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Advanced modeling of particle-laden gravity currents over flat and wavy surfaces using a residual-based variational multi-scale approach.

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Abstract. In this study, we introduce a numerical framework for simulating dilute particle-laden gravity currents at high Reynolds numbers (Re) using a residual-based variational multi-scale (RBVMS) approach. Our formulation effectively represents the coupling between velocity fine scales and the residual of the density concentration equation, ensuring a more accurate and robust representation of the underlying physical processes. We simulate lock-exchange particle-laden gravity currents across different Reynolds numbers to evaluate the proposed methodology. First, we validate the formulation by modeling gravity currents over flat terrain. Subsequently, we extend our analysis to particle-laden gravity currents over wavy terrains and varying wave heights to explore their influence on flow. Results indicate that floor roughness reduces the front velocity while significantly enhancing instabilities, leading to the earlier emergence of secondary instabilities. Additionally, roughness serves as an additional vorticity generation mechanism. Findings demonstrate that the (RBVMS) framework achieves high accuracy while requiring significantly low mesh resolution.

Keywords: Gravity currents, Residual-based variational multi-scale (RBVMS), Roughness floor

1 Introduction

Gravity currents - also known as density currents - are fluid flows driven by small differences in local density. These density variations create pressure gradients that initiate and sustain the flow. Such differences may arise from changes in salinity, temperature, or the presence of suspended particles, such as sediment. When particles contribute to the density contrast, the resulting flows are referred to as particle-laden or particle-driven currents. Particle-driven flows are a crucial subclass of buoyancy-driven fluid motions, characterized by horizontal propagation as a denser fluid displaces a lighter one. A classic example is the release of a heavier fluid behind a lock gate: once the gate is lifted, the denser fluid sinks and spreads horizontally along the bottom, forming a gravity current. Remarkably, even subtle density contrasts can generate substantial movement, underscoring the importance of these flows in natural and engineered systems. Density currents can dramatically alter flow dynamics, inducing sharp changes in velocity and generating regions of intense turbulence. These flows are prevalent across a wide spectrum of natural and human-influenced environments, often playing a pivotal role in the transport and dispersion of materials.

In this sense, sedimentation promoted by particle-laden flows can mold the seabed, producing different geological structures such as canyons, dunes, and ripples. They typically develop strong turbulence, which impacts directly the particles' ability to move relative to the carrying fluid, to settle, or to be re-entrained. Data recorded for turbidity currents in the ocean suggests Reynolds numbers of the order of 10^9 [1]. Depending on what prevails, settling or re-suspension, the current and its turbulent structures might evolve in an entirely different manner, and consequently, those flow changes affect the transport of particles. The spread of a gravity current depends on the boundary conditions, and two cases are usually distinguished based on whether the initial release is of the same width as the environment or not.

Recent research has begun to explore the behavior of density currents interacting with rough boundaries. While the majority of these investigations rely on numerical approaches, theoretical analyses and laboratory experiments remain relatively scarce. However, advancements in computational techniques and increased processing power have opened new possibilities. Today, it is feasible to conduct high-fidelity simulations—such as Large Eddy Simulations (LES) and Direct Numerical Simulations (DNS)—to study flow dynamics over complex geometries with greater accuracy and detail. Despite recent progress, accurately simulating turbulent density currents over rough surfaces remains a complex challenge. Several key factors contribute to this difficulty: Roughness-Induced Hydrodynamics: Surface roughness significantly affects flow behavior and cannot be oversimplified or neglected. Numerical models must consistently incorporate both roughness-induced drag and turbulence generation to capture realistic dynamics. Sensitivity to Roughness Parameters: The flow response is highly sensitive to the geometric characteristics of the roughness, such as the spacing and arrangement of roughness elements. Flow behavior differs markedly between idealized (regular and periodic) and irregular roughness configurations, complicating model generalization. Wake Effects: The wakes formed between roughness elements play a crucial role in shaping the overall flow structure. Accurately resolving these wake interactions is essential for reliable simulation outcomes. Currently, there is no universally accepted parameterization for near-wall flow over rough surfaces, which poses a major limitation for turbulence modeling—especially in Large Eddy Simulations (LES), where capturing small-scale dynamics near boundaries is critical.

