) technique. Therefore, the SEMM technique will be applied in a half-vehicle model, where the dynamics of a parent model will be updated with the dynamics of an overlay model of an equivalent system, but with lower density of degrees of freedom (DoFs). This procedure aims to create a hybrid model, which expands the dynamics of the overlay model to the denser DoFs of the parent model. The influence of the interface size, damping and signal noise on the final hybrid model will be evaluated in this paper.

Keywords: dynamic substructuring, model mixing, hybrid models, half-vehicle model

1 Introduction

Hybrid modeling methods allow the combination of models from equivalent systems of different natures, in order to eliminate the limitations and aggregate the benefits of each of these models. In this way, for example, it is possible to extend the real measured dynamics of an experimental model, which generally has a limited resolution of degrees of freedom (DoFs), to a numerical model of the same structure, with an inherently denser resolution of DoFs, but limited by the ability of accurately model the structures. The hybrid model generated is a updated numerical model, that faithfully represents the dynamics of the experimental model of the same structure.

Numerical models are limited by the ability to realistically model the geometry, material properties, boundary conditions, and contact properties, as these parameters are challenging to simulate real conditions. On the other hand, experimental models provide a true view of the behavior of a model, but are usually obtained with less dense spatial resolution, due to the limitations of available measurement resources and physical accessibility. For this reason, to correctly represent the dynamics of the real structure, numerical models are often updated from experimental models, thus forming a hybrid model.

The present work aims to carry out the study of the application of frequency-based substructuring (FBS) in system equivalent model mixing (SEMM) in the elaboration of hybrid systems. In this sense, the adopted methodology seeks to analyse a half-vehicle model, applying SEMM between two equivalent numerical models, in order to serve as an initial verification of the technique, and determining limitations and gain expertise for future applications of this method to be performed in future works. The specific objectives of this initial analysis is to determine the influence that the interface size, damping and signal noise have on the hybrid model generated by SEMM. The nature of the spurious peaks obtained, as well as the influence of damping on their formation is also of interest.

2 System Equivalent Model Mixing

System equivalent model mixing (SEMM) is a method introduced by Klaassen *et al.* [1], which, by coupling and decoupling substructures with the Langrange multiplier frequency-based susbtructuring (LM-FBS) method [2], aims to obtain a hybrid dynamic model by mixing equivalent models - usually numerical and experimental - of the same structure. The goal of SEMM, therefore, is to expand the dynamics contained in a model with fewer DoFs, but higher fidelity, to a model with a denser space of DoFs. This method can be implemented so that the dynamics of an experimental model can be expanded to a numerical model, in order to obtain data from previously unmeasurable DoFs of that experimental model, since experimental data is limited by the sensors and accessibility of all the parts of the structure in analysis.

The hybrid model is composed of three models: a parent model, a removed model, and an overlay model. The parent model incorporates all the DoFs of the final hybrid model and is typically numerical. The removed model is composed of a subset of the DoFs from the parent model, which will be removed from the dynamics of the parent model in order to be replaced by the dynamics of the overlay model. The overlay model provides the dynamical properties to be expanded, typically from a experimental model [3].

$$\begin{cases} \boldsymbol{u}(\omega) = \mathbf{Y}(\omega)\ [\boldsymbol{f}(\omega) + \boldsymbol{g}(\omega)]\,, \\ \mathbf{B}\,\boldsymbol{u}(\omega) = \mathbf{0}, \\ \mathbf{L}^{\mathrm{T}}\boldsymbol{g}(\omega) = \mathbf{0}. \end{cases} \tag{1}$$

The LM-FBS formulation, used to solve the equations from Eq. (1), consists of introducing unknown interface forces, by means of the Lagrange multipliers $\boldsymbol{\lambda}$ ($\boldsymbol{g} = -\mathbf{B}^{\mathrm{T}}\boldsymbol{\lambda}$), in such a way that the compatibility condition at the interface will be juxtaposed to the equation of dynamic balance of the subsystem [4]. This way, the equilibrium condition at interface is met, remaining only to guarantee the compatibility of displacements condition at the interface.