2 Physical Modeling: phenomenological equations

In this section we present, under an Eulerian-Eulerian framework, the governing equations for turbidity currents developed by combining mass and momentum balances with rheological phenomenological models. Through these currents, a mixture of sediments, encompassing different particle sizes, can be carried and eventually deposited on the sea bottom. Sediments are modeled as a continuum and described by the volumetric concentration. As there are no sharp limits to the volumetric sediment concentration defining dilute and nondilute flows, we invoke here Boussinesq's hypothesis, typically used for the dilute case, but assuming rheological relations that accommodate higher sediment concentrations. Assuming Boussinesq's hypothesis, we intend to propose an extension of the model capable of describing nondilute currents. The determination of its domain of validity is outside the scope of the present work, and here we aim at enhancing the understanding of the underlying physics, with particular emphasis on the sediment deposition mechanisms.

2.1 Incompressible flow coupled with particles transport

The spatial domain in which the flow takes place along the time interval $[0, t_f]$ is denoted by $\Omega \subset \mathbb{R}^{n_{dim}}$, where n_{dim} is the number of spatial dimensions, and Γ the boundary of Ω. A velocity-pressure non-conservative form of the Navier-Stokes (NS) equations describes the incompressible turbulent fluid flows carrying suspensions of sediments. We assume that for the scenarios analyzed, particle inertia and particle-particle interactions can be considered negligible. Moreover, we also apply the Boussinesq hypothesis which accounts for the fluid - particle interaction using a forcing term proportional to the local difference in the fluid density due to the presence of sediments. Fluid motion drives the sediment particles, but they are also endowed with extra mobility modeled by their settling velocity u_S, related to grain size sediment in the gravity direction, $\mathbf{e}^g$, uncoupled advection-diffusion equations model the sediment transport. The motion of each grain size, embedded in the mixture, is mapped to the fields $c = C/C_0$ the scaled concentrations, expressing the volume fraction occupied by each particle size. C and C_0 are, respectively, the actual concentration and the initial reference concentration or normalization value, the latter typically taken as the total initial volume fraction of the particles. Diffusion of the sediment is supposed to be quite small. Motivation for its inclusion in the modeling is often by numerical reasons [2]. Accordingly, the dimensionless equations that govern the particle-laden flow are:

Fluid: Incompressible Navier Stokes

$$\frac{\partial \mathbf{u}}{\partial \mathbf{t}} + \mathbf{u}\,.\nabla\mathbf{u} = -\nabla p + \frac{2}{\nu_f\sqrt{Gr}}\nabla.(\nu_m(c)\nabla^s\mathbf{u}) + \; c\;\mathbf{e}^g \;\text{ in }\; \Omega\times[0,t_f] \tag{1}$$

$$\nabla\,.\,\mathbf{u} = \mathbf{0} \;\text{ in }\; \Omega\times[0,t_f] \tag{2}$$

Sediment Transport: Advection + Diffusion

$$\frac{\partial c}{\partial t} + (\mathbf{u} + u_S\mathbf{e}^g).\nabla c = \nabla.\left(\frac{1}{Sc\sqrt{Gr}}\nabla c\right) \qquad \text{in }\; \Omega\times[0,t_f] \tag{3}$$

where $\mathbf{u}$, p and t are, respectively, non-dimensional velocity, pressure and time. Above, p, many times referred in the literature as the dynamic pressure, results after removing the hydrostatic component of the pressure. The rheological function $\nu_m(c)$ of the volumetric concentration, is the effective dynamic viscosity and u_S the particle settling velocity acting in the direction of gravity $\mathbf{e}^g$. Gr is the Grashoff number that expresses the ratio between buoyancy and viscous effects given by:

$$Gr = \left(\frac{u_b H \rho_f}{\nu_f}\right)^2 \tag{4}$$

with ν_f and ρ_f are, respectively, dynamic viscosity and the fluid density, H is a characteristic length of the flow and the buoyancy velocity;

$$u_b = \sqrt{gHc_0(\tilde{\rho}_p - \tilde{\rho}_f)/\tilde{\rho}_f}$$

where g stands for the gravity acceleration and $\tilde{\rho}_p$ and $\tilde{\rho}_f$ for, respectively, particles and fluid densities. The Reynolds number is such that $Re = Gr^2$. We assume that the different grains have the same density. A third dimensionless number, resulting from turning the governing equations into a non-dimensional form, is the Schmidt number, Sc, giving the ratio between diffusion and viscous effects:

$$Sc = \frac{\nu_f}{\kappa \rho_f} \tag{5}$$

where κ is the diffusion coefficient, supposed to be very small.