In order to perform the SEMM method with the LM-FBS formulation, the problem consists in performing a coupling with the overlay model and a decoupling of the removed model to the parent model. The block-diagonal equation of motion that determines these two steps is illustrated in Eq. (2):

$$\begin{bmatrix} \boldsymbol{u}^{\mathrm{par}} \\ \boldsymbol{u}^{\mathrm{rem}} \\ \boldsymbol{u}^{\mathrm{ov}} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{Y}^{\mathrm{par}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{Y}^{\mathrm{rem}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{Y}^{\mathrm{ov}} \end{bmatrix}}_{\mathbf{Y}} \begin{bmatrix} \boldsymbol{f}^{\mathrm{par}} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} - \begin{bmatrix} \boldsymbol{g}^{\mathrm{par}} \\ \boldsymbol{g}^{\mathrm{rem}} \\ \boldsymbol{g}^{\mathrm{ov}} \end{bmatrix}. \tag{2}$$

To ensure the compatibility of displacements between the interface DoFs b in order to guarantee that the interface conditions are satisfied, the Boolean mapping matrix $\mathbf{B}$ must be aquired. The following displacements, from the coupling and decoupling steps, are known to be equivalent:

$$\begin{aligned} \boldsymbol{u}_{\mathrm{b}}^{\mathrm{par}} &= \boldsymbol{u}^{\mathrm{rem}}, \\ \boldsymbol{u}^{\mathrm{rem}} &= \boldsymbol{u}^{\mathrm{ov}}. \end{aligned} \tag{3}$$

Equation (4) contains $\mathbf{B}$ for the two steps of the SEMM method. It can be seen that the first line $\mathbf{B}$ contains the compatibility for the decoupling between the reference and the removed model, while the second line ensures the compatibility for the coupling between the removed and overlay models.

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} & -\mathbf{I} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\mathbf{I} & \mathbf{I} \end{bmatrix}. \tag{4}$$

The coupled matrix $\overline{\mathbf{Y}}$ can be obtained from the decoupled admittance matrices $\mathbf{Y}$ and Boolean mapping matrices $\mathbf{B}$, with the classic LM-FBS equation:

$$\overline{\mathbf{Y}} = \mathbf{Y} - \mathbf{Y}\,\mathbf{B}^{\mathrm{T}} \left(\mathbf{B}\,\mathbf{Y}\,\mathbf{B}^{\mathrm{T}}\right)^{-1} \mathbf{B}\,\mathbf{Y}. \tag{5}$$

It can be observed from the dimensions of Eq. (5) that $\overline{\mathbf{Y}}$ has redundant DoFs, i.e., the interface GoLs appear three times in the system. To keep only the unique DoFs and obtain the redundancy-free coupled matrix $\mathbf{Y}^{\mathrm{SEMM}}$, Eq. (6) can be used, where $\mathbf{L}$ is the Boolean localisation matrix, that transforms the generalised DoFs to the physical DoFs, eliminating redundant columns and rows from $\overline{\mathbf{Y}}$:

$$\mathbf{Y}^{\text{SEMM}} = \mathbf{L}^{\dagger}\overline{\mathbf{Y}}\left(\mathbf{L}^{\text{T}}\right)^{\dagger}, \tag{6}$$

where $(\star)^{\dagger}$ denotes the pseudo-inverse.

3 Half-Vehicle System

The system of a half-car is a classic example in dynamic analysis, especially in the study of vehicular suspension, since the suspension is a fundamental component to ensure stability and comfort. The model of the half-vehicle, presented in Fig. 1, is formed by two wheels, m_1 and m_2 and a chassis m_3 with inertia J. The wheels are joined to the chassis by suspensions, which are modeled as a spring-damper set $k_1 - c_1$ and $k_2 - c_2$, and the contact of the wheels to the ground is made by the tires, which are also modeled as a spring-damper set $k_3 - c_3$ and $k_4 - c_4$. Thus, it can be seen that this system has four DoFs, referring to the vertical displacements of the wheels and the chassis, as well as the angular displacement of the chassis, as evidenced in Fig. 1.

The mass, damping and stiffness matrices, as well as the displacement vector for the system, are exposed in Eq. (7).