Essential and natural boundary conditions for Equation (1) are $\mathbf{u} = \mathbf{g}$ on $\Gamma_{\mathbf{g}}$ and $\mathbf{n} \cdot \left(-p\ \mathbf{I_d} + \frac{\nu_m}{\nu_f \sqrt{Gr}} \nabla \mathbf{u}\right) = \mathbf{h}$ on $\Gamma_{\mathbf{h}}$, where $\Gamma_{\mathbf{g}}$ and $\Gamma_{\mathbf{h}}$ are complementary subsets of the domain boundary Γ. Functions $\mathbf{g}$ and $\mathbf{h}$ are given, and $\mathbf{n}$ is the unit outward normal vector of Γ. A divergence-free velocity field $\mathbf{u}_0(\mathbf{x})$ is the initial condition for the velocity and $c_i(\mathbf{x}, 0)$ describing grain size composition and concentration of the suspended sediment in the beginning of the current have to be prescribed for the transport equation. For equation (3), boundary conditions modeling the transport of particles into and out the flow domain are:

$$\begin{aligned} c &= c_n \text{ on } \Gamma_n^{c_i} \\ u_S \mathbf{e}^g\ c - \left(\frac{1}{Sc\sqrt{Gr}}\right) \nabla c) \cdot \mathbf{n} &= 0 \text{ on } \Gamma_h^c \\ \frac{\partial c}{\partial t} - u_S \nabla c \cdot \mathbf{n} &= 0 \text{ on } \Gamma_{bottom} \end{aligned}$$

with $\Gamma = \Gamma_n^c \cup \Gamma_h^c \cup \Gamma_{bottom}$ and $\Gamma_n^c \cap \Gamma_h^c \cap \Gamma_{bottom} = \emptyset$.

The former, a Dirichlet condition, describes the quantity of sediment entering in the flow domain. The second and third boundary conditions are enforced to reproduce physical mechanisms of particle motion through the remaining boundary, either by diffusion or advection. Sedimentation is allowed at the bottom on Γ_{bottom}. This last condition implies a loss of sediment but does not take into account any modification of the bottom geometry by deposition.

2.2 Phenomenological modeling: closure equations

The interdependence of the motion of the two components of the mixture, fluid, and sediments, is expressed in the above balance equations through a buoyancy term. The buoyancy term is proportional to several factors. The total sediment concentration at each spatial point acting on the fluid, the deposition mechanism embedded in the bottom boundary condition, the convective velocity in the transport equation, and, the particular focus here, a phenomenological relation to describe the effect of the sediment concentration on the viscosity. The modeling of this modified rheological behavior introduces a closure equation to the mathematical problem. Here we adopt, despite the variety of possibilities found in the literature [3], the model of [4],

$$\nu_m(c) = \nu_0 \Big(1 - \frac{c}{c_m}\Big)^{-2.5 c_m} \tag{6}$$

where c_m is the maximum volumetric concentration. This particular model has been used in [5] to help in the understanding of turbulence modulation in nondilute sediment transport. The nonlinear phenomenological model described above is designed to approximate the rheological behavior of the mixture with limited accuracy. It was chosen to strike a balance between computational efficiency and realism, and was therefore adopted in this study.

The solver for the sediment transport coupled problem employed here relies on a weak formulation based on the Residual Based Variational Multiscale Method (RBVMS) introduced within the context of Finite Element Stabilized Methods. RBVMS have been used with success in the simulation of turbulent flows [6] and [7], free-surface flows [8] and multi-transport [9], two fundamental aspects of the present problem.

3 Results

In this section, we showed the assessment of the RBVMS formulation as the capacity of the phenomenological viscosity law to enhance the predictions of the turbidity currents, to do this we break down the analysis into two examples. We analyse density currents over the flat and wavy bottom covered with 3D elements. The difference between the densities of the heavy and the light fluid is set to less than 3% to match with the Boussinesq hypothesis. The initial condition, there is a domain filled with ambient fluid and a sediment fluid to be injected by a window with normalized concentration equal to 1.

3.1 Channel setups with flat and wavy floor

The computational setup, sketched in Figure 1, shown the initial configuration, $t = 0$ where the mixture of sediments and is initially in the lock area and the concentration of sediments and deposition for at $t = 20$ after mixing with the clear ambient water in the channel. The channel dimensions' are $L_x = 12$, $L_y = 4$, $L_z = 2$, where the volume is fulfilled with ambient fluid (lighter) and the sediments (heavier fluid) injected through a inlet window with $h = 0.25$ height, .No-slip boundary condition was applied at the bottom and wall faces of the thank, in the top face, we specify an open flux boundary condition. The simulation last 20 non-dimensional time units and we consider for Reynolds number, $Re = 5,000$. The domains was discretized by $1.5M$ tetrahedra with $.7M$ nodes. Those values, combined with the settling velocities and initial concentrations defined previously, lead to high Grashoff numbers which meets laboratory standards, but it does not correspond to typically values of turbidity currents in nature. The mixture is injected with a concentration reference $C_0 = 0.01$.

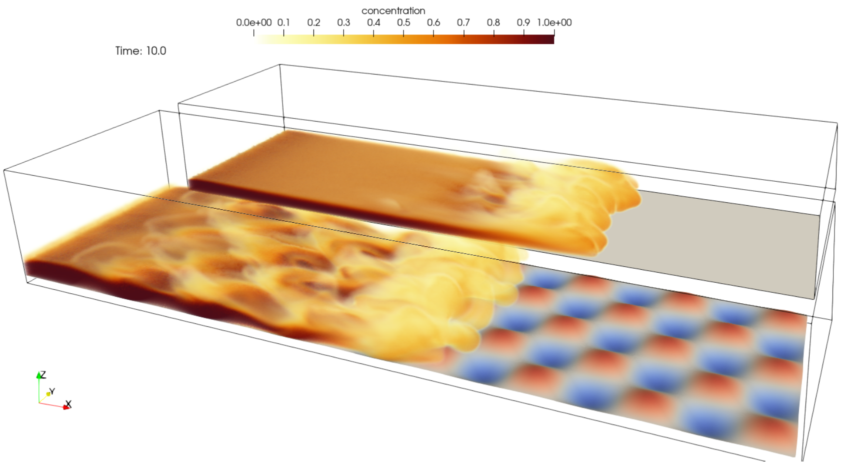

Figure 1. Channel setups with flat and wavy bottom

The Grashoff number reached allows us to see the formation of turbulent structures that interact with the sediments and are responsible for the deposition patterns showed in Figure 2.This figure illustrates the bottom complex pattern driven by the turbulent flow pattern resulting of total deposition. Figure 2 portrays a top view of the deposition, excess shear stresses and iso-contours of Q-criterium colored by vorticity for standard simulation with constant viscosity and considering the Krieger Dougherty viscosity law for the setup with flat bottom.

On the left the results we have the solutions with constant viscosity and in the right the ones where the viscosity is given by the Krieger and Dougherty nonlinear law. It is evident the effect of the nonlinear viscosity over the dynamics of the current, particularly near the bottom of the channel.

Although a single snapshot of flow and sediment transport reveals complex spatial patterns,