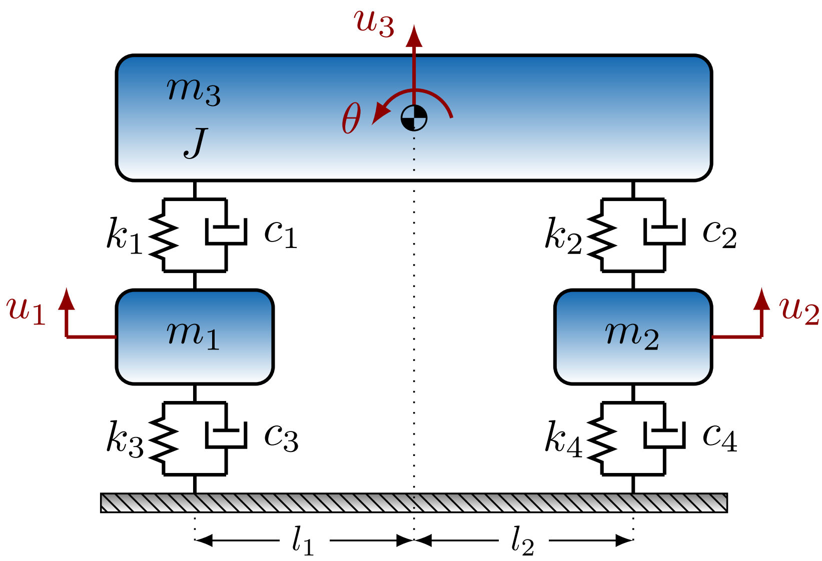

Figure 1. Schematic model of a half-vehicle with 4 DoFs

$$\boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \theta \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} c_1 + c_3 & 0 & -c_1 & -l_1 \cdot c_1 \\ 0 & c_2 + c_4 & -c_2 & l_2 \cdot c_2 \\ -c_1 & -c_2 & c_1 + c_2 & l_1 \cdot c_1 - l_2 \cdot c_2 \\ -l_1 \cdot c_1 & l_2 \cdot c_2 & l_1 \cdot c_1 - l_2 \cdot c_2 & l_1^2 . c_1 + l_2^2 . c_2 \end{bmatrix}, \tag{7a}$$

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & J \end{bmatrix} \text{ e } \mathbf{K} = \begin{bmatrix} k_1 + k_3 & 0 & -k_1 & -l_1 \cdot k_1 \\ 0 & k_2 + k_4 & -k_2 & l_2 \cdot k_2 \\ -k_1 & -k_2 & k_1 + k_2 & l_1 \cdot k_1 - l_2 \cdot k_2 \\ -l_1 \cdot k_1 & l_2 \cdot k_2 & l_1 \cdot k_1 - l_2 \cdot k_2 & l_1^2 \cdot k_1 + l_2^2 \cdot k_2 \end{bmatrix}. \tag{7b}$$

Using the presented half-vehicle model, for the purpose of exploring the scope of the SEMM method, it will be simulated the introduction of some “experimental” DoFs into a “numerical” model, which has all the DoFs of the system, in order to obtain a hybrid model that contains the dynamics of the numerical model updated with data introduced from the other model. However, all the data generated will be obtained numerically, and the values for mass, stiffness, and damping will be slightly different between the parent model and the overlay model, to simulate eventual differences between the “experimental” and “numerical” models.

The parent model is presented by a complete numerical model, while the overlay model, which is usually obtained experimentally, contains only the DoFs referring to the displacements u_1 and u_2, that corresponds to the responses of the of the wheels. In order to allow a comparison between the hybrid model with the real solution, a complete model with the parameters of the overlay model is also generated. The half-vehicle parameters, from Tab. 1, were used to generate the admittance matrices for the parent, removed, overlap, and real models by inverting the impedance matrix.

Table 1. Half-vehicle parameters

Parameter		Model	
		Parent	Real
Mass [kg]	m_1	100	90
	m_2	200	210
	m_3	300	320
Stiffness [kN/m]	k_1, k_2	8	8,3
	k_3, k_4	40	37
Inercia $\left[\text{kg.m}^2\right]$	J	250	270
Damping [N.s/m]	c_1	50	55
	c_2	60	65
	c_3, c_4	0	0
Length [m]	l_1, l_2	1	

Definition of the Models

In order to apply SEMM to the mid-vehicle system, the admittance matrices for the parent, removed, and overlapping models must be defined.