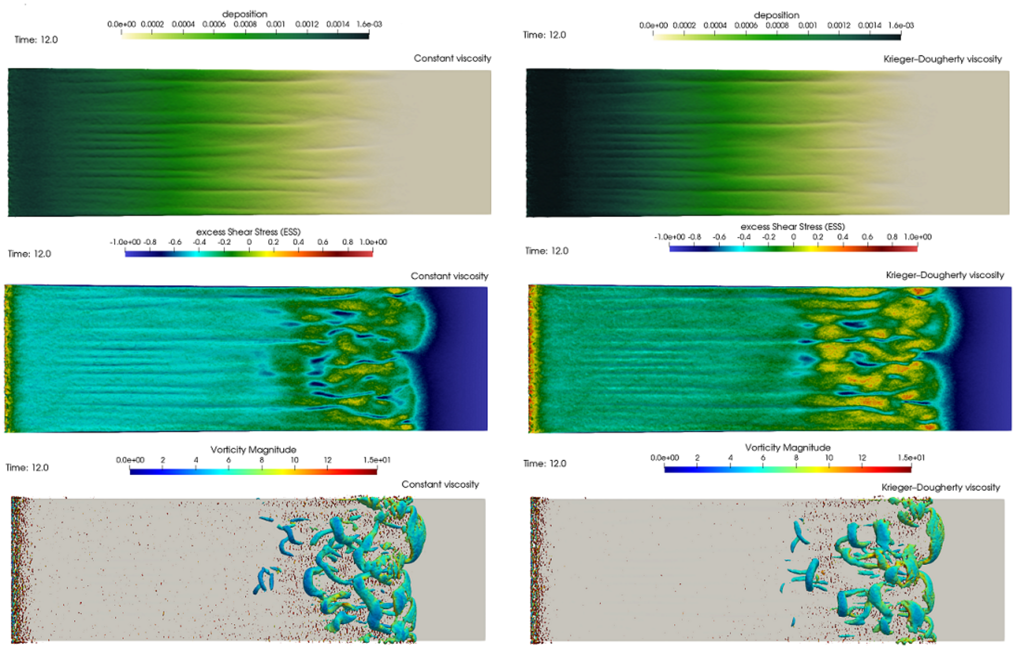

Figure 2. Flat bottom, from up to down, deposition of sediments, excess of shear stress and iso-contours of Q-criterium colored by vorticity for constant viscosity (left) and Krieger Dougherty viscocity law (right) at t= 12

the full simulation period highlights the highly three-dimensional nature of the flow, characterized by lobe and cleft instabilities—especially pronounced under constant viscosity conditions. Elevated shear stresses are observed near the inlet, where the sustained current dominates, while regions farther downstream show increased turbulence, which plays a key role in sediment re-suspension. The marked differences between the solutions underscore the critical influence of the phenomenological model on turbidity current dynamics.

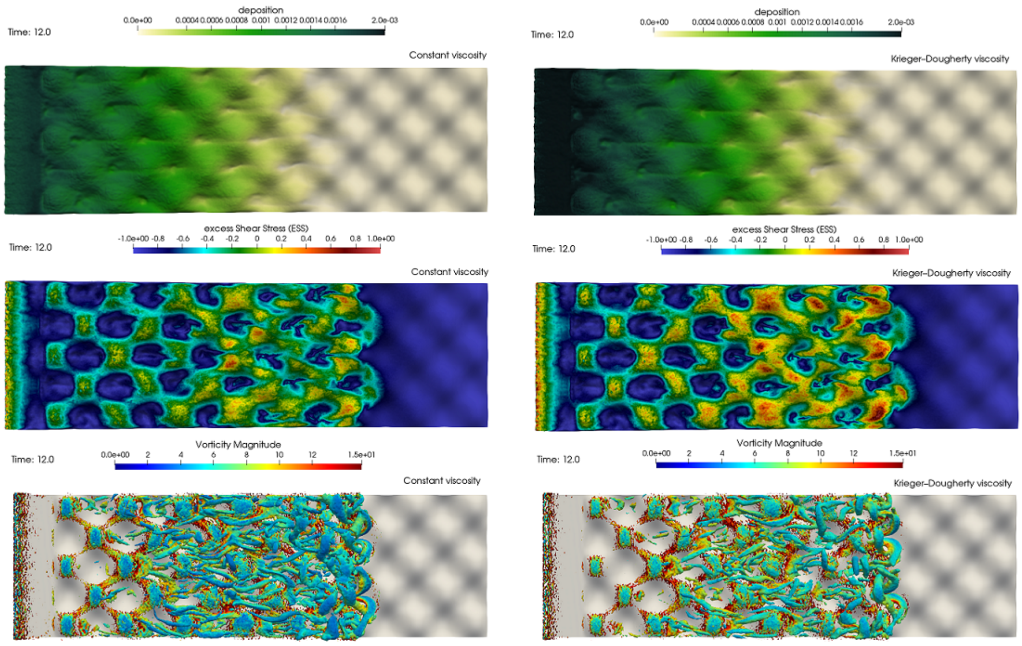

Figure 3. Wavy bottom, from up to down, deposition of sediments, excess of shear stress and iso-contours of Q-criterium colored by vorticity for constant viscosity (left) and Krieger Dougherty viscocity law (right) at t= 12

Figure 3 shows the particle-laden gravity currents over wavy bottom floor where effect of the height of the waves on the flow physics is analysed. Like a previous figure, same quantities re presented. The presence of waves at the bottom increases substantially to turbulence in the evolution of the current, this is evident trough the results of the Q-criterium.

3.2 Conclusions

The analyses are performed over a flat and wavy bottom and compared the results considering the effects of viscosity. It was found the flow behavior is noticeably changed, as the wave height bottom. The the current front speed affect the particle settling by the presence of bottom and by the viscosity too. The simulations over flat and wavy bottoms are performed and compared the results considering the Krieger Dougherty viscosity. It was found the flow behavior is noticeably changed, as the wave height increases. The wave height bottom decrease the current front speed and slow down the particle settling but in the other hand the viscosity augmented te erosion.

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