Parent Model - $\mathbf{Y}^{\text{par}}$

In order to obtain the parent model, from a initial model, in which the DoFs follow the order of the displacement vector of Eq. (7), the DoFs that will be introduced by the overlay model are grouped at the lower right of the matrix through elementary operations.

Removed Model - $\mathbf{Y}^{\text{rem}}$

The removed model corresponds to the DoFs of the parent model that will be decoupled. The admittance of the removed model is the block containing the interface DoFs of the parent model.

Overlay Model - $\mathbf{Y}^{\text{ov}}$

The overlay model contains the DoFs of a model equivalent to the parent model. SEMM is applied in order to expand the dynamics contained in this reduced model to the remaining DoFs, which have not been measured. The DoFs of the overlay model are the ones that define the DoFs that form the interface of the parent and removed models.

The Hybrid Model

The block-diagonal admittance matrix of the decoupled hybrid system can be assembled, as seen on Eq. (2), in which the processes of coupling the DoFs of the overlay model and decoupling the excessive DoFs of the parent model (removed model) are highlighted. Eq. (8) is introduced in order to help visualise the SEMM process, highlighting the interface DoFs and the transformation from the initial model to the parent model. In this sense, Eq. (8) represents, in fact, the decoupled block-diagonal admittance matrix of the hybrid model, with the corresponding displacement vector.

$$\boldsymbol{u} = \begin{bmatrix} u_3^{\text{par}} \\ \theta^{\text{par}} \\ u_1^{\text{par}} \\ u_2^{\text{par}} \\ u_1^{\text{rem}} \\ u_2^{\text{rem}} \\ u_1^{\text{ov}} \\ u_2^{\text{ov}} \end{bmatrix}, \quad \mathbf{Y} = \begin{bmatrix} Y_{33}^{\text{par}} & Y_{34}^{\text{par}} & Y_{31}^{\text{par}} & Y_{32}^{\text{par}} & 0 & 0 & 0 & 0 \\ Y_{43}^{\text{par}} & Y_{44}^{\text{par}} & Y_{41}^{\text{par}} & Y_{42}^{\text{par}} & 0 & 0 & 0 & 0 \\ Y_{13}^{\text{par}} & Y_{14}^{\text{par}} & Y_{11}^{\text{par}} & Y_{12}^{\text{par}} & 0 & 0 & 0 & 0 \\ Y_{23}^{\text{par}} & Y_{24}^{\text{par}} & Y_{21}^{\text{par}} & Y_{22}^{\text{par}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -Y_{11}^{\text{rem}} & -Y_{12}^{\text{rem}} & 0 & 0 \\ 0 & 0 & 0 & 0 & -Y_{21}^{\text{rem}} & -Y_{22}^{\text{rem}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Y_{11}^{\text{ov}} & Y_{12}^{\text{ov}} \\ 0 & 0 & 0 & 0 & 0 & 0 & Y_{21}^{\text{ov}} & Y_{22}^{\text{ov}} \end{bmatrix}. \tag{8}$$

To obtain the hybrid model through SEMM now comes down to solving the coupling and decoupling processes by the LM-FBS method. Through the SEMM formulation, the constraints are imposed for the decoupling step between the parent model and the removed model and for the coupling step between the removed model and the overlay model. The Boolean matrix $\mathbf{B}$ can be generated:

$$\underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}}_{\mathbf{B}} \begin{bmatrix} u_3^{\text{par}} \\ \theta^{\text{par}} \\ u_1^{\text{par}} \\ u_2^{\text{par}} \\ u_1^{\text{rem}} \\ u_2^{\text{rem}} \\ u_1^{\text{ov}} \\ u_2^{\text{ov}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{9}$$

Therefore, (9), the coupled matrix $\overline{\mathbf{Y}}$ can be obtained by Eq. (5). Next, in order to eliminate the four redundant DoFs arising from the dual coupling, the Boolean location matrix $\mathbf{L}$ is to be determined. Using Eq. (6), in sequence, to determine $\mathbf{Y}^{\text{SEMM}}$, the admittance matrix of the hybrid system without the redundant rows and columns. The relationship between the physical coordinates and their respective generalised coordinates can be seen by Eq. (10):

$$\begin{bmatrix} u_3^{\text{par}} \\ \theta^{\text{par}} \\ u_1^{\text{par}} \\ u_2^{\text{par}} \\ u_1^{\text{rem}} \\ u_2^{\text{rem}} \\ u_1^{\text{ov}} \\ u_2^{\text{ov}} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\mathbf{L}} \begin{bmatrix} q_3 \\ q_4 \\ q_1 \\ q_2 \end{bmatrix}. \tag{10}$$

4 Results

Once the $\mathbf{Y}^{\text{SEMM}}$ matrix is obtained, the FRFs of the hybrid model obtained by the SEMM method can be compared with the FRFs of the parent model and the real model. Figure 2 contains the Bode diagram of the admittance $\text{Y}_{3\theta}^{\text{SEMM}}$ for these three cases. Note that, initially, an undamped analysis is performed, so that when generating the FRFs for the three models, the dampings from Tab. 1 were all considered to be zero. The FRFs of $Y^{SEMM}_{3\theta}$ were chosen since that they represent the DoFs that do not belong to the overlay model and show how the dynamics from the DoFs of the overlay model influence the dynamics of the other Dofs of the parent model.

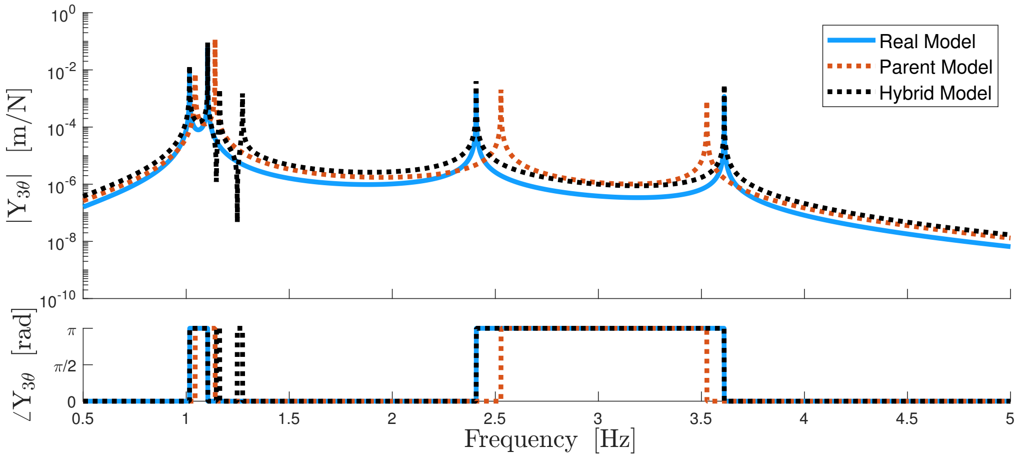

Figure 2. Basic SEMM

In the graph from Fig. 2, the FRF of the parent model corresponds to the response of the half-vehicle model with the data from the left column of Tab. 1, while the FRF of the real model corresponds to the response with the data from the right column of Tab. 1 and the FRF of the hybrid model corresponds to the response obtained by the SEMM method, when the FRFs corresponding to u_1 and u_2 of the real model were introduced into the parent model, i.e., it is the parent model updated with information from some measurements of the real model.

It can be seen that the SEMM method was indeed able to update the dynamics of the parent model to reflect the behaviour of the real model, and the resonance peaks were the same, even for the case of the admittance of the DoFs that do not belong in the overlay model, which shows that the overlay model is able to update the dynamics of the parent model as a whole. However, there was the occurrence of the spurious peaks. Therefore, applying the extended SEMM method, as demonstrated in [1] as a way of eliminating these peaks, Fig. 3 contains the Bode diagram for $Y^{SEMM}_{3\theta}$ with the extended interface between the parent and removed models.

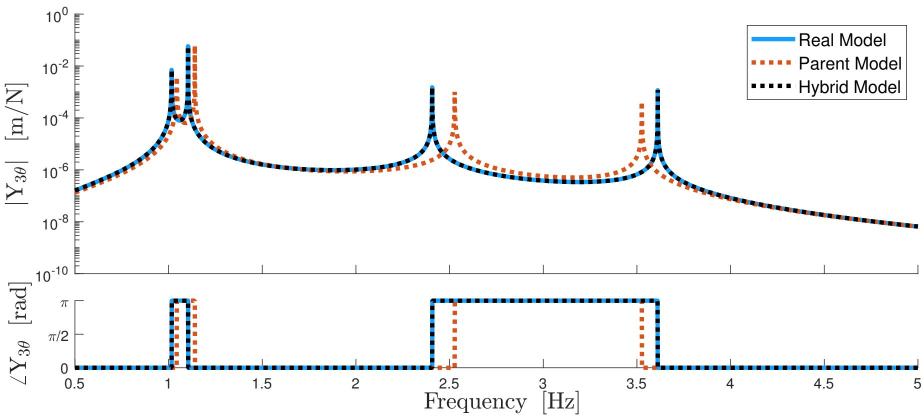

Figure 3. SEMM with extended interface

Analysing Fig. 3, the effect of the extended interface was able to remove the spurious peaks and the response of the hybrid model becomes equal to that of the real model. This extended formulation of the SEMM method, therefore, is the most commonly used in practice, since it generally provides better results than the basic formulation.

To analyse various real experiments, the numerically obtained FRFs are often intentionally contaminated with random noise to resemble the experimental data. To analyse the applicability of the SEMM method when the overlay model is actually obtained experimentally, we add a white noise with a signal-to-noise ratio of 80 dB, as recommended by Brincker and Rixen [5], in the FRFs of the overlay model before performing the coupling step.

Figure 4 contains the Bode diagram of the admittance $Y_{3\theta}^{SEMM}$ for the undamped system when the overlay model has noise and the extended SEMM was applied.

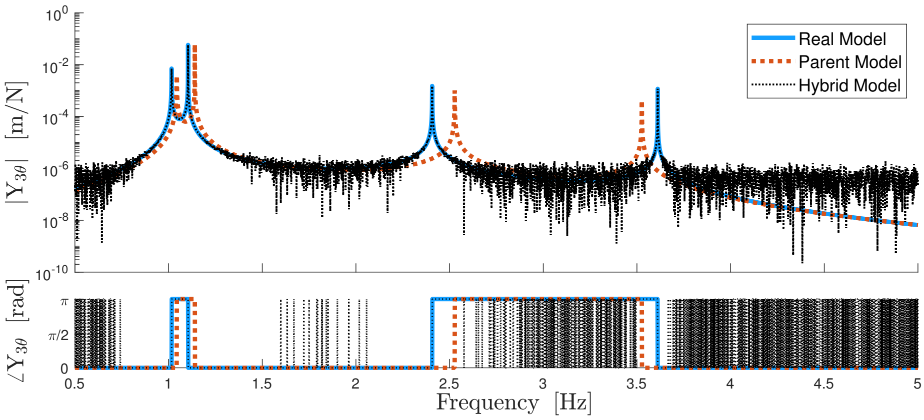

Figure 4. SEMM with noise on the overlay model

It can be seen from the FRFs from Fig. 4 that even when the signal of the overlay model has a fair amount of noise, the SEMM method is able to precisely generate the hybrid model that reflects the dynamics of the real model.

5 Conclusion

In this paper, the novel system equivalent model mixing technique was explored as a mean to produce hybrid models. With the half-vehicle model analysed, the technique was applied with different interface sizes, damping and noise conditions. The results demonstrated that a larger interface is recommended, as it eliminates the spurious peaks that can happen with the basic SEMM implementation. Furthermore, it was verified that damping and signal noise were not a factor that influenced the efficiency of the method, since the hybrid models maintained a high fidelity when compared to the real model they were trying to replicate. Finally, through this example, it was demonstrated how SEMM is capable of expanding the dynamics of o model with a sparse density of DoFs to a model with a higher resolution of DoFs. As a result of this analysis, the potential that this technique has as a tool in dynamics analyses is verified. In a next stage, this method is visioned to be applied for the updating of joint dynamics, in order to determine a methodology for the identification of joint dynamics.

Acknowledgements. The present study was financed by Brazilian National Conseil of Research and Development (Conselho Nacional de Pesquisa e Desenvolvimento - CNPq) with partnership with University of Brasilia by Edital 2021/2022 PROIC/UnB-CNPq. The authors acknowledge to CNPq and UnB by resources that make possible the present work.

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